CARIBBEAN EXAMINATIONS COUNCIL

CSEC®
Mathematics

SYLLABUS
SPECIMEN PAPER
MARK SCHEME
SUBJECT REPORTS

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CSEC® Mathematics Free Resources

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CSEC Mathematics

The CSEC Mathematics syllabus addresses the personal development and educational needs of Caribbean students by encapsulating a variety of skills integral to everyday life and prerequisites for entering environments of work and academia. These skills include critical and creative thinking, problem solving, logical reasoning, modelling ability, team work, decision making, research techniques, information communication and technological competencies for life-long learning.

The syllabus also uniquely details a smooth progression of concepts that caters for students with primary or rudimentary knowledge of mathematics, and it can be easily subdivided to match the curricula of the different grades within the local high schools. Moreover, it is centrally positioned within the CXC sequence of examinations bridging the CPEA and CCSLC with the Additional and **CAPE®** Mathematics syllabuses. The competencies and certification acquired upon completion of this course of study is comparable with the mathematics curricula of high schools world-wide.

The syllabus is divided into nine (9) Sections:

SECTION 1 – NUMBER THEORY AND COMPUTATION

SECTION 2 – CONSUMER ARITHMETIC

SECTION 3 – SETS

SECTION 4 – MEASUREMENT

SECTION 5 – STATISTICS

SECTION 6 – ALGEBRA

SECTION 7 – RELATIONS, FUNCTIONS AND GRAPHS

SECTION 8 – GEOMETRY AND TRIGONOMETRY

SECTION 9 – VECTORS AND MATRICES

CARIBBEAN EXAMINATIONS COUNCIL

Caribbean Secondary Education Certificate®

SYLLABUS

MATHEMATICS

CXC 05/G/SYLL 16

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CSEC®

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Contents

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Please note that the syllabus has been revised and notable amendments are indicated by italics and vertical lines.

First Published in 1977
Revised in 1981
Revised in 1985
Revised in 1992
Revised in 2001
Revised in 2008
Revised in 2016

Mathematics Syllabus

♦ RATIONALE

The Caribbean society is an integral part of an ever-changing world. The impact of globalisation on most societies encourages this diverse Caribbean region to revisit the education and career opportunities of our current and future citizens. A common denominator of the Caribbean societies is to create among its citizens a plethora of quality leadership with the acumen required to make meaningful projections and innovations for further development. Further, learning appropriate problem-solving techniques, inherent to the study of mathematics, is vital for such leaders. Mathematics promotes intellectual development, is utilitarian and applicable to all disciplines. Additionally, its aesthetics and epistemological approaches provide solutions fit for any purpose. Therefore, Mathematics is the essential tool to empower people with the knowledge, competencies and attitudes which are precursors for this dynamic world.

*The syllabus addresses the personal development and educational needs of Caribbean students by encapsulating a variety of skills integral to everyday life and prerequisites for entering environments of work and academia. These skills include critical and creative thinking, problem solving, logical reasoning, modelling ability, team work, decision making, research techniques, information communication and technological competencies for life-long learning. The syllabus also uniquely details a smooth progression of concepts that caters for students with primary or rudimentary knowledge of mathematics, and it can be easily subdivided to match the curricula of the different grades within the local high schools. Moreover, it is centrally positioned within the **CXC®** sequence of examinations bridging the CPEA and CCSLC with the Additional and **CAPE®** Mathematics syllabuses. Additionally, the competencies and certification acquired upon completion of this course of study is comparable with the mathematics curricula of high schools world-wide. In consideration of educational support, the syllabus provides teachers with useful approaches and techniques, and it points to resources which are suitable for every learning style.*

This syllabus will contribute to the development of the Ideal Caribbean Person as articulated by the CARICOM Heads of Government in the following areas: "demonstrate multiple literacies, independent and critical thinking and innovative application of science and technology to problem solving. Such a person should also demonstrate a positive work attitude and value and display creative imagination and entrepreneurship". In keeping with the UNESCO Pillars of Learning, on completion of this course of study, students will learn to do, learn to be and learn to transform themselves and society.

♦ AIMS

This syllabus aims to:

1. make Mathematics relevant to the interests and experiences of students *by* helping them to recognise Mathematics in *the local and global* environment;

2. help students appreciate the use of mathematics as a form of communication;

3. help students acquire a range of mathematical techniques and skills and to foster and maintain the awareness of the importance of accuracy;

4. help students develop positive attitudes, such as open-mindedness, *resourcefulness*, persistence and a spirit of enquiry;

5. prepare students for the use of Mathematics in further studies;

6. *help students foster a 'spirit of collaboration', not only with their peers but with others within the wider community*;

7. *help students apply the knowledge and skills acquired to solve problems in everyday situations; and,*

8. *integrate Information Communication and Technology (ICT) tools and skills in the teaching and learning processes.*

♦ ORGANISATION OF THE SYLLABUS

The syllabus is arranged as a set of topics *as outlined below*, and each topic is defined by its specific objectives and content*/explanatory notes*. It is expected that students would be able to master the specific objectives and related content after pursuing a course in Mathematics over five years of secondary schooling.

SECTION 1: *NUMBER THEORY AND COMPUTATION*

SECTION 2: CONSUMER ARITHMETIC

SECTION 3: SETS

SECTION 4: MEASUREMENT

SECTION 5: STATISTICS

SECTION 6: ALGEBRA

SECTION 7: RELATIONS, FUNCTIONS AND GRAPHS

SECTION 8: GEOMETRY AND TRIGONOMETRY

SECTION 9: VECTORS AND MATRICES

♦ FORMAT OF THE EXAMINATIONS

The examination will consist of two papers: Paper 01, an objective type paper and Paper 02, an essay or problem solving type paper.

Paper 01
(1 hour 30 minutes)

The Paper will consist of 60 multiple-choice items, *from all Sections of the syllabus as outlined below.*

Sections	**No. of items**
Number Theory and Computation	*6*
Consumer Arithmetic	*8*
Sets	*6*
Measurement	*8*
Statistics	*6*
Algebra	*6*
Relations, Functions and Graphs	*8*
Geometry and Trigonometry	*8*
Vectors and Matrices	*4*
Total	***60***

Each item will be allocated one mark.

Paper 02
(2 hours and 40 minutes)

The Paper consists *of ten compulsory structured type questions.*

The marks allocated to the topics are:

Sections	**No. of marks**
Number Theory, Consumer Arithmetic and Computation	*9*
Measurement	*9*
Statistics	*9*
Algebra	*10*
Relations, Functions and Graphs	*20*
**Investigation*	*10*
Geometry and Trigonometry	*21*
Vectors and Matrices	*12*
Total	***100***

* The investigation question may be set on any combination of objectives in the syllabus.

SCHOOL BASED ASSESSMENT: Paper 031 and Paper 032

Paper 031 (20 per cent of Total Assessment)

Paper 031 comprises a project.

The project requires candidates to demonstrate the practical application of Mathematics in everyday life. In essence it should allow candidates to probe, describe and explain a mathematical area of interest and communicate the findings using mathematical symbols, language and tools. The topic(s) chosen may be from any section or combination of different sections of the syllabus.

*See Guidelines for School Based Assessment on pages **43 – 47**.*

Paper 032 (Alternative to Paper 031)
(1 hour)

This paper is an alternative to Paper 031 and is intended for private candidates. This paper comprises two compulsory questions. The topic(s) tested may be from any section or combination of different sections of the syllabus.

♦ CERTIFICATION AND PROFILE DIMENSIONS

The subject will be examined for certification at the General Proficiency.

In each paper, items and questions will be classified, according to the kind of cognitive demand made, as follows:

Knowledge require the recall of rules, procedures, definitions and facts, that is, items characterised by rote memory as well as simple computations and constructions.

Comprehension requires algorithmic thinking that involves translation from one mathematical mode to another. Use of algorithms and the application of these algorithms to familiar problem situations.

Reasoning requires:

(a) translation of non-routine problems into mathematical symbols and then choosing suitable algorithms to solve the problems;

(b) combination of two or more algorithms to solve problems;

(c) use of an algorithm or part of an algorithm, in a reverse order, to solve a problem;

(d) inferences and generalisations from given data;

(e) *justification of results or statement*; and,

(f) analysis and synthesis.

Candidates' performance will be reported under Knowledge, Comprehension and Reasoning.

♦ WEIGHTING OF PAPER AND PROFILES

The percentage weighting of the examination components and profiles is as follows:

PROFILES	***PAPER 01***	***PAPER 02***	***PAPER 03***	***TOTAL (%)***
Knowledge (K)	*18*	*30*	*6* ***(12)***	***60 (30%***
Comprehension (C)	*24*	*40*	*8* ***(16)***	***80 (40%)***
Reasoning (R)	*18*	*30*	*6* ***(12)***	***60 (30%)***
TOTAL	***60***	***100***	***20 (40)***	***200***
%	*30%*	*50%*	*20%*	*100%*

♦ REGULATIONS FOR RESIT CANDIDATES

Resit candidates must complete Papers 01 and 02 and Paper 03 of the examination for the year for which they re-register.

Resit candidates may opt to complete the School-Based Assessment (SBA) or may opt to re-use their previous SBA score which satisfies the condition below.

A candidate who rewrites the examination within two years may reuse the moderated SBA score earned in the previous sitting within the preceding two years. Candidates reusing SBA scores in this way must register as "Resit candidates" and provide their previous candidate number.

All resit candidates may register through schools, recognised educational institutions, or the Local Registrar's Office.

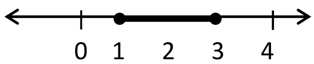
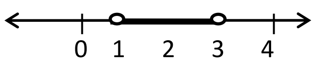
♦ REGULATIONS FOR PRIVATE CANDIDATES

Private candidates must be registered for the examination through the Local Registrar in their respective territories and will be required to sit Papers 01, 02 and 032,

Paper 032 is designed for candidates whose work cannot be monitored by tutors in recognised educational institutions. The Paper will be of 1 hour duration and will consist of two questions.

♦ SYMBOLS USED ON THE EXAMINATION PAPERS

The symbols shown below will be used on examination papers. Candidates, however, may make use of any symbol or nomenclature provided that such use is consistent and understandable in the given context. Measurement will be given in S I Units.

SYMBOL	MEANING
Sets	
U	universal set
$\cup$	*union of sets*
$\cap$	*intersection of sets*
$\in$	*element of*
$\{\}$ *or* ϕ	the null (empty) set
$\subset$	subset of
A'	complement of set A
$\{x: \ldots\}$	the set of all x such that . . .
Relations, Functions and Graphs	
$y \propto x^n$	y varies as x^n
$f(x)$	*value of the function* f *at* x
$f^{-1}(x)$	*the inverse of the function* $f(x)$
$gf(x)$, $g[f(x)]$	*composite function of the functions* f *and* g
$g^2(x)$	$g[g(x)]$
	$\{x : 1 \leq x \leq 3\}$
	$\{x : 1 < x < 3\}$

Number Theory

$\mathbb{W}$	the set of whole numbers
$\mathbb{N}$	the set of natural (counting) numbers
$\mathbb{Z}$	the set of integers, where $\begin{cases} \mathbb{Z}^+ \text{ are positive integers} \\ \mathbb{Z}^- \text{ are negative integers} \end{cases}$
$\mathbb{Q}$	the set of rational numbers
$\mathbb{R}$	the set of real numbers
$5.\dot{4}3\dot{2}$	5.432 432 432 . . .
$9.87\dot{2}\dot{1}$	9.87212121 . . .

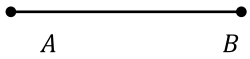
Measurement

05:00 h.	5:00 a.m.
13:15 h.	1:15 p.m.
7 mm ± 0.5 mm	7 mm to the nearest millimetre
10 m/s or 10 ms^{-1}	10 metres per second

Geometry

For transformations these symbols will be used.

M	reflection
R_θ	rotation through θ^o
T	translation
G	glide reflection
E	enlargement
MR_θ	rotation through θ followed by reflection
$\measuredangle$, $\angle$, $\wedge$	angle
$\equiv$	is congruent to
$\overleftrightarrow{AB}$	line AB

	ray AB
	line segment AB

Vectors and Matrices

$\underline{a}$ or **a**	vector **a**
$\overrightarrow{AB}$	vector A to B
$\lvert\overrightarrow{AB}\rvert$	magnitude of vector $\overrightarrow{AB}$
$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ or $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$	*the matrix A*
$\lvert A\rvert$ or det(A)	*the determinant of a matrix A*
Adj(A)	*the adjoint of a matrix A.*
A^{-1}	inverse of *a* matrix A
$I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$	*identity matrix under multiplication*
$\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$	*zero matrix or identity matrix under addition*

Other Symbols

=	is equal to
$\geq$	is greater than or equal to
$\leq$	is less than or equal to
$\cong$	is approximately equal to
$\Rightarrow$	implies
$A \Rightarrow B$	if A, then B or, A implies B
$A \Leftrightarrow B$	$\left.\begin{matrix} If\ A\ then\ B \\ and \\ If\ B\ then\ A \end{matrix}\right\}$ or A is equivalent to B

♦ FORMULAE AND TABLES PROVIDED IN THE EXAMINATION

LIST OF FORMULAE

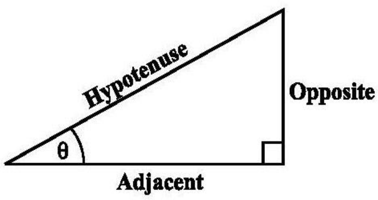
Volume of a prism	$V = Ah$ where A is the area of a cross section and h is the perpendicular length.
Volume of cylinder	$V = \pi r^2 h$ where r is the radius of the base and h is the perpendicular height.
Volume of a right pyramid	$V = \frac{1}{3}Ah$ where A is the area of the base and h is the perpendicular height.
Circumference	$C = 2\pi r$ where r is the radius of the circle.
Arc length	$S = \frac{\theta}{360} \times 2\pi r$ where θ is the angle subtended by the arc, measured in degrees.
Area of a circle	$A = \pi r^2$ where r is the radius of the circle.
Area of a sector	$A = \frac{\theta}{360} \times \pi r^2$ where θ is the angle of the sector, measured in degrees.
Area of trapezium	$A = \frac{1}{2}(a + b)\,h$ where a and b are the lengths of the parallel sides and h is the perpendicular distance between the parallel sides.
Roots of quadratic equations	If $ax^2 + bx + c = 0$, then $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Trigonometric ratios

$\sin\theta = \frac{\text{opposite side}}{\text{hypotenuse}}$

$\cos\theta = \frac{\text{adjacent side}}{\text{hypotenuse}}$

$\tan\theta = \frac{\text{opposite side}}{\text{adjacent side}}$



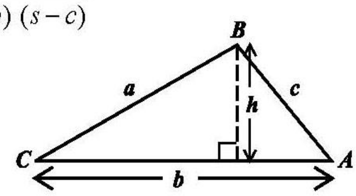
Area of triangle

Area of $\Delta = \frac{1}{2}bh$ where b is the length of the base and h is the perpendicular height.

Area of $\Delta ABC = \frac{1}{2}ab\sin C$

Area of $\Delta ABC = \sqrt{s\,(s-a)\,(s-b)\,(s-c)}$

where $s = \frac{a+b+c}{2}$



Sine rule $\quad \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

Cosine rule $\quad a^2 = b^2 + c^2 - 2bc\cos A$

♦ USE OF ELECTRONIC CALCULATORS

Candidates are expected to have an electronic *nonprogrammable* calculator and are encouraged to use such a calculator in Paper 02.

Guidelines for the use of electronic calculators are listed below.

1. Silent, electronic handheld calculators may be used.
2. Calculators should be battery or solar powered.
3. Candidates are responsible for ensuring that calculators are in working condition.
4. Candidates are permitted to bring a set of spare batteries in the examination room.
5. **No** compensation will be given to candidates because of faulty calculators.
6. **No** help or advice is permitted on the use or repair of calculators during the examination.
7. Sharing calculators is **not** permitted in the examination room.
8. Instruction manuals and external storage media (for example, card, tape, disk, smartcard or plug-in modules) are **not** permitted in the examination room.
9. Calculators with graphical display, data bank, dictionary or language translation are **not** allowed.
10. Calculators that have the capability of communication with any agency in or outside of the examination room **are prohibited**.

♦ SECTION 1: *NUMBER THEORY AND COMPUTATION*

GENERAL OBJECTIVES

On completion of this Section, students should:

1. demonstrate computational skills;
2. be aware of the importance of accuracy in computation;
3. appreciate the need for numeracy in everyday life;
4. demonstrate the ability to make estimates fit for purpose;
5. understand and appreciate the decimal numeration system;
6. appreciate the development of different numeration systems;
7. demonstrate the ability to use rational approximations of real numbers;
8. demonstrate the ability to use number properties to solve problems; and,
9. develop the ability to use patterns, trends and investigative skills.

SPECIFIC OBJECTIVES	CONTENT/ *EXPLANATORY NOTES*
Students should be able to:	
1. distinguish among sets of numbers;	*Sets of numbers:* *natural numbers* $\mathbb{N} = \{1, 2, 3, \ldots\}$*;* *whole numbers* $\mathbb{W} = \{0, 1, 2, 3, \ldots\}$*;* *integers* $\mathbb{Z} = \{\ldots -2, -1, 0, 1, 2, \ldots\}$*;* *rational numbers* $\mathbb{Q} = \{\frac{p}{q} :$ p and q are integers, $q \neq o\}$*;* *irrational numbers (numbers that cannot be expressed as terminating or recurring decimals, for example, numbers such as* π *and* $\sqrt{2}$*);* *real numbers* $\mathbb{R} = \{\text{the union of rational and irrational numbers}\}$*;* *inclusion relations, for example,* $\mathbb{N} \subset \mathbb{W} \subset \mathbb{Z} \subset \mathbb{Q} \subset \mathbb{R}$*; and,* *sequences of numbers that have a recognisable pattern; factors and multiples; square numbers; even numbers; odd numbers; prime numbers; composite numbers.*

SECTION 1: *NUMBER THEORY AND COMPUTATION (cont'd)*

SPECIFIC OBJECTIVES	CONTENT/*EXPLANATORY NOTES*
Students should be able to:	
2. *compute powers of real numbers of the form* x^a, where $a \in \mathbb{Q}$ *;*	*Including squares, square roots, cubes, cube roots.*
3. *evaluate numerical expressions* using any of the four basic operations *on* real numbers;	Addition, multiplication, subtraction and division of whole numbers, fractions and decimals; order of operations.
4. convert among fractions, per cents and decimals;	Conversion of fractions to decimals and percents, conversion of decimal to fractions and percents, conversion of percents to decimals and fractions.
5. list the set of factors and multiples of a given integer;	*Positive and negative factors of an integer.*
6. compute the H.C.F. or L.C.M. of two or more positive integers;	*Highest common factors and lowest common multiples.*
7. state the value of a digit of a numeral in a given base;	*Place value and face value of numbers in bases 2, 4, 8, and 10.*
8. convert from one set of units to another;	Conversion using conversion scales, converting within the metric scales, 12-hour and 24-hour clock.
9. express a value to a given number of: (a) significant figures; *and,* (b) decimal places.	1, 2 or 3 significant figures. 0, 1, 2 or 3 decimal places.
10. use properties of numbers and operations in computational tasks;	Properties of operations such as closure, associativity, additive and multiplicative identities and inverses, commutativity and distributivity.
11. write any rational number in *scientific notation*;	Scientific notation. *For example* $759000 = 7.59 \times 10^5$
12. calculate any fraction or percentage of a given quantity;	Fractions and percentages of a whole. The whole given a fraction or percentage.

SECTION 1: *NUMBER THEORY AND COMPUTATION (cont'd)*

SPECIFIC OBJECTIVES	CONTENT/ *EXPLANATORY NOTES*
Students should be able to:	
13. express one quantity as a fraction or percentage of another;	Comparing two quantities using fractions and percentages.
14. compare quantities;	Ratio, proportion *and rates*.
15. order a set of real numbers;	*Rearranging a set of real numbers in ascending or descending order. For example* $1.1, \frac{7}{2}, \sqrt{2}, 1.45, \pi$ *in ascending order is* $1.1, \sqrt{2}, 1.45, \pi, \frac{7}{2}$.
16. *compute* terms of a sequence given a rule;	
17. derive an appropriate rule given the terms of a sequence;	
18. divide a quantity in a given ratio; *and,*	Ratio, proportion of no more than three parts.
19. solve problems involving concepts in number theory and computation.	*Including ratio, rates and proportion.*

<u>Suggested Teaching and Learning Activities</u>

To facilitate students' attainment of the objectives of this Section, teachers are advised to engage students in the teaching and learning activities listed below.

1. Encourage the use of:

(a) calculators;

(b) games and quizzes; (for example, to investigate whether a given number is rational or irrational);

(c) appropriate software;

(d) examples of computation drawn from current affairs;

(e) the use recipes in teaching ratio and proportion; *and,*

(f) online demonstrative videos.

SECTION 1: *NUMBER THEORY AND COMPUTATION (cont'd)*

2. *Explore the link between mathematics and other disciplines, for example:*

   (a) *Music: the octave;*

   (b) *Sciences and Nature: periodic tables, counting petals, leaves and other random natural events;*

   (c) *Art and Geography: enlargement of photos as compared with ratio and proportion;*

   (d) *Architecture: number patterns and lighting patterns, ratio of width to length to height of a building or building part;*

   (e) *Health and Family Life: nutrition facts of food products; and,*

   (f) *Business Studies: using approximations in transactions, finding percentages of investments and capital.*

3. *Engage the students in the history of numbers.*

4. *Teachers can engage students in the process of "mental computation". The use of divisibility tests and other ready reckoners and properties such as associativity.*

5. *In the development of mental computation in the classroom, teachers can provide oral or written questions and encourage students to explain how they arrived at their answers and to compare their problem-solving strategies with those of their classmates. Below are two examples.*

   (a) *A flight departs on a journey at 0800 hours. After 30 minutes of flying time the journey is $\frac{1}{3}$ complete. Estimate the arrival time of the flight assuming the flight was at constant speed throughout the journey.*

   (b) *In a cricket game, at the end of the fifth over the run rate of a team is 4.6 runs per over. If the team continues to score at the same rate, determine the projected score at the end of the twentieth over.*

♦ SECTION *2:* CONSUMER ARITHMETIC

GENERAL OBJECTIVES

On completion of this Section, students should*:*

1. develop the ability to perform the calculations required in normal business transactions, and in computing their own budgets;

2. appreciate the need for both accuracy and speed in calculations;

3. appreciate the advantages and disadvantages of different ways of investing money;

4. appreciate that business arithmetic is indispensable in everyday life; *and,*

5. demonstrate the ability to use concepts in consumer arithmetic to describe, model and solve real-world problems.

SPECIFIC OBJECTIVES	**CONTENT/ *EXPLANATORY NOTES***
Students should be able to:	
1. calculate: (a) discount; (b) sales tax; (c) profit; *and,* (d) loss;	
2. calculate (a) percentage profit; *and,* (b) percentage loss;	
3. express a profit, loss, discount, markup and purchase tax, as a percentage of some value;	
4. solve problems involving marked price, selling price, cost price, profit, loss or discount;	
5. solve problems involving payments by instalments as in the case of hire purchase and mortgages;	

SPECIFIC OBJECTIVES	**CONTENT/ *EXPLANATORY NOTES***
Students should be able to:	
6. solve problems involving simple interest;	Principal, time, rate, amount.
7. solve problems involving compound interest;	*Formulae may be used in computing compound interest. The use of calculators is encouraged.*
8. solve problems involving appreciation and depreciation;	
9. solve problems involving measures and money; *and,*	Currency conversion.
10. solve problems involving:	
(a) rates and taxes;	
(b) utilities;	
(c) invoices and shopping bills;	
(d) salaries and wages; *and,*	
(e) insurance and investments.	

Suggested Teaching and Learning Activities

To facilitate students' attainment of the objectives of this Section, teachers are advised to engage students in the teaching and learning activities listed below.

1. Encourage the use of:

(a) *calculators;*

(b) *games and quizzes;*

(c) *appropriate software (for example, create an excel document to calculate utility bills and net salary);*

(d) *examples of consumer arithmetic drawn from current affairs;*

(e) *online videos;*

(f) *advertisement clippings for comparing prices; comparing prices and determining best buy; calculating hire purchase; and,*

(g) *bills and financial forms, for example, calculating utility cost and completing tax forms.*

2. Solve problems using the straight line and reducing balance method.

3. Conduct surveys or solve problems based on comparative shopping; finding total price for an item purchased online.

4. Encourage in-class role play of market situations and Cambio.

5. Examine and verifying premiums and interest using amortisation tables.

♦ SECTION 3: SETS

GENERAL OBJECTIVES

On completion of this Section, students should:

1. demonstrate the ability to communicate using set language and concepts;

2. demonstrate the ability to reason logically; and,

3. appreciate the importance and utility of sets in analysing and solving real-world problems.

SPECIFIC OBJECTIVES	**CONTENT/*EXPLANATORY NOTES***
Students should be able to:	
1. explain concepts relating to sets;	Examples and non-examples of sets, description of sets using words, membership of a set, cardinality of a set, finite and infinite sets, universal set, empty set, complement of a set, subsets.
2. represent a set in various forms;	*Representation of a set. For example*, (a) Description: the set A comprising the first three natural numbers. (b) Set builder notation: $A = \{x : 0 < x < 4, x \in \mathbb{N}\}$; (c) Listing: $A = \{1,2,3\}$
3. list subsets of a given set;	*Identifying the subsets as well as determining the* number of subsets of a set with n elements.
4. determine elements in intersections, unions and complements of sets;	Intersection and union of not more than three sets. Apply the result $n(A \cup B) = n(A) + n(B) - n(A \cap B)$.
5. describe relationships among sets using set notation and symbols;	Universal, complement, subsets, equal and equivalent sets, intersection, disjoint sets and union of sets.
6. *draw* Venn diagrams to represent relationships among sets;	Not more than 4 sets including the universal set.
7. *use Venn diagrams to represent the relationships among sets;* and,	

8. solve problems in Number Theory, Algebra and Geometry using concepts in Set Theory.

Suggested Teaching and Learning Activities

To facilitate students' attainment of the objectives of this Section, teachers are advised to engage students in the teaching and learning activities listed below.

1. *Encourage the use of:*

   (a) *calculators;*

   (b) *games and quizzes;*

   (c) *appropriate software (for example, whiteboard apps for drawing and labelling Venn diagrams);*

   (d) *examples of sets drawn from current affairs;*

   (e) *the use of recipes in teaching sets; and,*

   (f) *online demonstrative videos.*

2. *Explore the link between sets and other disciplines, for example:*

   (a) *Music: types of instruments, classification of songs;*

   (b) *Sciences: periodic tables; find the number of elements in a naturally occurring set based on characteristics of other sets;*

   (c) *Art and Geography: classifying regions according to soil type or altitude;*

   (d) *Architecture: classifying buildings according to the style of roof, shape of building, number of floors, historical design;*

   (e) *Health and family life: use medical records to categorise patients by disease, identify intersections for example hypertension and diabetes; and,*

   (f) *Business studies: types of businesses, types of products and services.*

3. *Engage in activities which assign students to groups based on their interests (sets) noting those in more than one group (intersection) and those not in a group (complement).*

4. *Use graphic organisers for comparing and/or classifying sets of items: the set of real numbers, plane figures, solids.*

♦ SECTION *4*: MEASUREMENT

GENERAL OBJECTIVES

On completion of this Section, students should:

1. understand that the attributes of geometrical objects can be quantified using measurement;

2. appreciate that all measurements are approximate and that the relative accuracy of a measurement is dependent on the measuring instrument and the measurement process; *and,*

3. demonstrate the ability to use concepts in measurement to model and solve real-world problems.

SPECIFIC OBJECTIVES	CONTENT/ *EXPLANATORY NOTES*
Students should be able to:	
1. convert units of length**,** mass, area, volume, capacity;	*Refer to Sec 1, SO8.*
2. use the appropriate SI unit of measure for area, volume, capacity, mass, temperature and time (24-hour clock) and other derived quantities;	*Refer to Sec 1, SO8.*
3. *determine* the perimeter of a *plane shape*;	*Estimating and measuring the perimeter of compound and irregular shapes.* *Calculating the perimeter of polygons and circles.*
4. calculate the length of an arc of a circle;	*Perimeter of sector of a circle*
5. estimate the area of plane *shapes*;	*Finding the area of plane shapes without using formulae.*
6. calculate the area of polygons and circles;	
7. calculate the area of a sector of a circle;	
8. calculate the area of a triangle given two sides and the angle they form;	Use of formulae. Including given two sides and included angle.
9. calculate the area of a segment of a circle;	

SPECIFIC OBJECTIVES	**CONTENT/*EXPLANATORY NOTES***
Students should be able to:	
10. calculate the surface area of solids;	Prisms including cubes and cylinders; right pyramids including cones; spheres. Surface area of sphere, $A = 4\pi r^2$.
11. calculate the volume of solids;	Prism including cube and cuboid, cylinder, right pyramid, cone and sphere. Volume of sphere, $V = \frac{4}{3}\pi r^3$
12. solve problems involving the relations among time, distance and speed;	*Average speed.*
13. estimate the margin of error for a given measurement;	Sources of error. Maximum and minimum measurements.
14. use *scales* and scale drawings to determine distances and areas; *and,*	*(Link to Geography)*
15. solve problems involving measurement.	*Perimeter, area and volume of compound shapes and solids.*

Suggested Teaching and Learning Activities

To facilitate students' attainment of the objectives of this Section, teachers are advised to engage students in the teaching and learning activities listed below.

1. *Encourage the use of:*

(a) *calculators;*

(b) *games and quizzes;*

(c) *appropriate software (for example, use of white board software for sketching diagrams; Mobile apps: Math Ref with list of formulae); and,*

(d) *online demonstrative videos.*

2. *Explore the link between measurement and other disciplines, for example:*

   (a) *Music: creating music with bottles of water where the height/volume of water results in a particular tone;*

   (b) *Sciences and nature: area of naturally occurring surfaces; length of the beach or water edge; experiments in calculating speed, distance or time; plot a graph to show the cooling rate of boiling water as time elapse; use various measurement instruments from a science laboratory;*

   (c) *Art and Geography: use of rain gauge, map reading, measuring distances on map including irregular paths;*

   (d) *Architecture: finding the perimeter, area or volume of structures: roof, wall, floor, room, column, eaves;*

   (e) *Health and family life: experiment to calculate BMI; and,*

   (f) *Business studies: determining amounts to buy given various units of lengths, area and volume.*

3. *Teacher-made resources: grid for finding area of irregular shapes, rubric for peer to peer assessment by students.*

4. *Engage students in investigating the value of pi and the area of the circle.*

♦ SECTION *5*: STATISTICS

GENERAL OBJECTIVES

On completion of this Section, students should:

1. appreciate the advantages and disadvantages of the various ways of presenting and representing data;

2. appreciate the necessity for taking precautions in collecting, analysing and interpreting statistical data, and making inferences;

3. demonstrate the ability to use concepts in statistics and probability to describe, model and solve real-world problems; and,

4. *understand the four levels/scales of measurement that inform the collection of data.*

SPECIFIC OBJECTIVES	CONTENT/*EXPLANATORY NOTES*
Students should be able to:	
1. *differentiate between sample and population attributes;*	*Sample statistics and population parameters.*
2. construct a frequency table for a given set of data;	*Discrete and continuous variables.* *Ungrouped and grouped data*.
3. determine class features for a given set of data;	Class interval, class boundaries, class limits, class midpoint, class width.
4. construct statistical diagrams;	Pie charts, bar charts, line graphs, histograms with bars of equal width and frequency polygons.
5. determine measures of central tendency for raw, ungrouped and grouped data;	*Ungrouped data*: mean, median and mode *Grouped data*: modal *class, median class and the estimate of the mean.*
6. determine when it is most appropriate to use the mean, median and mode as the average for a set of data;	*Levels of measurement (measurement scales): nominal, ordinal, interval and ratio.* *Sets with extreme values or recurring values.*
7. determine the measures of dispersion (spread) for raw, ungrouped and grouped data;	Range, interquartile range and semi-interquartile range; *estimating these measures for grouped data.*
8. *use standard deviation to compare sets of data;*	*No calculation of the standard deviation will be required.*

SPECIFIC OBJECTIVES	**CONTENT/*EXPLANATORY NOTES***
Students should be able to:	
9. draw cumulative frequency curve (Ogive);	Appropriate scales for axes. Class boundaries as domain.
10. analyse statistical diagrams;	*Finding the mean, mode, median,* range, quartiles*, interquartile range, semi-interquartile range; trends and patterns.*
11. determine the proportion or percentage of the sample above or below a given value from raw data, frequency table or cumulative frequency curve;	
12. identify the sample space for simple experiment;	*Including the use of coins, dice and playing cards.* *The use of contingency tables.*
13. determine experimental and theoretical probabilities of simple events; and,	*The use of contingency tables.* *Addition for exclusive events; multiplication for independent events.*
14. make inference(s) from statistics.	Raw data, tables, diagrams, summary statistics.

<u>Suggested Teaching and Learning Activities</u>

To facilitate students' attainment of the objectives of this Section, teachers are advised to engage students in the teaching and learning activities listed below.

1. *Encourage the use of:*

(a) *calculators;*

(b) *games and quizzes;*

(c) *appropriate software (for example, use of applications to generate and solve statistical problems including charts and calculating measures of central tendencies);*

(d) *examples of statistics drawn from newspapers, magazines and other sources of current affairs; and,*

(e) *online demonstrative videos.*

2. *Explore the link between statistics and other disciplines, for example:*

   (a) *Music: comparing record sales of various artistes, number of weeks artistes are in the top ten chart;*

   (b) *Sciences: statistics (birth and death rates) on various types of populations: people, plants and animals;*

   (c) *Geography: track rain fall, population count and density;*

   (d) *Art and Architecture: average floor size of rooms in a buildings, house lots;*

   (e) *Health and family life: monitoring weight and height, average amounts of calories, nutritional facts; and,*

   (f) *Business studies: Gross Domestic Product, predicting sales, purchasing decision.*

3. Discuss when it is most appropriate to use Nominal, Ordinal, Interval or Ratio scales.

4. Use the class of students as the population and extract samples to investigate concepts such as bias and sampling, measures of central tendencies and spread.

♦ SECTION *6*: ALGEBRA

GENERAL OBJECTIVES

On completion of this Section, students should:

1. *appreciate the use of algebra as a language and a form of communication;*

2. *appreciate the role of symbols and algebraic techniques in solving problems in mathematics and related fields; and,*

3. *demonstrate the ability to reason* with abstract entities*.*

SPECIFIC OBJECTIVES	**CONTENT/*EXPLANATORY NOTES***
Students should be able to:	
1. use symbols to represent numbers, operations, variables and relations;	Symbolic representation.
2. *translate between algebraic symbols and worded expressions;*	
3. *evaluate* arithmetic operations involving directed numbers;	
4. *simplify algebraic expressions using* the four basic operations;	
5. substitute numbers for *variables* in algebraic expressions;	
6. *evaluate expressions involving* binary operations (other than the four basic operations);	*Commutative, associative and distributive properties.*
7. apply the distributive law to factorise or expand algebraic expressions;	*For example,* $x(a+b) = ax + bx$ *and* $(a+b)(x+y) = ax + bx + ay + by$.

SPECIFIC OBJECTIVES	**CONTENT/ *EXPLANATORY NOTES***
Students should be able to:	
8. simplify algebraic fractions;	*The four basic operation on algebraic fractions.*
9. use the laws of indices to manipulate expressions with integral indices*;*	For $m \in \mathbb{Z}$, $n \in \mathbb{Z}$. *(i)* $x^m \times x^n = x^{m+n}$ *(ii)* $\frac{x^m}{x^n} = x^{m-n}$ *(iii)* $\left(x^m\right)^n = x^{m \times n}$ *(iv)* $x^{-m} = \frac{1}{x^m}$
10. solve linear equations in one unknown;	
11. solve simultaneous linear equations, in two unknowns, algebraically;	
12. solve a simple linear inequality in one unknown;	
13. change the subject of formulae;	*Including equations* involving roots and powers.
14. factorise algebraic expressions;	*Expressions of the type:* $a^2 - b^2$; $a^2 \pm 2ab + b^2$ $ax + bx + ay + by$ $ax^2 + bx + c$ where a, b, and c are integers and $a \neq 0$
15. *rewrite a quadratic expression in the form* $a(x+h)^2 + k$;	*Completing the square of a quadratic expression.*
16. *solve quadratic equations algebraically;*	*Formula and by methods of factorisation and completing the square.*

17.	solve word problems;	Linear equation, Linear inequalities, two simultaneous linear equations, quadratic equations. *Applications to other subjects for example demand and supply functions of business studies.*
18.	solve a pair of equations in two variables when one equation is quadratic or non-linear and the other linear;	
19.	prove two algebraic expressions to be identical;	*Equations vs. identities.*
20.	represent direct and *inverse* variation symbolically; and,	y varies directly as $x: y \propto x, y = kx$ y varies inversely as $x: y \propto \frac{1}{x}, y = \frac{k}{x}$
21.	solve *problems* involving direct variation and inverse *variation.*	

Suggested Teaching and Learning Activities

To facilitate students' attainment of the objectives of this Section, teachers are advised to engage students in the teaching and learning activities listed below.

1. *Encourage the use of:*

(a) *calculators;*

(b) *games and quizzes;*

(c) *appropriate software (for example, equation solving apps);*

(d) *examples of algebraic problems drawn from real-life situations; and,*

(e) *online demonstrative videos.*

2. *Explore the link between algebra and other disciplines, for example:*

(a) *Music: the use of music symbols;*

(b) *Sciences and nature: rearranging scientific formulae;*

(c) *Architecture: determine the size or amounts of tiles/windows/doors of a floor or wall; and,*

(d) *Business studies: solving equations to determine profit/loss, demand and supply.*

3. *Introduce students to symbolic representation using examples drawn from everyday life such as safety symbols, road signs and other familiar informational and warning signs.*

4. *Promote appropriate use of variables. For example, differentiate between* $5\ m$ *as an abbreviation for 5 metres and* $5m$*, where* m *represent the number of mangoes bought.*

5. *Explore the concept of equality through the use of:*

   (a) *Pan Balance activities with numbers* $(8 + 4 = x - 2)$ *and shapes; and,*

   (b) *Hands-on Algebra.*

6. *Use manipulatives such as integer chips, algebra tiles and other appropriate materials to develop the understanding of:*

   (a) *Operations with integers.*

   (b) *Simplifying algebraic expressions (adding/subtracting like terms).*

   (c) *Multiplying binomials of power 1.*

   (d) *Solving linear equations with one unknown.*

   (e) *Rearranging an equation/formula.*

7. *Conduct labs to assist students in the efficient use of calculators. For example: to explore the order of operations, to evaluate expressions with exponents and roots.*

♦ SECTION 7: RELATIONS, FUNCTIONS AND GRAPHS

GENERAL OBJECTIVES

On completion of this Section, students should:

1. appreciate the importance of relations in Mathematics;

2. appreciate that many mathematical relations may be represented in symbolic form, tabular or pictorial form; and,

3. appreciate the usefulness of concepts in relations, functions and graphs to solve real-world problems.

SPECIFIC OBJECTIVES	**CONTENT/ *EXPLANATORY NOTES***
Students should be able to:	
1. explain basic concepts associated with relations;	Concept of a relation, types of relations, examples and non-examples of relations, domain, range, image, co-domain.
2. represent a relation in various ways;	Set of ordered pairs, arrow diagrams, graphically, algebraically.
3. state the characteristics that define a function;	Concept of a function, examples and non-examples of functions.
4. use functional notation;	For example, $f : x \rightarrow x^2$; or $f(x) = x^2$ as well as $y = f(x)$ for given domains. *The inverse function* $f^{-1}(x)$. *Composite functions* $fg = f[g(x)]$.
5. distinguish between a relation and a function;	Ordered pairs, arrow diagram, graphically (vertical line test).
6. draw graphs of linear functions;	Concept of linear function, types of linear function ($y = c$; $x = k$; $y = mx + c$; where m, c and k are real numbers).
7. determine the intercepts of the graph of linear functions;	x-intercepts and y-intercepts, graphically and algebraically.
8. determine the gradient of a straight line;	Definition of gradient/*slope*.

SECTION 7: RELATIONS, FUNCTIONS AND GRAPHS (cont'd)

SPECIFIC OBJECTIVES	CONTENT/ *EXPLANATORY NOTES*
Students should be able to:	
9. determine the equation of a straight line;	Using: (a) the graph of the line; (b) the co-ordinates of two points on the line; (c) the gradient and one point on the line; and, (d) one point on the line or its gradient, and its relationship to another line.
10. solve problems involving the gradient of parallel and perpendicular lines;	
11. determine from co-ordinates on a line segment: (a) the length; and, (b) the co-ordinates of the midpoint.	The concept of magnitude or length, concept of midpoint.
12. *solve a pair of simultaneous linear equations in two unknowns graphically;*	*Intersection of graphs.*
13. represent the solution of linear inequalities in one variable using: (a) set notation; (b) the number line; and, (c) graph.	
14. draw a graph to represent a linear inequality in two variables;	

SPECIFIC OBJECTIVES	CONTENT/*EXPLANATORY NOTES*
Students should be able to:	
15. use linear programming techniques to graphically *solve* problems involving two variables;	
16. derive the composition of functions;	Composite function of no more than two functions, for example, fg, f^2 given f and g. Non-commutativity of composite functions ($fg \neq gf$) *in general*.
17. state the relationship between a function and its inverse;	The concept of the inverse of a function; *The composition of inverse functions* $f(x)$ and $f^{-1}(x)$ *is commutative and results in* x.
18. derive the inverse of a function;	$f^{-1}, (fg)^{-1}$
19. evaluate a function $f(x)$ at a given value of x;	$f(a), f^{-1}(a), fg(a)$, where $a \in \mathbb{R}$.
20. *draw and use the graph of a quadratic function to identify its features:*	*Roots of the equation.*
(a) an element of the domain that has a given image;	
(b) the image of a given element in the domain;	
(c) the maximum or minimum value of the function; and,	
(d) the equation of the axis of symmetry.	

SPECIFIC OBJECTIVES	CONTENT/*EXPLANATORY NOTES*
Students should be able to:	
21. *interpret the graph of a quadratic function to determine*: (a) the interval of the domain for which the elements of the range may be greater than or less than a given point; (b) an estimate of the value of the gradient at a given point; and, (c) intercepts of the function.	*Concepts of gradient of a curve at a point, tangent, turning point. Roots of the function*.
22. determine the *equation of the* axis of symmetry and the maximum or minimum value of a quadratic function expressed in the form $a(x + h)2 + k$;	
23. sketch the graph of a quadratic function expressed in the form $y = a(x + h)2 + k$ and determine the number of roots;	
24. draw graphs of non-linear functions;	$y = ax^n$ where $n = -1, -2$ and $+3$ *and a is a constant*. Including distance-time and speed-time.
25. *interpret graphs of functions;* and,	Including distance-time graphs and speed-time graphs.
26. *solve problems involving graphs of linear and non-linear functions.*	

Suggested Teaching and Learning Activities

To facilitate students' attainment of the objectives of this Section, teachers are advised to engage students in the teaching and learning activities listed below.

1. *Encourage the use of:*

   (a) *calculators;*

   (b) *games and quizzes;*

   (c) *appropriate software (for example, the use of graphing apps in demonstrating properties of graphs);*

   (d) *examples of functions and graphs obtained from magazines and newspapers;* and,

   (e) *online demonstrative videos.*

2. *Explore the link between relations, functions and graphs and other disciplines, for example:*

   (a) *Music: create a mapping of the number of beats to the music notes;*

   (b) *Sciences: plot graphs of sound waves, path of a projectile such as a shot putt, 2-dimensional graph of a terrain;*

   (c) *Art and Geography: identifying locations on a map using coordinate systems, the use of GPS technology;*

   (d) *Architecture: gradient of a roof, ramp;*

   (e) *Health and family life: plotting a graph of weight against time and finding the rate using the gradient of a function;* and,

   (f) *Business studies: finding marginal cost using the concept of gradient, break even analysis.*

3. *Students can be provided with samples of ordered pairs and be required to determine the domain, the range and whether the relation is or is not a function.*

4. *Encourage students to describe a function based on its properties and not the independent variable.*

5. *Use functions machines to show input and output.*

6. *Demonstrate relationships between a function and its inverse: for example doubling will undo halving, geometric interpretation as a reflection in the line $y = x$.*

7. *Relate reverse processes of real life situations to functions and their inverses, for example, the route from home to school.*

8. *Use real life examples of items that fit related categories to identify common characteristics as an analogy to linear programing.*

♦ SECTION *8*: GEOMETRY AND TRIGONOMETRY

GENERAL OBJECTIVES

On completion of this Section, students should:

1. appreciate the notion of space as a set of points with subsets of that set (space) having properties related to other mathematical systems;

2. understand the properties and relationship among geometrical objects;

3. understand the properties of transformations;

4. demonstrate the ability to use geometrical concepts to model and solve real world problems; and,

5. appreciate the power of trigonometrical methods in solving authentic problems.

SPECIFIC OBJECTIVES	**CONTENT/*EXPLANATORY NOTES***
Students should be able to:	
1. explain concepts relating to geometry;	Points, lines, parallel lines, intersecting lines and perpendicular lines, line segments, rays, curves, planes; types of angles; number of faces, edges and vertices.
2. draw and measure angles and line segments accurately using appropriate instruments;	
3. construct lines**,** angles, and polygons using appropriate instruments;	Parallel and perpendicular lines. *Bisecting line segments and angles.* *Constructing a line perpendicular to another line, L, from a point that is not on the line, L.* Triangles, quadrilaterals, regular and irregular polygons. Angles include $30^o, 45^o, 60^o, 90^o, 120^o$ *and their combinations.*
4. identify the type(s) of symmetry possessed by a given plane figure;	Line(s) of symmetry, rotational symmetry, order of rotational symmetry.

SPECIFIC OBJECTIVES	**CONTENT/*EXPLANATORY NOTES***
Students should be able to:	
5. solve geometric problems using properties of:	
(a) lines, angles, and polygons;	*Determining and justifying the measure of angles: adjacent angles, angles at a point, supplementary angles, complementary angles, vertically opposite angles.* *Parallel lines and transversals, alternate angles, corresponding angles, co-interior angles.* *Triangles: Equilateral, Isosceles, scalene, obtuse, right, acute.* *Quadrilaterals: Square, rectangle, rhombus, kite, parallelogram, trapezium.* *Other polygons.*
(b) congruent triangles;	*Cases of congruency*.
(c) similar figures;	*Properties of similar triangles*
(d) faces, edges and vertices of solids; and,	
(e) classes of solids.	Prisms, pyramids, cylinders, cones, sphere.
6. *solve geometric problems using properties of circles and circle theorems;*	*Radius, diameter, chord, circumference, arc, tangent, segment, sector, semicircle, pi.* *Determining and justifying angles using the circle theorems:* *The angle which an arc of a circle subtends at the centre of a circle is twice the angle it subtends at any point on the remaining part of the circumference.* *Angles at the circumference in the same segment of a circle and subtended by the same arc/chord are equal.* *The angle at the circumference subtended by the diameter is a right angle.* *The opposite angles of a cyclic quadrilateral are supplementary.*

SECTION *8*: GEOMETRY AND TRIGONOMETRY (cont'd)

SPECIFIC OBJECTIVES	**CONTENT/*EXPLANATORY NOTES***
Students should be able to:	
	The exterior angle of a cyclic quadrilateral is equal to the interior opposite angle. *The angle between a tangent to a circle and a chord through the point of contact is equal to the angle in the alternate segment.* *A tangent of a circle is perpendicular to the radius/diameter of that circle at the point of contact.* *The lengths of two tangents from an external point to the points of contact on the circle are equal.* *The line joining the centre of a circle to the midpoint of a chord is perpendicular to the chord.*
7. represent translations in a plane using vectors;	Column matrix notation $\begin{pmatrix} x \\ y \end{pmatrix}$.
8. determine and represent the location of: (a) the image of an object under a transformation; and, (b) an object given the image under a transformation.	Translation in the plane. Reflection in a line in that plane. Rotation about a point (the centre of rotation) in that plane. Enlargement in the plane.
9. state the relationship between an object and its image in the plane under geometric transformations;	*Orientation*, similarity, congruency.
10. describe a transformation given an object and its image;	*Translation: vector notation.* *Reflection: mirror line/ axis of symmetry.* *Rotation: centre of rotation, angle of rotation, direction of rotation.* *Enlargement: centre, scale factor k such that* $\lvert k\rvert > 1 \text{ or } 0 < \lvert k\rvert < 1$.
11. locate the image of *an object* under a combination of transformations;	*Combination of any two of:* (a) *enlargement;* (b) *translation;* (c) *rotation; and,* (d) *reflection.*

SPECIFIC OBJECTIVES	CONTENT/*EXPLANATORY NOTES*
Students should be able to:	
12. use Pythagoras' theorem to solve problems;	
13. define the trigonometric ratios of acute angles in a right triangle;	Sine, Cosine, Tangent.
14. relate objects in the physical world to geometric objects;	Angle of elevation, angle of depression, bearing.
15. *apply the trigonometric ratios to solve problems;*	*Spatial* geometry and scale drawing, angles of elevation and depression.
16. *use* the sine and cosine rules *to solve* problems involving triangles; and,	
17. solve problems involving bearings.	*Relative position of two points given the bearing of one point with respect to the other; bearing of one point relative to another point given the position of the points. Bearing written in 3-digit format for example* 060^o.

Suggested Teaching and Learning Activities

To facilitate students' attainment of the objectives of this Section, teachers are advised to engage students in the teaching and learning activities listed below.

1. *Encourage the use of:*

   (a) *calculators;*

   (b) *games and quizzes;*

   (c) *appropriate software (for example, 3-D sketching software, 2-D apps such as Geogebra);*

   (d) *Concrete models of geometric figures in common places; and,*

   (e) *online demonstrative videos.*

SECTION *8*: GEOMETRY AND TRIGONOMETRY (cont'd)

2. *Explore the link between geometry and trigonometry and other disciplines, for example:*

   (a) *Music: Exploring geometric properties of musical instruments;*

   (b) *Sciences: orbital locus of planets, galaxies; geometry in nature: leaves, shells, waves, spherical objects;*

   (c) *Geography: the use of bearings;*

   (d) *Art and Architecture: geometry of structures, triangles, circles; using geometric figures to create art such as paintings, tessellations; symmetry, similarity and congruency in structures such as the roof; and,*

   (e) *Health and family life: the geometry of postures in exercise and athletics.*

3. *Explore concepts of elevation, depression, bearings in real life situations.*

4. *Estimate distances and area using geometry, pictures with a known distance.*

5. *Engage students in activities of detecting which of two objects is taller.*

6. *Construction of shapes for art work such as collages.*

7. *Use instruments and strings to locate points of a defined locus.*

♦ SECTION *9*: VECTORS AND MATRICES

GENERAL OBJECTIVES

On completion of this Section, students should:

1. demonstrate the ability to use vector notation and concepts to model and solve real-world problems;

2. develop awareness of the existence of certain mathematical objects, such as matrices, that do not satisfy the same rules of operation as the real number system; and,

3. appreciate the use of vectors and matrices in representing certain types of linear transformations in the plane.

SPECIFIC OBJECTIVES	CONTENT/*EXPLANATORY NOTES*
Students should be able to:	
1. *explain* concepts associated with vectors;	Concept of a vector, magnitude, *unit vector*, direction, scalar. *Scalar multiples: parallel vectors, equal vectors, inverse vectors.*
2. *simplify expressions involving vectors*;	*Vector algebra: addition, subtraction, scalar multiplication.* *Vector geometry: triangle law, parallelogram law.*
3. *write the position vector of a point* $P(a, b)$ *as* $\overrightarrow{OP} = \binom{a}{b}$ where O is the origin $(0,0)$;	Displacement and position vectors; *including the use of* co-ordinates in the x-y plane to identify and determine displacement and position vectors.
4. *determine* the magnitude of a vector;	*Including unit vectors.*
5. *determine the direction of a vector;*	
6. use vectors to solve problems in geometry;	*Points in a straight line, Parallel lines; displacement, velocity, weight.*
7. *explain basic* concepts associated with matrices;	Concept of a matrix, row, column, square, identity rectangular, order.
8. *solve problems involving matrix operations*;	*Addition and subtraction of matrices of the same order. Scalar multiples.* *Multiplication of conformable matrices.* *Equality, non-commutativity of matrix multiplication.*

SECTION *9*: VECTORS AND MATRICES (cont'd)

SPECIFIC OBJECTIVES	CONTENT/ *EXPLANATORY NOTES*
Students should be able to:	
9. *evaluate* the determinant of a '2 x 2' matrix;	
10. *define the multiplicative inverse of a non-singular square matrix;*	*Identity for the square matrices.*
11. obtain the inverse of a non-singular '2 x 2' matrix;	Determinant and adjoint of a matrix.
12. *determine a* '2 x 2' *matrix* associated with a specified transformation; and,	*Transformation which is equivalent to the composition of two linear transformations in a plane (where the origin remains fixed).* (a) *Reflection in: the x-axis, y-axis,* the lines $y = x$ and $y = -x$. (b) *Rotation in a clockwise and anticlockwise direction about the origin; the general rotation matrix.* (c) *Enlargement with centre at the origin.*
13. *use* matrices to solve simple problems in *Arithmetic, Algebra and Geometry.*	*Data matrices, equality*. Use of matrices to solve linear simultaneous equations *with two unknowns.* *Problems involving determinants are restricted to 2x2 matrices. Matrices of order greater than 'mxn' will not be set, where $m \leq 4$, $n \leq 4$.*

<u>Suggested Teaching and Learning Activities</u>

To facilitate students' attainment of the objectives of this Section, teachers are advised to engage students in the teaching and learning activities listed below.

1. *Encourage the use of:*

(a) *calculators;*

(b) *games and quizzes;*

(c) *appropriate software (for example, the use of matrix solver apps);*

(d) *data matrix that are extracted from sources such as grades spread sheet; and,*

(e) *online demonstrative videos.*

2. *Explore the link between vectors and matrices and other disciplines, for example:*

(a) *Sciences & Nature: the effects of a river current as a vector quantity;*

(b) *Art & Geography: dividing an image/photo into a matrix of smaller images for enlargement; and,*

(c) *Architecture: representing items in the class room such as a tile on the floor using vector notation.*

3. *Tabulate data into matrix form.*

4. *Finding hidden treasures using clues given as vectors.*

5. *The use of matrices as operators in transformation.*

♦ GUIDELINES FOR THE SCHOOL-BASED ASSESSMENT

RATIONALE

School-Based Assessment (SBA) is an integral part of student assessment in the course covered by this syllabus. It is intended to assist students in acquiring certain knowledge, skills and attitudes that are critical to the subject. The activities for the School-Based Assessment are linked to the "Suggested Practical Activities" and should form part of the learning activities to enable the student to achieve the objectives of the syllabus.

During the course of study of the subject, students obtain marks for the competencies they develop and demonstrate in undertaking their SBA assignments. These marks contribute to the final marks and grades that are awarded to students for their performance in the examination.

The guidelines provided in this syllabus for selecting appropriate tasks are intended to assist teachers and students in selecting assignments that are valid for the purpose of the SBA. These guidelines are also intended to assist teachers in awarding marks according to the degree of achievement in the SBA component of the course. In order to ensure that the scores awarded by teachers are not out of line with the ***CXC®*** *standards, the Council undertakes the moderation of a sample of SBA assignments marked by each teacher.*

School-Based Assessment provides an opportunity to individualise a part of the curriculum to meet the needs of students. It facilitates feedback to the students at various stages of the experience. This helps to build the self-confidence of the students as they proceed with their studies. School-Based Assessment further facilitates the development of critical skills and that allows the students to function more effectively in their chosen vocation. School-Based Assessment' therefore, makes a significant and unique contribution to the development of relevant skills by the students. It also provides an instrument for testing them and rewarding them for their achievements.

The Caribbean Examinations Council seeks to ensure that the School Based Assessment scores are valid and reliable estimates of accomplishment. The guidelines provided in this syllabus are intended to assist in doing so.

THE PROJECT

The project may require candidates to collect data or demonstrate the application of Mathematics in everyday situations. The length of the report should be maximum 1000 words, not including appropriate quotations, sources, charts, graphs, tables, pictures, references and appendices.

The activities related to the Project should be integrated into the classroom instruction so as to enable the candidates to learn and practice the skills needed to complete the project.

Some time in class should be allocated for general discussion of project work; allowing for discussion between teacher and student, and student and student.

Role of the Teacher

The role of the teacher is to:

1. *Guide students in identifying suitable topics for the project for the School Based Assessment.*

2. *Provide guidance throughout the project and guide the candidate through the SBA by helping to resolve any issues that may arise.*

3. *Ensure that the project is developed as a continuous exercise that occurs during scheduled class hours as well as outside class times.*

4. *Assess the project and record the marks. Hardcopies of the completed documents should be kept by both the teacher and the student. The teacher should use the mark scheme provided by* ***CXC®*** *and include comments pertinent to the conduct of the assessment.*

Assignment

The School Based Assessment consists of ONE project to be marked by the teacher in accordance with ***CXC®*** *guidelines.*

Why a project?

The study of mathematics is essential to everyday life. It pervades almost every aspect of our daily activities: planning a picnic, baking a cake, comparing performances in a 100 metre race, shopping for groceries, all require applying mathematical concepts and principles to investigate, describe, explain or predict some real world phenomena.

However, to those engaged in learning mathematics in secondary schools, the links between mathematics and the real world are often not recognised or at least not identified and practised. The purpose of a project is to encourage students to apply mathematical concepts, skills and procedures to investigate, describe and explain real world phenomena, to practise problem-solving, and to evaluate results. And these experiences are to be realised by encouraging all students to:

1. *Define problems in personal ways, especially by how the problems were motivated;*

2. *Discuss the problems with (a) their teachers, (b) their classmates and (c) their parents and knowledgeable adults;*

3. *Develop ways of solving the problems of interest and of curiosity;*

4. *Record their problems and the ways they would attempt to find solutions using:*

words	*charts*	*models*
symbols	*tables*	*algorithms*
diagrams	*figures*	

5. *Develop positive attitudes towards the methods of mathematics, use of mathematics and the enjoyment associated with knowing mathematics and solving mathematical problems; and,*

6. *Extend mathematical processes and products to exploring and understanding other subjects on the school curriculum.*

Sample Areas of Research May Include:

Hire Purchase or Cash?

1. *An explanation of the advantages and disadvantages of hire purchase, using data collected from at least two stores.*

Sports Utility Vehicles versus Cars

2. *Comparative analyses of the costs of different types of vehicles, considering fuel economy, maintenance and features.*

What features should be included in your project?

1. *Explaining the mathematical ideas contained in your project.*

2. *Carrying our practical tasks, using one or more of the following:*

   (a) *Ruler, compasses, protractor*

   (b) *Drawing, constructing, measuring*

   (c) *Counting, looking for patterns, weighing*

   (d) *Calculators, computers, other technological devices*

3. *Performing calculations*

   (a) *Mentally*

   (b) *With paper and pencil*

   (c) *Using calculators*

4. *Responding orally to mathematics questions asked by the teacher, peers or other interested persons.*

5. *Identifying sections of project which:*

   (a) *are to be done* _inside normal class time_ *and* _scheduled by teacher_*; and,*

   (b) *may be done* _outside_ *normal class time at student's convenience*

Guidelines for students (and their teachers)

Main Activities for the Project

1. *Stating the task(s) you which to undertake - its nature, scope and focus*

2. *Planning*

    (a) *What you will do?*

    (b) *How you will do it?*

    (c) *What materials you will need?*

    (d) *What procedures you will use?*

3. *Carrying out the plan, procedures or activities*

4. *Recording what you did, how you did it, and why you did what you did, using words (including mathematical words and phrases); diagrams, tables, figures or charts*

5. *Conclusion*

    (a) *Your findings*

    (b) *Comments on your findings*

    (c) *How to improve your findings*

    (d) *Making your findings more useful*

Seven strategies that students use when they are searching for solutions to problems in mathematics.

1. *Using simple numbers*

    - *Replace the original numbers with very simple numbers and try to find a solution. After the solution has been found, try the solution with the original numbers.*

2. *Sketch a simple diagram*

    - *Attempt to understand the problem using your insight from geometry and the physical arrangements of the spaces around you.*

3. Make a table of the results

- *Try to discover any pattern within the table. You may reduce the size of the table, rotate the table and view its content from a second perspective.*

4. Guess and check

5. Look for patterns

- *Try to create patterns by combining numbers in novel ways*

6. Use algebraic symbols to express ideas

7. Make full use of calculators:

(a) Explore number ideas, number patterns and number sense

(b) Highlight estimation skills associated with arithmetic operations (such as ×, ÷, √,)

(c) Check calculations associated with a wide range of applications (from real life)

(d) Focus on the processes associated with problem-solving

ASSESSMENT CRITERIA

The project will be presented in the form of a report and will have the following parts.

1. Project Title
2. Introduction
3. Method of Data Collection
4. Presentation of Data
5. Analysis of Data
6. Discussion of Findings
7. Conclusion

It will be marked out of a total of 20 marks and the marks will be allocated to each task and profile as outlined below.

Project Descriptors	**Mark**			
	K	**C**	**R**	**Total**
Project Title				**1**
• *Title is clear and concise and relates to a real-world problem*	1			
Introduction				**4**
• *Objectives are clearly stated and relate to title*	1			
• *Comprehensive description of the project*		2		
• *Limited description of the project*		(1)		
• *Detailed contents page with page numbers*	1			
Method of Data Collection				**2**
• *Data collection method is clearly described, appropriate and without flaws, where the data is genuinely obtained.*		2		
• *Data collection method is stated*		(1)		
Presentation of Data				**5**
• *Data, which is genuinely obtained, is accurate and well organised*		2		
• *Data, which is genuinely obtained, is presented but is not well organised*		(1)		
• *Tables/graphs/diagrams/formulas/proofs included, correctly labelled, clearly and logically stated, used appropriately and reflect the data collected.*		2		
• *Tables/graphs/diagrams/formulas/proofs included and reflect the data collected*		(1)		
• *Accurate use of mathematical concepts*	1			
Analysis of Data				**2**
• *Detailed analysis of findings done, which is coherent and reflects the data collected and presented.*			2	
• *Limited analysis of findings that reflects the data collected and presented.*			(1)	
Discussion of findings				**2**
• *Statement of findings clearly stated*			1	
• *Statement of findings follows from data collected*			1	
Conclusion				**2**
• *Conclusion was based on findings and related to the purpose of the project*			2	
• *Conclusion related to the purpose of the project*			(1)	
Overall Presentation				**2**
• *Information was communicated logically using correct grammar*	2			
• *Information was poorly organised or difficult to understand at times*	(1)			
Maximum for each profile	**6**	**8**	**6**	**20**

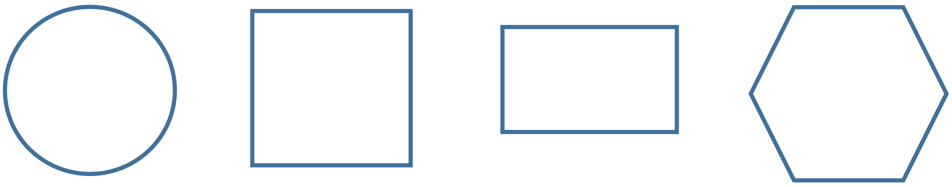
EXEMPLAR

Project Title: *Designing a Basketball Hoop - Why Use a Circle?*

Introduction: *The purpose of this project was to determine the most suitable shape for a basketball hoop. The number of goals scored using the traditional hoop was compared to the number scored using square, rectangular and hexagonal hoops.*

Data Collection: *The area enclosed by the circular hoop was calculated and hoops were made using frames to enclose an area. The dimensions of the frames were calculated to ensure that a standard basketball could pass through each frame.*

The area enclosed by a standard basketball hoop is 1641 cm^2. Efforts were made to use dimensions which would give this approximate area. The hoops in the different shapes were made with the enclosed areas shown.



Circle	*Square*	*Rectangle*	*Hexagon*
1641 cm^2	*1640 cm^2*	*1640 cm^2*	*1644 cm^2*

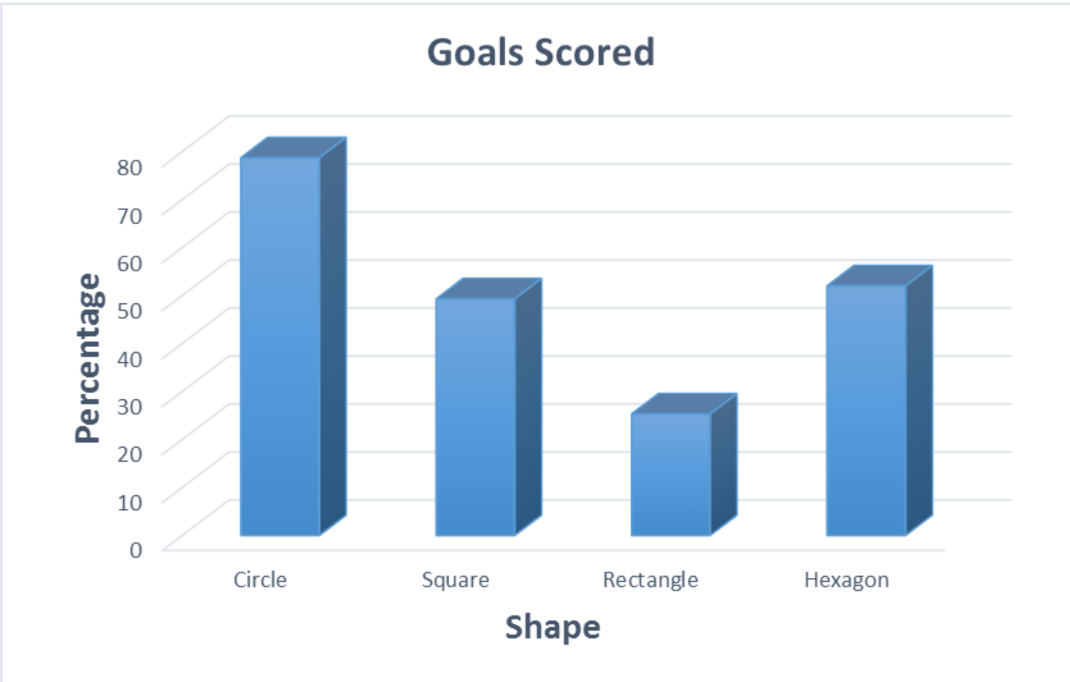
Data Collection Sheet

Name of Student:	____
Shape	***No. of goals***
Circle	
Square	
Rectangle	
Hexagon	

Presentation Data: *The table below shows the number of goals scored by each student, using of each of the hoops. Each student made 25 goal attempts for each shape. Hence, there were a total of 300 goal attempts made.*

	Number of Goals Scored				
Student	***Circle***	***Square***	***Rectangle***	***Hexagon***	***Total***
Alan	*22*	*14*	*09*	*15*	***60***
Briana	*20*	*12*	*06*	*10*	***48***
Chris	*17*	*11*	*04*	*14*	***46***
Total	***59***	***37***	***19***	***39***	***154***
% success by shape	***78.7***	***49.3***	***25.3***	***52.0***	
% of scored goals (out of 154)	***38.3***	***24.0***	***12.3***	***25.3***	

The graph below shows the percentage of goals scored for each of the shapes.



Analysis of Data: *The data collected from the experiment revealed that of the three students, Alan scored the most goals and Chris the least. Although some students were more successful in scoring, for each student, the most goals were scored with the standard basketball hoop which was in the shape of a circle where the success rate was 79 per cent. Overall, out of the 154 goals scored 38.3 per cent were using the circle, 25.3 per cent with the hexagon, 24 per cent with the square and 12.3 per cent with the rectangle.*

Discussion of Findings/ Conclusion:	*While it is possible to construct a basketball hoop using many different shapes, all shapes will not give the same results. A rectangular shaped hoop is the least suitable shape and the circular hoop, the most preferred.* *Hence, in constructing a basketball hoop, the most appropriate shape to ensure success in scoring goals is a circle.*

Procedures for Reporting and Submitting the School Based Assessment

*Teachers are required to record the mark awarded to each candidate under the appropriate profile dimension on the mark sheet provided by CXC. The completed mark sheets should be submitted to **CXC®** no later than April 30 of the year of the examination.*

Note: The school is advised to keep a copy of the project of each candidate as well as copies of the mark sheets.

*Teachers will be required to submit to **CXC®** copies of the projects of a sample of candidates as indicated by CXC. The sample will be re-marked by **CXC®** for moderation purposes.*

Moderation of School Based Assessment

The candidate's performance on the project will be moderated. The standard and range of marks awarded by the teacher will be adjusted where appropriate. However, the rank order assigned by the teacher will be adjusted only in special circumstances and then only after consideration of the data provided by the sample of marked projects submitted by the teacher and re-marked by CXC.

PAPER 032

(a) *This paper consists of two questions based on topics from any section or combination of different sections of the syllabus. The duration of the paper is **1 hour**.*

(b) *All questions are compulsory and will require an extended response.*

(c) *The paper carries a maximum of **20** marks. Marks will be awarded for Knowledge, Comprehension and Reasoning as follows:*

*Knowledge: the recall of rules, procedures, definitions and facts; simple computations. (**6** marks)*

*Comprehension: algorithmic thinking, use of algorithms and the application of algorithms to problem situations. (**8** marks)*

*Reasoning: translation of non-routine problems into mathematical symbols; making inferences and generalisations from given data; analysing and synthesising. (**6** marks)*

♦ RECOMMENDED TEXTS

Buckwell, G., Solomon, R., and Chung Harris, T.	*CXC Mathematics for Today 1*. Oxford: Macmillan Education, 2005.
Chandler, S., Smith, E., Ali, F., Layne, C. and Mothersill, A.	*Mathematics for CSEC.* United Kingdom: Nelson Thorne Limited, 2008.
Golberg, N.	*Mathematics for the Caribbean 4.* Oxford: Oxford University Press, 2006.
Greer and Layne	*Certificate Mathematics*, *A Revision Course for the Caribbean*. United Kingdom: Nelson Thrones Limited, 2001.
Layne, Ali, Bostock, Chandler, Shepherd and Ali.	*STP Caribbean Mathematics for CXC Book 4*. United Kingdom: Nelson Thrones Limited, 2005.
Toolsie, R.	*Mathematics, A Complete Course Volume 1.* Caribbean Educational Publisher Limited, 2006.
Toolsie, R.	*Mathematics, A Complete Course Volume 2.* Caribbean Educational Publisher Limited, 2006.

Websites

http://mathworld.wolfram.com/
http://plus.maths.org/
http://nrich.maths.org/public/
http://mathforum.org/
http://www.ies.co.jp/math/java/

♦ GLOSSARY OF EXAMINATION TERMS

KEY TO ABBREVIATIONS

K - Knowledge
C - Comprehension
R - Reasoning

WORD	DEFINITION	NOTES
Analyse	examine in detail	
Annotate	add a brief note to a label	Simple phrase or a few words only.
Apply	use knowledge/principles to solve problems	Make inferences/conclusions.
Assess	present reasons for the importance of particular structures, relationships or processes	Compare the advantages and disadvantages or the merits and demerits of a particular structure, relationship or process.
Calculate	arrive at the solution to a numerical problem	Steps should be shown; units must be included.
Classify	divide into groups according to observable characteristics	
Comment	state opinion or view with supporting reasons	
Compare	state similarities and differences	An explanation of the significance of each similarity and difference stated may be required for comparisons which are other than structural.
Construct	use a specific format to make and/or draw a graph, histogram, pie chart or other representation using data or material provided or drawn from practical investigations, build (for example, a model), draw scale diagram	Such representations should normally bear a title, appropriate headings and legend.
Deduce	make a logical connection between two or more pieces of information; use data to arrive at a conclusion	

WORD	DEFINITION	NOTES
Define	state concisely the meaning of a word or term	This should include the defining equation/formula where relevant.
Demonstrate	show; direct attention to...	
Derive	to deduce, determine or extract from data by a set of logical steps some relationship, formula or result	This relationship may be general or specific.
Describe	provide detailed factual information of the appearance or arrangement of a specific structure or a sequence of a specific process	Description may be in words, drawings or diagrams or any appropriate combination. Drawings or diagrams should be annotated to show appropriate detail where necessary.
Determine	find the value of a physical quantity	
Design	plan and present with appropriate practical detail	Where hypotheses are stated or when tests are to be conducted, possible outcomes should be clearly stated and/or the way in which data will be analysed and presented.
Develop	expand or elaborate an idea or argument with supporting reasons	
Diagram	simplified representation showing the relationship between components	
Differentiate/ Distinguish (between/ among)	state or explain briefly those differences between or among items which can be used to define the items or place them into separate categories	
Discuss	present reasoned argument; consider points both for and against; explain the relative merits of a case	
Draw	make a line representation from specimens or apparatus which shows an accurate relation between the parts	In the case of drawings from specimens, the magnification must always be stated.
Estimate	make an approximate quantitative judgement	
Evaluate	weigh evidence and make judgements based on given criteria	The use of logical supporting reasons for a particular point of view is more important than the view held; usually both sides of an argument should be considered.
Explain	give reasons based on recall; account for	

WORD	DEFINITION	NOTES
Find	locate a feature or obtain as from a graph	
Formulate	devise a hypothesis	
Identify	name or point out specific components or features	
Illustrate	show clearly by using appropriate examples or diagrams, sketches	
Interpret	explain the meaning of	
Investigate	use simple systematic procedures to observe, record data and draw logical conclusions	
Justify	explain the correctness of	
Label	add names to identify structures or parts indicated by pointers	
List	itemise without detail	
Measure	take accurate quantitative readings using appropriate instruments	
Name	give only the name of	No additional information is required.
Note	write down observations	
Observe	pay attention to details which characterise a specimen, reaction or change taking place; to examine and note scientifically	Observations may involve all the senses and/or extensions of them but would normally exclude the sense of taste.
Outline	give basic steps only	
Plan	prepare to conduct an investigation	
Predict	use information provided to arrive at a likely conclusion or suggest a possible outcome	
Record	write an accurate description of the full range of observations made during a given procedure	This includes the values for any variable being investigated; where appropriate, recorded data may be depicted in graphs, histograms or tables.

WORD	DEFINITION	NOTES
Relate	show connections between; explain how one set of facts or data depend on others or are determined by them	
Sketch	make a simple freehand diagram showing relevant proportions and any important details	
State	provide factual information in concise terms outlining explanations	
Suggest	offer an explanation deduced from information provided or previous knowledge. (... a hypothesis; provide a generalisation which offers a likely explanation for a set of data or observations.)	No correct or incorrect solution is presumed but suggestions must be acceptable within the limits of scientific knowledge.
Use	apply knowledge/principles to solve problems	Make inferences/conclusions.

♦ GLOSSARY OF MATHEMATICAL TERMS

WORD	MEANING
Abscissa	The x-coordinate in a Cartesian coordinate system.
Absolute value	The absolute value of a real number x, denoted by $\|x\|$, is defined by $\|x\| = x$ if $x > 0$ and $\|x\| = -x$ if $x < 0$. For example, $\|-4\| = 4$.
Acceleration	The rate of change of velocity with respect to time.
Acute Angle	An angle whose measure is greater than 0 degrees and less than 90 degrees. **Acute triangle** is a triangle with all three of its angles being acute.
Adjacent	Being next to or adjoining **Adjacent angles** are two angles that have the same vertex and share a common arm. In a right triangle the **adjacent side**, with respect to an acute angle, is the shorter side which, together with the hypotenuse, forms the given acute angle.
Adjoint Matrix	The adjoint of a 2×2 matrix A, denoted $Adj(A)$, satisfies the following: If $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$, then $Adj(A) = \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$.
Algebraic Expression	A combination of numbers, variables and algebraic operations. For example $\frac{3x^4}{17} + 5(y - \sqrt{16z})$ is an algebraic expression.
Algebraic Term	An algebraic expression that is strictly a multiplication of constants and variables. For example the algebraic expression $6x^3 - 3x^2 + 5x$ contains three algebraic terms: $(6x^3)$, $(-3x^2)$ and $(5x)$.
Algorithm	A process consisting of a specific sequence of operations to solve a certain types of problems. See **Heuristic**.
Alternate Interior Angles	Angles located inside a set of parallel lines and on opposite sides of the transversal. Also known as 'Z-angles'.
Altitude	The altitude of a triangle is the perpendicular distance of a vertex to the line of the side opposite. A triangle has three altitudes.
Appreciation	An increase in value of an asset that is not due to altering its state.
Approximate	To find the value of a quantity within a specified degree of accuracy.
Arc	A portion of a circle; also a portion of any curve. **See circle.**
Area	The area of a plane figure is a measure of how much of that plane is enclosed by the figure.

WORD	MEANING
Arithmetic Mean	The average of a set of values found by dividing the sum of the values by the amount of values.
Arithmetic Sequence	A sequence of elements, a_1, a_2, a_3,….., such that the difference of successive terms is a constant d. For example, the sequence $\{2, 5, 8, 11, 14, \ldots\}$ has common difference 3.
Associative Property	A binary operation $\circ$ on a set S is associative if, for all a, b and c in S, $(a \circ b) \circ c = a \circ (b \circ c)$.
Asymptotes	A straight line is said to be an asymptote of a curve if the curve has the property of becoming and staying arbitrarily close to the line as the distance from the origin increases to infinity.
Average	The average of a set of values is the number which represents the usual or typical value in that set. Average is synonymous with measures of central tendency. These include the mean, mode and median.
Axis of symmetry	A line that passes through a figure such that the portion of the figure on one side of the line is the mirror image of the portion on the other side of the line.
Bar Graph	A diagram showing a system of connections or interrelations between two or more things by using bars.
Base	1. The base of a polygon is one of its sides; for example, a side of a triangle. 2. The base of a solid is one of its faces; for example, the flat face of a cylinder. 3. The base of a number system is the number of digits it contains; for example, the base of the binary system is two.
Bimodal	Having two modes, which are equally the most frequently occurring numbers in a list.
Binary Numbers	Numbers written in the base two number system. The digits used are 0 and 1. For example, 11011_2.
Binomial	An algebraic expression consisting of the sum or difference of two terms.
Bisector	To cut something in half. For example, an angle bisector is a line that divides one angle into two angles of equal size.
Capacity	The maximum amount that something can contain.

WORD	MEANING
Cartesian Plane	A plane with a point selected as an origin, some length selected as a unit of distance, and two perpendicular lines that intersect at the origin, with positive and negative directions selected on each line. Traditionally, the lines are called x (drawn from left to right, with positive direction to the right of the origin) and y (drawn from bottom to top, with positive direction upward of the origin). Coordinates of a point are determined by the distance of this point from the lines, and the signs of the coordinates are determined by whether the point is in the positive or in the negative direction from the origin. A line segment that connects two points on a curve.
Chord	**The Diameter** of a circle is a special chord that passes through the centre of the circle.
Circle	The set of points in a plane that are all a fixed distance from a given point which is called the centre. **The Circumference** of a circle is the distance along the circle; it's a special name for the perimeter of the circle.
Class Interval	Non-overlapping intervals, which together contain every piece of data in a survey.
Coefficients	The constant multiplicative factor of a mathematical object. For example, in the expression 4d+5 t^2 +3s, the 4, 5, and 3 are coefficients for the variables d, t^2, and s respectively.
Collinear	A set of points are said to be collinear if they all lie on the same straight line.
Commutative Property	Reversing the order in which two objects are being added or multiplied will yield the same result. For all real numbers a and b, a+b=b+a and ab=ba.
Complement	The complement of a set A is another set of all the elements outside of set A but within the universal set.
Complementary Angles	Two angles that have a sum of 90 degrees.
Composite Function	A function consisting of two or more functions such that the output of one function is the input of the other function. For example, in the composite function $f\big(g(x)\big)$ the input of f is g.
Composite Numbers	Numbers that have more than two factors. For example, 6 and 20 are composite numbers while 7 and 41 are not.
Compound Interest	A system of calculating interest on the sum of the initial amount invested together with the interest previously awarded; if A is the initial sum invested in an account and r is the rate of interest per period invested, then the total after n periods is $A(1+r)^n$.

WORD	MEANING
Congruent	Two shapes in the plane or in space are congruent if they are identical. That is, if one shape is placed on the other they match exactly.
Coordinates	A unique order of numbers that identifies a point on the coordinate plane. On the Cartesian two dimensional plane the first number in the ordered pair identifies the position with regard to the horizontal (x-axis) while the second number identifies the position relative to the vertical (y-axis).
Coplanar	A set of points is coplanar if the points all lie in the same plane.
Corresponding Angles	Two angles in the same relative position on two parallel lines when those lines are cut by a transversal.
Decimal Number	A number written in base ten.
Degrees	A degree is a unit of measure of angles where one degree is $\frac{1}{360}$ of a complete revolution.
Depreciation	The rate which the value of an asset diminishes due only to wear and tear.
Diagonal	The diagonal of a polygon is a straight line joining two of its nonadjacent vertices.
Discontinuous Graph	A line in a graph that is interrupted, or has breaks in it.
Discrete	A set of values are said to be discrete if they are all distinct and separated from each other. For example, the set of shoe sizes where the elements of this set can only take on a limited and distinct set of values.
Disjoint	Two sets are disjoint if they have no common elements; their intersection is empty.
Distributive Property	Summing two numbers and then multiplying by a third number yields the same value as multiplying both numbers by the third number and then adding. In algebraic terms, for all real numbers a, b, and c, $a(b+c) = ab + ac$.
Domain of the function f	The set of objects x for which f(x) is defined.
Element of a set	A member of or an object in a set.
Empty Set	The empty set is the set that has no elements; it is denoted with the symbol Ø.
Equally Likely	In probability, when there are the same chances for more than one event to happen, the events are equally likely to occur. For example, if

WORD	MEANING
	someone flips a fair coin, the chances of getting heads or tails are the same. There are equally likely chances of getting heads or tails.
Equation	A statement that says that two mathematical expressions have the same value.
Equilateral Triangle	A triangle with three equal sides. Equilateral triangles have three equal angles of measure 60 degrees.
Estimate	The best guess for an unknown quantity arrived at after considering all the information given in a problem.
Event	In probability, an event is a set of outcomes of an experiment. For example, the even A may be defined as obtaining two heads from tossing a coin twice.
Expected Value	The average amount that is predicted if an experiment is repeated many times.
Experimental Probability	The chances of something happening, based on repeated testing and observing results. It is the ratio of the number of times an event occurred to the number of times tested. For example, to find the experimental probability of winning a game, one must play the game many times, then divide by the number of games won by the total number of games played.
Exponent	The power to which a number of variables is raised.
Exponential Function	A function that has the form $y=a^x$, where a is any real number and is called the base.
Exterior Angle	The exterior angle of a polygon is an angle formed by a side and a line which is the extension of an adjacent side.
Factors	1. The factors of a whole number are those numbers by which it can be divided without leaving a remainder. 2. The factors of an algebraic expression A are those expressions which, when multiplied together, results in A. For example x and $(3-y)$ are the factors of $3x-xy$.
Factorise	The process of rewriting an algebraic expression as a product of its factors. For example, $4x^2-4y^2$ when factorised may be written as $(2x-2y)(2x+2y)$. To **factorise completely** is to rewrite an expression as a product of prime factors. For example, $4x^2-4y^2$ when factorised completely is $4(x-y)(x+y)$.
Frequency	The number of items occurring in a given category.
Frustum	The frustum is a portion of a cone or pyramid bounded by two faces parallel to the base.

WORD	MEANING
Function	A correspondence in which each member of one set is mapped unto a member of another set.
Graph	A visual representation of data that displays the relationship among variables, usually cast along x and y axes.
Histogram	A bar graph with no spaces between the bars where the area of the bars is proportional to the corresponding frequencies. If the bars have the same width, then the heights are proportional to the frequencies.
Hypotenuse	The side of the Right triangle that is opposite the right angle. It is the longest of the three sides.
Identity	1. An equation that is true for every possible value of the variables. For example, $x^2 - 1 = (x-1)(x+1)$ is an identity while $x^2 - 1 = 3$ is not, as it is only true for the values ± 2. 2. The **identity element** of an operation is a number such that when operated on with any other number results in the other number. For example, the identity element under addition of real numbers is zero; the identity element under multiplication of 2 x 2 matrices is $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$.
Inequality	A relationship between two quantities indicating that one is strictly less than or less than or equal to the other.
Infinity	The symbol ∞ indicating a limitless quantity. For example, the result of a nonzero number divided by zero is infinity.
Integers	The set consisting of the positive and negative whole numbers and zero, for example, {… -2, -1, 0, 1, 2,…}.
Intercept	The **x-intercept** of a graph is the value of x where the curve crosses the x-axis. The **y-intercept** of a graph is the y value where the curve crosses the y-axis.
Intersection	The intersection of two sets is the set of elements which are common in both sets.
Inverse	The inverse of an element under an operation is another element which when operated on with the first element results in the identity. For example, the inverse of a real number under addition is the negative of that number.
Irrational Number	A number that cannot be represented as an exact ratio of two integers. For example, π or the square root of 2.

WORD	MEANING
Isosceles Triangle	A triangle that has two equal sides.
Like Terms	Two terms are like terms if all parts of both, except for the numerical coefficient, are the same.
Limit	The target value that terms in a sequence of numbers are getting closer to. This limit is not necessarily ever reached; the numbers in the sequence eventually get arbitrarily close to the limit.
Line Graph	A diagram showing a system of connections or interrelations between two or more things by using lines.
Line symmetry	If a figure is divided by a line and both divisions are mirrors of each other, the figure has line symmetry. The line that divides the figure is the line of symmetry.
Linear Equation	An equation containing linear expressions.
Linear Expression	An expression of the form $ax + b$ where x is a variable of power one and a and b are constants, or in more variables, an expression of the form $ax + by + c$, $ax + by + cz + d$ where a, b, c and d are constants.
Magnitude	The length of a vector.
Matrix	A rectangular arrangement of numbers in rows and columns.
Mean	In statistics, the average obtained by dividing the sum of two or more quantities by the number of these quantities.
Median	In statistics, the quantity designating the middle value in a set of numbers which have been arranged in ascending or descending order.
Mode	In statistics, the value that occurs most frequently in a given set of numbers.
Multimodal distribution	A distribution with more than one mode. For example, the set {2, 4, 3, 5, 3, 6, 5, 2, 5, 3} has modal values 3 and 5.
Multiples	The product of multiplying a number by a whole number. For example, multiples of 5 are 10, 15, 20 or any number that can be evenly divided by 5.
Natural Numbers	The set of the counting numbers, that is, N = {1, 2, 3, 4…}
Negative Numbers	Numbers less than zero. In graphing, numbers to the left of zero. Negative numbers are represented by placing a minus sign (-) in front of the number. For example, $-3, -0.5, -\frac{14}{9}$ are negative numbers.

WORD	MEANING
Obtuse Angle	An angle whose measure is greater than 90 degrees but less than 180 degrees.
Obtuse Triangle	A triangle containing one obtuse angle.
Ordered Pair	A set of numbers where the order in which the numbers are written has an agreed-upon meaning. For example, points on the Cartesian plane are represented by ordered pairs such as P(4,7) where 4 is the x-value and 7 the y-value.
Origin	In the Cartesian coordinate plane, the origin is the point at which the horizontal and vertical axes intersect, at zero (0,0).
Parallel	Given distinct lines in the plane that are infinite in both directions, the lines are parallel if they never meet. Two distinct lines in the coordinate plane are parallel if and only if they have the same slope.
Parallelogram	A quadrilateral that contains two pairs of parallel sides.
Pattern	Characteristic(s) observed in one item that may be repeated in similar or identical manners in other items.
Percent	A ratio that compares a number to one hundred. The symbol for per cent is %.
Perpendicular	Two lines are said to be perpendicular to each other if they form a 90 degrees angle.
Pi	The designated name for the ratio of the circumference of a circle to its diameter.
Pie Chart	A chart made by plotting the numeric values of a set of quantities as a set of adjacent circular wedges where the arc lengths are proportional to the total amount. All wedges taken together comprise an entire disk.
Pie Graph	A diagram showing a system of connections or interrelations between two or more things by using a circle divided into segments that look like pieces of pie.
Polygon	A closed plane figure formed by three or more line segments.
Polyhedra	Any solid figure with an outer surface composed of polygon faces.
Polynominal	An algebraic expression involving a sum of algebraic terms with nonnegative integer powers. For example, $2x^3 + 3x^2 - x + 6$ is a polynomial in one variable.
Population	In statistics population is the set of all items under consideration.
Prime	A natural number p greater than 1 is prime if and only if the only positive integer factors of p are 1 and p. The first seven primes are 2, 3, 5, 7, 11, 13, 17.

WORD	MEANING
Probability	The measure of how likely it is for an event to occur. The probability of an event is always a number between zero and 1.
Proportion	1. A relationship between two ratios in which the first ratio is always equal to the second. Usually of the form $\frac{a}{b}=\frac{c}{d}$. 2. The fraction of a part and the whole. If two parts of a whole are in the ratio 2:7, then the corresponding proportions are $\frac{2}{9}$ and $\frac{7}{9}$ respectively.
Protractor	An instrument used for drawing and measuring angles.
Pythagorean Theorem	The Pythagorean Theorem states that the square of the hypotenuse is equal to the sum of the squares of the two sides, or $a^2+b^2=c^2$, where c is the hypotenuse.
Quadrant	The four parts of the coordinate plane divided by the x and y axes. Each of these quadrants has a number designation. First quadrant contains all the points with positive x and positive y coordinates. Second quadrant contains all the points with negative x and positive y coordinates. The third quadrant contains all the points with both coordinates negative. Fourth quadrant contains all the points with positive x and negative y coordinates.
Quadratic Function	A function given by a polynomial of degree 2.
Quadrilateral	A polygon that has four sides.
Quartiles	Consider a set of numbers arranged in ascending or descending order. The quartiles are the three numbers which divide the set into four parts of equal amount of numbers. The first quartile in a list of numbers is the number such that a quarter of the numbers is below it. The second quartile is the median. The third quartile is the number such that three quarters of the numbers are below it.
Quotient	The result of division.
Radical	The radical symbol (√) is used to indicate the taking of a root of a number. $\sqrt[q]{x}$ means the q^th root of x; if q=2 then it is usually written as $\sqrt{x}$. For example $\sqrt[5]{243}=3$, $\sqrt[4]{16}=2$. The radical always means to take the positive value. For example, both 5 and -5 satisfy the equation $x^2=25$, but $\sqrt{25}=5$.
Range	The range of a set of numbers is the difference between the largest value in the set and the smallest value in the set. Note that the range is a single number, not many numbers.
Range of Function f	The set of all the numbers f(x) for x in the domain of f.

WORD	MEANING
Ratio	A comparison expressed as a fraction. For example, the ratio of three boys to two girls in a class is written as $\frac{3}{2}$ or 3:2.
Rational Numbers	Numbers that can be expressed as the quotient of two integers, for example, $\frac{7}{3}$, $\frac{5}{11}$, $\frac{-5}{13}$, $7 = \frac{7}{1}$.
Ray	A straight line that begins at a point and continues outward in one direction.
Real Numbers	The union of the set of rational numbers and the set of irrational numbers.
Reciprocal	The reciprocal of a number **a** is equal to $\frac{1}{a}$ where $a \neq 0$.
Regular Polygon	A polygon whose side lengths are all the same and whose interior angle measures are all the same.
Rhombus	A parallelogram with four congruent sides.
Right Angle	An angle of 90 degrees.
Right Triangle	A triangle containing an angle of 90 degrees.
Rotate	The turning of an object (or co-ordinate system) by an angle about a fixed point.
Root	1. The root of an equation is the same as the solution of that equation. For example, if y=f(x), then the roots are the values of x for which y=0. Graphically, the roots are the x-intercepts of the graph. 2. The n^{th} root of a real number x is a number which, when multiplied by itself n times, gives x. If n is odd then there is one root for every value of x; if n is even there are two roots (one positive and one negative) for positive values of x and no real roots for negative values of x. The positive root is called the **Principal root** and is represented by the radical sign (√). For example, the principal square root of 9 is written as $\sqrt{9} = 3$ but the square roots of 9 are $\pm\sqrt{9} = \pm 3$.
Sample	A group of items chosen from a population.
Sample Space	The set of outcomes of a probability experiment. Also called probability space.
Scalar	A quantity which has size but no direction.
Scalene Triangle	A triangle with no two sides equal. A scalene triangle has no two angles equal.

WORD	MEANING
Scientific Notation	A shorthand way of writing very large or very small numbers. A number expressed in scientific notation is expressed as a decimal number between 1 and 10 multiplied by a power of 10 (for example, 7000 = $7\text{x}10^3$ or 0.0000019 = $1.9\text{x}10^{-6}$).
Sector	The sector of a circle is a closed figure formed by an arc and two radii of the circle.
Segment	1. A line segment is a piece of a line with two end points. 2. A segment of a circle is a closed figure formed by an arc and a chord.
Sequence	A set of numbers with a prescribed order.
Set	A set is a well-defined collection of things, without regard to their order.
Significant Digits	The amount of digits required for calculations or measurements to be close enough to the actual value. Some rules in determining the number of digits considered significant in a number: - The leftmost non-zero digit is the first significant digit. - Zeros between two non-zero digits are significant. Trailing zeros to the right of the decimal point are considered significant.
Similar	Two figures are said to be similar when all corresponding angles are equal. If two shapes are similar, then the corresponding sides are in the same ratio.
Simple Event	A non-decomposable outcome of a probability experiment.
Simple Interest	An interest of a fixed amount calculated on the initial investment.
Simultaneous Equations	A system (set) of equations that must all be true at the same time.
Solid	A three dimensional geometric figure that completely encloses a volume of space.
Square Matrix	A matrix with equal number of rows and columns.
Square Root	The square root of a positive real number n is the number m such that $m^2 = n$. For example, the square roots of 16 are 4 and -4.
Subset	A subset of a given set is a collection of things that belong to the original set. For example, the subsets of $A = \{a, b\}$ are: $\{a\}, \{b\}, \{a, b\}$, and the null set.

WORD	MEANING
Surface Area	The sum of the areas of the surfaces of a solid.
Statistical Inference	The process of estimating unobservable characteristics of a population using information obtained from a sample.
Symmetry	Two points A and B are symmetric with respect to a line if the line is a perpendicular bisector of the segment AB.
Tangent	A line is a tangent to a curve at a point A if it just touches the curve at A in such a way that it remains on one side of the curve at A. A tangent to a circle intersects the circle only once.
Translate	In a tessellation, to translate an object means repeating it by sliding it over a certain distance in a certain direction.
Translation	A rigid motion of the plane or space of the form X goes to X + V for a fixed vector V.
Transversal	In geometry, given two or more lines in the plane a transversal is a line distinct from the original lines and intersects each of the given lines in a single point.
Theoretical Probability	The chances of events happening as determined by calculating results that would occur under ideal circumstances. For example, the theoretical probability of rolling a 4 on a fair four-sided die is ¼ or 25%, because there is one chance in four to roll a 4, and under ideal circumstances one out of every four rolls would be a 4.
Trapezoid	A quadrilateral with exactly one pair of parallel sides.
Trigonometry	The study of triangles. Three trigonometric functions defined for either acute angles in the right triangle are: Sine of the angle x is the ratio of the side opposite the angle and the hypotenuse. In short, $\sin x = \frac{O}{H}$; Cosine of the angle x is the ratio of the short side adjacent to the angle and the hypotenuse. In short, $\cos x = \frac{A}{H}$; Tangent of the angle x is the ratio of the side opposite the angle and the short side adjacent to the angle. In short $\tan x = \frac{O}{A}$
Union of Sets	The union of two or more sets is the set of all the elements contained in all the sets. The symbol for union is U.
Unit Vector	A vector of length 1.

WORD	MEANING
Variable	A placeholder in an algebraic expression, for example, in $3x + y = 23$, x and y are variables.
Vector	Quantity that has magnitude (length) and direction. It may be represented as a directed line segment.
Velocity	The rate of change of position overtime in a given direction is velocity, calculated by dividing directed distance by time.
Venn Diagram	A diagram where sets are represented as simple geometric figures, with overlapping and similarity of sets represented by intersections and unions of the figures.
Vertex	The vertex of an angle is the point where the two sides of the angle meet.
Volume	A measure of the number of cubic units of space an object occupies.

Western Zone Office
21 June 2016

CARIBBEAN EXAMINATIONS COUNCIL

Caribbean Secondary Education Certificate®
CSEC®

MATHEMATICS

Specimen Papers and Mark Schemes/Keys

<u>Specimen Papers:</u>

Paper 01
Paper 02
Paper 032

<u>Mark Schemes and Key:</u>

Paper 01
Paper 02
Paper 032

The Specimen Papers

Paper 01

1. The Paper 01 consists of 60 items. However, there are 30 items in this specimen paper. The Specimen represents the syllabus topics and the profiles in the same ratio as they will occur on the Paper 01 for the revised syllabus.

2. Vectors and Matrices will be tested on Paper 01.

Paper 02

1. The Paper 02 will be marked electronically. The structure of the actual paper has been modified to allow candidates to write in the spaces following each part of a question. However, in an effort to limit the number of pages in the specimen paper, the spaces for the working were omitted.

2. The topic Sets will no longer be tested on Paper 02.

Paper 032

1. This is a new paper and is designed for candidates whose work cannot be monitored by tutors in recognised educational institutions. The Paper will be of one hour duration and will consist of two questions.

2. The Paper 032 will be marked electronically and the structure, as presented in the specimen, will allow candidates to write in the spaces following each part of a question.

C A R I B B E A N   E X A M I N A T I O N S   C O U N C I L

CARIBBEAN SECONDARY EDUCATION CERTIFICATE®
EXAMINATION

SPECIMEN
MULTIPLE CHOICE QUESTIONS
FOR

MATHEMATICS

READ THE FOLLOWING DIRECTIONS CAREFULLY

Each item in this test has four suggested answers lettered (A), (B), (C), (D). Read each item you are about to answer and decide which choice is best.

Sample Item

$2a + 6a =$

(A) $8a$
(B) $8a^2$
(C) $12a$
(D) $12a^2$

Sample Answer

● (B) (C) (D)

The best answer to this item is "8a", so answer space (A) has been shaded.

There are 30 items in this specimen paper. However, the Paper 01 test consists of 60 items.

LIST OF FORMULAE

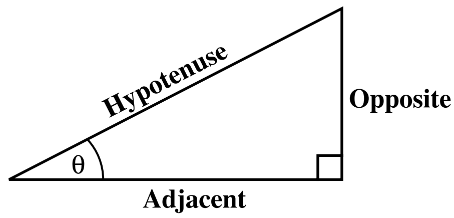
Volume of a prism	$V = Ah$ where A is the area of a cross-section and h is the perpendicular length.
Volume of cylinder	$V = \pi r^2 h$ where r is the radius of the base and h is the perpendicular height.
Volume of a right pyramid	$V = \frac{1}{3}Ah$ where A is the area of the base and h is the perpendicular height.
Circumference	$C = 2\pi r$ where r is the radius of the circle.
Arc length	$S = \frac{\theta}{360} \times 2\pi r$ where θ is the angle subtended by the arc, measured in degrees.
Area of a circle	$A = \pi r^2$ where r is the radius of the circle.
Area of a sector	$A = \frac{\theta}{360} \times \pi r^2$ where θ is the angle of the sector, measured in degrees.
Area of trapezium	$A = \frac{1}{2}(a + b)\,h$ where a and b are the lengths of the parallel sides and h is the perpendicular distance between the parallel sides.
Roots of quadratic equations	If $ax^2 + bx + c = 0$, then $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Trigonometric ratios

$$\sin\theta = \frac{\text{opposite side}}{\text{hypotenuse}}$$

$$\cos\theta = \frac{\text{adjacent side}}{\text{hypotenuse}}$$

$$\tan\theta = \frac{\text{opposite side}}{\text{adjacent side}}$$



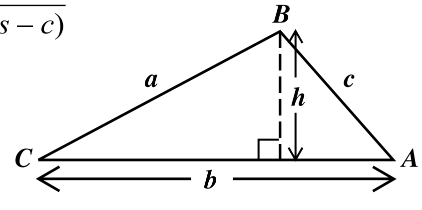
Area of triangle

Area of $\Delta = \frac{1}{2}bh$ where b is the length of the base and h is the perpendicular height.

$$\text{Area of } \Delta\, ABC = \frac{1}{2}ab\sin C$$

$$\text{Area of } \Delta\, ABC = \sqrt{s(s-a)(s-b)(s-c)}$$

where $s = \frac{a+b+c}{2}$



Sine rule

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Cosine rule

$$a^2 = b^2 + c^2 - 2bc\cos A$$

1. The number 2 747 written to 3 significant figures is

(A) 2 740
(B) 2 750
(C) 274
(D) 275

2. Expressed in scientific notation 0.045×10^{-3} is

(A) 4.5×10^{-1}
(B) 4.5×10^{-4}
(C) 4.5×10^{-5}
(D) 4.5×10^{-6}

3. The value of $\frac{(5+2)^3}{5^2 - 2^2}$ in its simplest form is

(A) $\frac{8}{21}$

(B) $\frac{7}{3}$

(C) $\frac{7}{2}$

(D) $\frac{49}{3}$

4. How much simple interest is due on a loan of $1 200 for two years if the annual rate of interest is $5\frac{1}{2}$ per cent?

(A) $120.00
(B) $132.00
(C) $264.00
(D) $330.00

Item **5** refers to the chart shown below.

Rate on Fixed Deposits	
2014	7.8%
2015	7.5%

5. How much more interest did a fixed deposit of $10 000 earn in 2015 than in 2014?

(A) $ 0.30
(B) $ 3.00
(C) $30.00
(D) $33.00

6. Tom bought a pen for $60. He sold it to gain 20% on his cost. How much money did he gain?

(A) $12
(B) $40
(C) $72
(D) $80

7. The Water Authority charges $10.00 per month for the meter rent, $25.00 for the first 100 litres and $1.00 for each additional 10 litres.

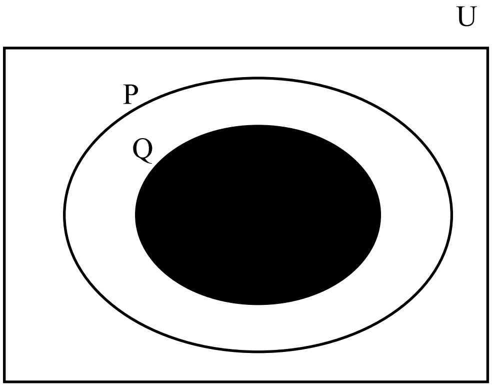
What is the total bill for 250 litres used in one month?

(A) $25.00
(B) $35.00
(C) $40.00
(D) $50.00

8. Which of the following sets has an infinite number of members?

(A) {factors of 20}
(B) {multiples of 20}
(C) {prime numbers less than 20}
(D) {odd numbers between 10 and 20}

Item **9** refers to the Venn diagram below.



9. The shaded area in the Venn diagram above represents

(A) P'
(B) Q'
(C) $P \cup Q$
(D) $P \cap Q$

10. P and Q are two finite sets such that n(P) = 7, n(Q) = 5 and n(P ∩ Q) = 3. What is n(P ∪ Q)?

(A) 6
(B) 9
(C) 15
(D) 18

11. How many litres of water would a container whose volume is 36 cm^3 hold?

(A) 0.036
(B) 0.36
(C) 36
(D) 3600

12. A man leaves home at 22:15 hrs and reaches his destination at 04:00 hrs. On the following day, in the same time zone. How long did the journey take?

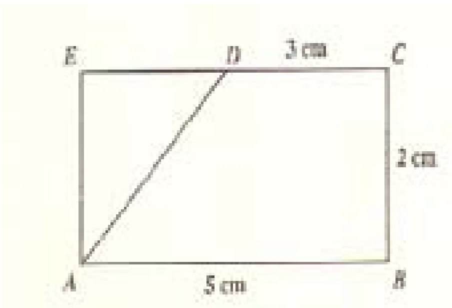
(A) 5 hrs

(B) $5\frac{3}{4}$ hrs

(C) 6 hrs

(D) $6\frac{1}{4}$ hrs

Item **13** refers to the trapezium below, **not drawn to scale**.



13. *ABCD* is a trapezium and *ADE* is a triangle. Angles *B*, *C* and *E* are right angles.

The area of the trapezium *ABCD* is

(A) 8 cm^2
(B) 16 cm^2
(C) 30 cm^2
(D) 32 cm^2

14. A circular hole with diameter 6 cm is cut from a circular piece of card with a diameter of 12 cm. The area of the remaining card, in cm^2, is

(A) 6π
(B) 27π
(C) 36π
(D) 108π

15. The mean of ten numbers is 58. If one of the numbers is 40, what is the mean of the other nine?

(A) 18
(B) 60
(C) 162
(D) 540

Items **16–17** refer to the table below which shows the distribution of the ages of 25 children in a choir.

Age	11	12	13	14	15	16
No. of children	6	3	5	4	4	3

16. What is the probability that a child chosen at random is AT LEAST 13 years old?

(A) $\frac{4}{25}$

(B) $\frac{9}{25}$

(C) $\frac{14}{25}$

(D) $\frac{16}{25}$

17. What is the mode of this distribution?

(A) 4
(B) 6
(C) 11
(D) 16

18. Seven times the product of two numbers, a and b, may be written as

(A) $7ab$
(B) $7a + b$
(C) $7a + 7b$
(D) $49\ ab$

19. If $2(x - 1) - 3x = 6$, then $x =$

(A) -8
(B) -4
(C) 4
(D) 8

20. $\frac{3x+1}{2} - \frac{x+1}{4} =$

(A) $\frac{7x+3}{4}$

(B) $\frac{7x+1}{4}$

(C) $\frac{5x+1}{4}$

(D) $\frac{5x+3}{4}$

21. The equation of the line which passes through the point $(0, 2)$ and has a gradient of $\frac{1}{3}$ is

(A) $y = 3x$

(B) $y = 3x + 2$

(C) $y = \frac{1}{3}x$

(D) $y = \frac{1}{3}x + 2$

22. If g is a function such that $g(x) = 2x + 1$, which of the following coordinates satisfies the function?

(A) $(-3, -5)$
(B) $(-6, 11)$
(C) $(5, 2)$
(D) $(13, 6)$

23. What is the gradient of a line which passes through the points $(-4, 3)$ and $(-2, 5)$?

(A) -4

(B) $\frac{-1}{3}$

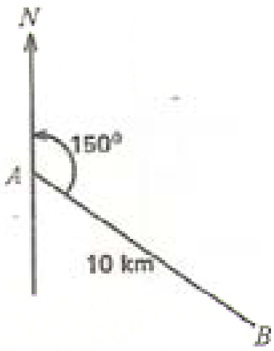
(C) $\frac{1}{3}$

(D) 1

24. If $f(x) = 2x - 3$ and $g(x) = 3x + 1$, then $fg(-2)$ is

(A) -13
(B) -7
(C) 5
(D) 20

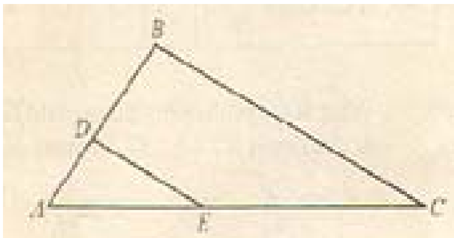
Item **25** refers to the diagram below, **not drawn to scale**.



25. A plane travels from point A on a bearing 150^0 to point B which is 10 km from A. How far east of A is B?

(A) $10 \tan 30^0$
(B) $10 \cos 30^0$
(C) $10 \cos 60^0$
(D) $10 \sin 60^0$

Item **26** refers to the diagram below, **not drawn to scale**.



26. Triangle ABC is an enlargement of triangle ADE such that

$$\frac{AD}{DB} = \frac{AE}{EC} = \frac{1}{2}$$

If the area $ABC = 36$ cm^2, then the area of $DECB$, in cm^2, is

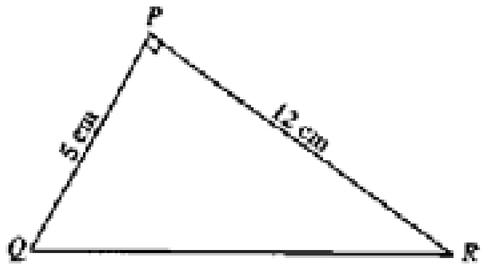
(A) 18
(B) 24
(C) 27
(D) 32

27. The point $P(2, -3)$ is rotated about the origin through an angle of 90° in an anti-clockwise direction.

What are the coordinates of the image P?

(A) (3, 2)
(B) (2, 3)
(C) (−3, 2)
(D) (3, −2)

<u>Item **28**</u> refers to the triangle PQR below, **not drawn to scale**.



28. If angle QPR = 90°, PR = 12 cm and PQ = 5 cm then the length of QR, in cm, is

(A) 7
(B) 11
(C) 13
(D) 17

29. Given that P and Q are points with coordinates $P(1, 3)$ and $Q(-1, 4)$, the position vector $\overrightarrow{PQ}$ is

(A) $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$

(B) $\begin{pmatrix} 0 \\ 7 \end{pmatrix}$

(C) $\begin{pmatrix} -2 \\ 1 \end{pmatrix}$

(D) $\begin{pmatrix} 2 \\ -1 \end{pmatrix}$

30. The transformation matrix $Q = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$ represents

(A) a 180° rotation about (0, 2)
(B) a reflection in the line $x = 2$
(C) a reflection in the line $y = 2$
(D) an enlargement by scale factor 2

IF YOU FINISH BEFORE TIME IS CALLED, CHECK YOUR WORK ON THIS TEST.

CARIBBEAN EXAMINATIONS COUNCIL

SECONDARY EDUCATION CERTIFICATE

MATHEMATICS

SPECIMEN PAPER 01

Item Number	Specific Objective	Key
1	Number Theory and Computation 9(a)	B
2	Number Theory and Computation 11	C
3	Number Theory and Computation 2	D
4	Consumer Arithmetic 6	B
5	Consumer Arithmetic 10 (e)	C
6	Consumer Arithmetic 4	A
7	Consumer Arithmetic 10 (b)	D
8	Sets 1	B
9	Sets 7	D
10	Sets 4	B
11	Measurement 1	A
12	Measurement 12	B
13	Measurement 6	A
14	Measurement 6	B
15	Statistics 5	B
16	Statistics 13	D
17	Statistics 5	C
18	Algebra 2	A
19	Algebra 10	A
20	Algebra 8	C
21	Relations, Functions & Graphs 9	D
22	Relations, Functions & Graphs 4	A
23	Relations, Functions & Graphs 8	D
24	Relations, Functions & Graphs 19	A
25	Geometry & Trigonometry 13	C
26	Geometry & Trigonometry 5 (c)	D
27	Geometry & Trigonometry 11 (c)	A
28	Geometry & Trigonometry 12	C
29	Vectors & Matrices 3	C
30	Vectors & Matrices 12	D

TEST CODE **01234020/SPEC**

FORM TP 01234020/SPEC

C A R I B B E A N   E X A M I N A T I O N S   C O U N C I L

CARIBBEAN SECONDARY EDUCATION CERTIFICATE® EXAMINATION

MATHEMATICS

SPECIMEN PAPER

Paper 02 – General Proficiency

2 hours 40 minutes

READ THE FOLLOWING INSTRUCTIONS CAREFULLY.

1. This paper consists of TWO sections: I and II.
2. Section I has SEVEN questions and Section II has THREE questions.
3. Answer ALL questions from the TWO sections.
4. Write your answers in the booklet provided.
5. Do NOT write in the margins.
6. All working must be clearly shown.
7. Unless otherwise stated in the question, any numerical answer that is not exact MUST be written to THREE significant figures.
8. A list of formulae is provided on page 4 of this booklet.
9. If you need to rewrite any answer and there is not enough space to do so on the original page, you must use the extra page(s) provided at the back of this booklet. **Remember to draw a line through your original answer**.
10. **If you use the extra page(s) you MUST write the question number clearly in the box provided at the top of the extra page(s) and, where relevant, include the question part beside the answer.**

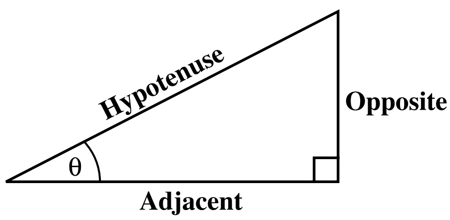
Examination Materials Permitted

Silent, non-programmable electronic calculator
Mathematical instruments

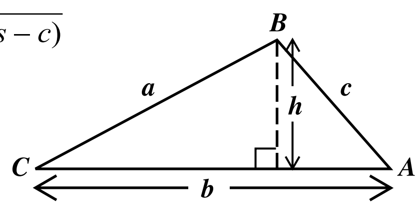
DO NOT TURN THIS PAGE UNTIL YOU ARE TOLD TO DO SO.

LIST OF FORMULAE

Volume of a prism	$V = Ah$ where A is the area of a cross-section and h is the perpendicular length.
Volume of cylinder	$V = \pi r^2 h$ where r is the radius of the base and h is the perpendicular height.
Volume of a right pyramid	$V = \frac{1}{3} Ah$ where A is the area of the base and h is the perpendicular height.
Circumference	$C = 2\pi r$ where r is the radius of the circle.
Arc length	$S = \frac{\theta}{360} \times 2\pi r$ where θ is the angle subtended by the arc, measured in degrees.
Area of a circle	$A = \pi r^2$ where r is the radius of the circle.
Area of a sector	$A = \frac{\theta}{360} \times \pi r^2$ where θ is the angle of the sector, measured in degrees.
Area of trapezium	$A = \frac{1}{2}(a + b)\,h$ where a and b are the lengths of the parallel sides and h is the perpendicular distance between the parallel sides.
Roots of quadratic equations	If $ax^2 + bx + c = 0$, then $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2\text{a}}$
Trigonometric ratios	$\sin\theta = \dfrac{\text{opposite side}}{\text{hypotenuse}}$ $\cos\theta = \dfrac{\text{adjacent side}}{\text{hypotenuse}}$ $\tan\theta = \dfrac{\text{opposite side}}{\text{adjacent side}}$



Area of triangle	Area of $\Delta = \frac{1}{2} bh$ where b is the length of the base and h is the perpendicular height. Area of $\Delta\, ABC = \frac{1}{2} ab \sin C$ Area of $\Delta\, ABC = \sqrt{s(s-a)(s-b)(s-c)}$ where $s = \dfrac{a+b+c}{2}$
Sine rule	$\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$
Cosine rule	$a^2 = b^2 + c^2 - 2bc \cos A$



SECTION I

Answer ALL questions in this section.

All working must be clearly shown.

1. (a) Using a calculator, determine the value of

$$\frac{(3.29)^2 - 5.5}{\sqrt{1.5 \times 0.06}}$$

giving your answer to 1 decimal place. **(3 marks)**

(b) The table below shows rates of exchange.

US\$1.00 = TT\$6.45
BBD\$1.00 = TT\$3.00

(i) Using the table, calculate the amount in BBD dollars equivalent to US\$1.00. **(2 marks)**

(ii) Gail exchanged BBD\$1806.00 for US dollars. Calculate the amount she received in US dollars. **(1 mark)**

(c) The cash price of a laptop is \$4799.00. It can be bought on hire purchase by making a deposit of \$540.00 and 12 monthly instalments of \$374.98 EACH.

(iii) Calculate the TOTAL hire purchase price of the laptop. **(2 marks)**

(iv) Calculate the amount saved by purchasing the laptop at the cash price. **(1 mark)**

Total 9 marks

GO ON TO THE NEXT PAGE

2. (a) Simplify: $p^3q^2 \times pq^5$ **(1 mark)**

(b) If $a * b = 2a - 5b$, calculate the value of

(i) 3 * 4 **(1 mark)**

(ii) (3 * 4) * 1 **(1 mark)**

(c) Factorize completely: $3x + 6y - x^2 - 2xy$ **(2 marks)**

(d) A string of length 14 cm is cut into two pieces. The length of the first piece is x cm. The second piece is 5 cm **longer** than half the length of the first piece.

(i) State in terms of x, the length of the second piece of string. **(1 mark)**

(ii) Write an expression, in terms of x, to represent the TOTAL length of the two pieces of string. **(1 mark)**

(iii) Hence, calculate the length of the first piece of string. **(2 marks)**

Total 9 marks

3. (a) Using a ruler, a pencil and a pair of compasses, construct a rhombus, PQRS, in which PR = 6 cm and RPQ = 60°. **(4 marks)**

(b) **Use the graph on page 5 to answer this question.**

The diagram shows triangle JKL and its image J´K´L´ after an enlargement.

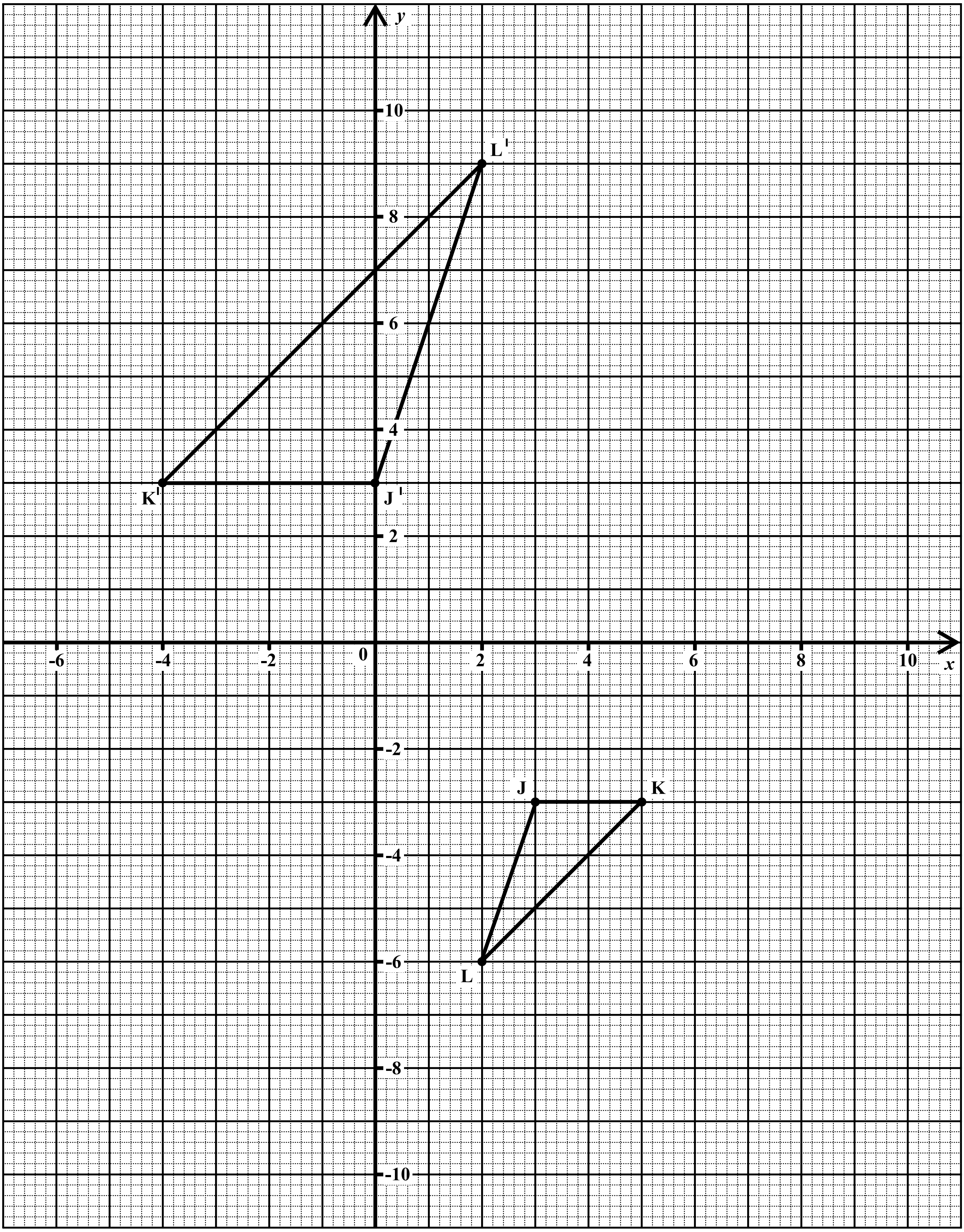
(i) Draw lines on your diagram to locate the point G, the centre of the enlargement. **(1 mark)**

(ii) State the coordinates of the point G. **(1 mark)**

(iii) State the scale factor of the enlargement. **(1 mark)**

(iv) On your diagram, show the point J´´, the image of the point J´, after a reflection in the line $x = 4$. **(2 marks)**

Total 9 marks


y
10
8
6
4
2
-2
-4
-6
-8
-10
-6
-4
-2
0
2
4
6
8
10
x
L'
K'
J'
J
K
L

4. (a) The line BC passes through the point A(−5, 3) and has a gradient of $\frac{2}{5}$.

(i) Write the equation of the line BC in the form $y = mx + c$. **(2 marks)**

(ii) Determine the equation of the line which passes through the origin and is perpendicular to the line BC. **(2 marks)**

(b) The functions f and g are defined as:

$f(x) = \dfrac{2x-1}{x+3}$ $\qquad\qquad$ $g(x) = 4x - 5$

(i) Determine f g(3). **(2 marks)**

(ii) Derive an expression for $f^{-1}(x)$. **(3 marks)**

Total 9 marks

5. **A graph sheet is provided for this question.** The table below shows the time spent, to the nearest minute, by 25 students at the school canteen.

	Time spent at the bookstore (minutes)	Frequency	Cumulative Frequency
	6–10	2	2
	11–15	4	6
	16–30	5	11
(i)	21–25	_____	20
(ii)	26–30	4	_____
	31–35	1	25

(a) Complete the rows numbered (i) and (ii). **(2 marks)**

(b) On your graph on page 7, draw a cumulative frequency curve of the time spent at the canteen, using a scale of **2 cm to represent 5 minutes** on the horizontal axis and **2 cm to represent 5 students** on the vertical axis. **(4 marks)**

(c) **Use the graph drawn at (b)** to estimate

(i) the median time spent at the canteen **(1 mark)**

(ii) the probability that a student selected at random spent LESS than 24 minutes at the canteen. **(2 marks)**

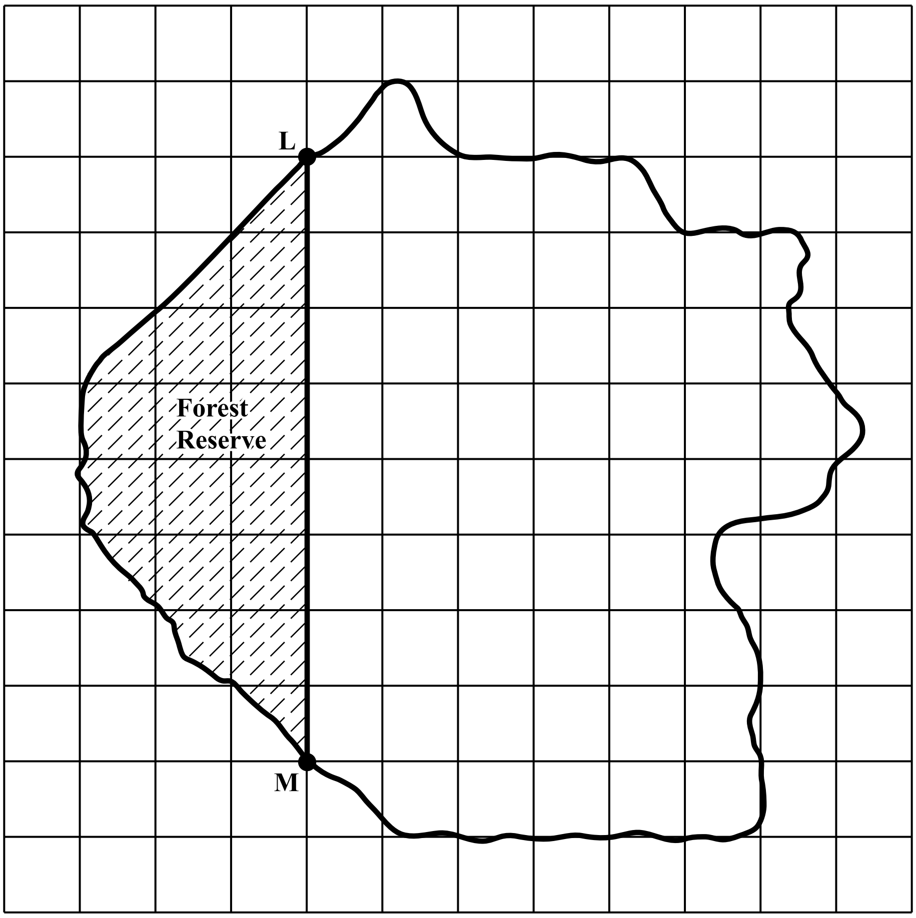
Total 9 marks

6. (a) In this question, use $\pi = \frac{22}{7}$.

(i) A piece of wire is bent to form a square of area 121 cm^2. Calculate the perimeter of the square. **(2 marks)**

(ii) The same piece of wire is bent to form a circle. Calculate the radius of the circle. **(2 marks)**

(b) The diagram below shows a map of an island drawn on a grid of 1 cm squares. The map is drawn to a scale of 1:50 000.



(i) L and M are two tracking stations. State, in centimetres, the distance LM on the map. **(1 mark)**

(ii) Calculate the ACTUAL distance, in kilometres, from L to M on the island. **(2 marks)**

(iii) Calcuate the ACTUAL area, in km^2, of the forest reserve, given that 1×10^{10} cm^2 = 1 km^2. **(2 marks)**

Total 9 marks

GO ON TO THE NEXT PAGE

7. The table below represents the calculation of the sum of the cubes of the first **n** natural numbers. Information is missing from some rows of the table.

(a) Study the pattern in the table and complete the rows marked (i), (ii) and (iii).

	n	**Series**	**Sum**	**Formula**
	1	1^3	1	$\frac{1^2}{4}(1+1)^2$
	2	$1^3 + 2^3$	9	$\frac{2^2}{4}(1+2)^2$
	3	$1^3 + 2^3 + 3^3$	36	$\frac{3^2}{4}(1+3)^2$
	4	$1^3 + 2^3 + 3^3 + 4^3$	100	$\frac{4^2}{4}(1+4)^2$
(i)	5	....	....	....
	6	$1^3 + 2^3 + 3^3 + 4^3 + 5^3 + 6^3$	441	$\frac{6^2}{4}(1+6)^2$
(ii)	....		....	$\frac{8^2}{4}(1+8)^2$
(iii)	**n**			....

(6 marks)

(b) It was further noted that:

$1 + 2 = 3 = \sqrt{9}$

$1 + 2 + 3 = 6 = \sqrt{36}$

$1 + 2 + 3 + 4 = 10 = \sqrt{100}$

Using information from the table above and the pattern in the three statements above, determine

(i) the value of x for which $1 + 2 + 3 + 4 + 5 + 6 = \sqrt{x}$ **(2 marks)**

(ii) a formula in terms of **n** for the series: $1 + 2 + 3 + 4 + ... + n$ **(2 marks)**

Total 10 marks

GO ON TO THE NEXT PAGE

SECTION II

Answer ALL questions in this section.

All working must be clearly shown.

8. (a) Solve the pair of simultaneous equations:

$$x^2 = 4 - y$$

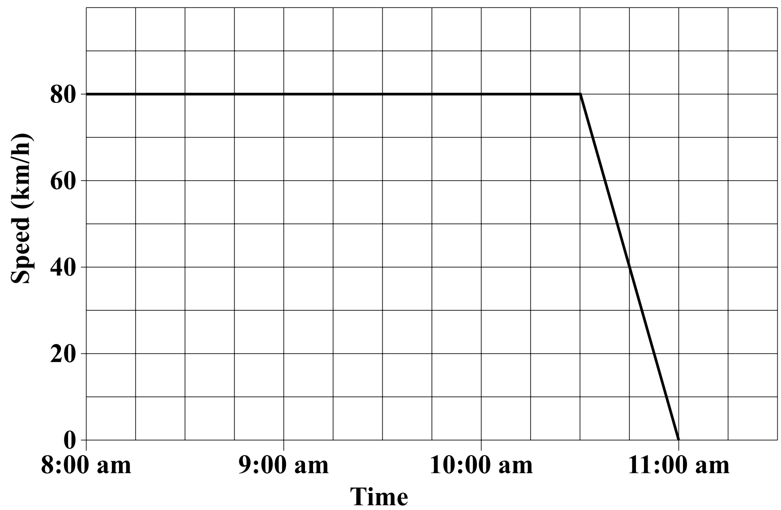
$$x = y + 2$$ **(4 marks)**

(b) (i) Express $3x^2 + 2x + 1$ in the form $a(x + p)^2 + q$ where a, p and q are real numbers. **(2 marks)**

(ii) Hence, determine for $f(x) = 3x^2 + 2x + 1$

- the minimum value for $f(x)$
- the equation of the axis of symmetry. **(2 marks)**

(c) The speed–time graph below shows the journey of a car from 8:00 a.m. to 11:00 a.m.



Using the graph, determine

(i) the time at which the speed of the car was 40 km/h **(1 mark)**

(ii) the TOTAL distance the car travelled for the entire journey **(2 marks)**

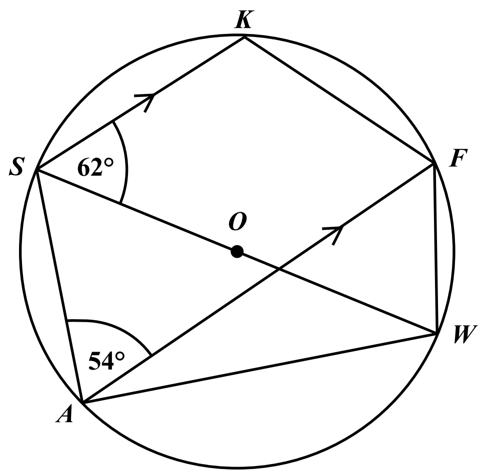
(iii) the average speed of the car for the entire journey. **(1 mark)**

Total 12 marks

GO ON TO THE NEXT PAGE

9. (a) In the diagram below, **not drawn to scale**, O is the centre of the circle. The lines SK and AF are parallel.

$\angle KSW = 62°$ $\angle SAF = 54°$



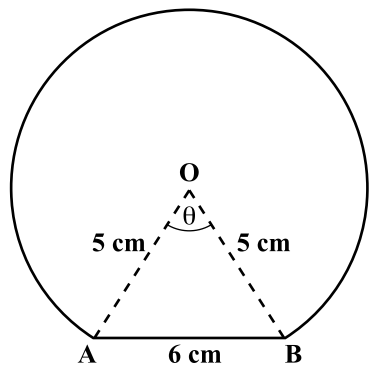
Calculate, **giving reasons for your answer**, the measure of:

(i) $\angle FAW$ **(2 marks)**

(ii) $\angle SKF$ **(2 marks)**

(iii) $\angle ASW$ **(2 marks)**

(b) A machine produces circular discs of diameter 10 cm. The machine malfunctions and cuts a disc to produce the shape in the figure below, **not drawn to scale**, with centre, O.



Use $\pi = 3.14$

Determine

(i) the measure of angle θ **(2 marks)**

(ii) the area of triangle AOB **(2 marks)**

(iii) the area of the disc that was cut off. **(2 marks)**

Total 12 marks

10. (a) The vertices of a quadrilateral, OPQR are (0,0), (4,2), (6,10) and (2,8) respectively.

(i) **Using a vector method**, express in the form $\begin{pmatrix} x \\ y \end{pmatrix}$ the vectors

- $\overrightarrow{OP}$
- $\overrightarrow{RQ}$ **(2 marks)**

(ii) Calculate $|\overrightarrow{OP}|$, the magnitude of $\overrightarrow{OP}$. **(2 marks)**

(iii) State ONE geometrical relationship between the line segments OP and RQ. **(2 marks)**

(b) The matrix, K, maps the point S (1,4) onto S′ (−4,−1) and the point T (3,5) onto T′ (–5,–3).

Given that K = $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$,

(i) express as a matrix equation, the relationship between K, S, S′, T and T′. **(2 marks)**

(ii) hence, determine the values of a, b, c and d. **(3 marks)**

(iii) describe COMPLETELY the geometric transformation which is represented by the matrix K. **(2 marks)**

Total 12 marks

END OF TEST

IF YOU FINISH BEFORE TIME IS CALLED, CHECK YOUR WORK ON THIS TEST.

CARIBBEAN EXAMINATIONS COUNCIL
SECONDARY EDUCATION CERTIFICATE EXAMINATION
MATHEMATICS

Specimen Paper

Table of Specifications

Ques.	Specific Objectives	P1	P2	P3	Total
1	Number Theory and Computation: 2, 3, 9 (b) Consumer Arithmetic: 5, 9	4	3	2	**9**
2	Algebra: 2, 6, 9, 10	3	4	2	**9**
3	Geometry and Trigonometry: 2, 3, 8 (a), 10	3	3	3	**9**
4	Relations, Functions & Graphs: 9 (c), 10, 18, 19	3	3	3	**9**
5	Statistics: 2, 5, 7, 9, 11	3	3	3	**9**
6	Measurement: 2, 3, 14, 15	2	4	3	**9**
7	Investigation	3	2	5	**10**
8	Algebra: 15, 18 Relations, Functions & Graphs: 22, 25, 26	3	6	3	**12**
9	Geometry and Trigonometry: 6, 15, 16,	3	6	3	**12**
10	Vectors & Matrices: 3, 4, 11, 12, 13	3	6	3	**12**
	TOTAL	**30**	**40**	**30**	**100**

01234020/SPECIMEN PAPER/MS/2015

C A R I B B E A N   E X A M I N A T I O N S   C O U N C I L

CARIBBEAN SECONDARY EDUCATION CERTIFICATE®

EXAMINATION

MATHEMATICS

PAPER 02 – GENERAL PROFICIENCY

MARK SCHEME

SPECIMEN PAPER 2015

MATHEMATICS

PAPER 02 - GENERAL PROFICIENCY

SPECIMEN

MARK SCHEME

QUESTION: 1

	PROFILES			Total
	K	C	R	
(a) $\frac{(3.29)^2 - 5.5}{\sqrt{1.5 \times 0.06}}$				
$= \frac{10.8241 - 5.5}{0.3}$	1			
= 17.747	1			
= 17.7 to 1 decimal place	1			
	3	**–**	**–**	**3**
(b) (i) $\text{US } \$ \ 1.00 = \text{BD } \$ \left(\frac{1.00}{3.00}\right) \times 6.45$			1	
= BD \$ 2.15	1			
(ii) Amount Gail received				
$= \text{US } \$ \left(\frac{1806.00}{2.15}\right)$		1		
= US \$ 840.00				
	1	**1**	**1**	**3**
(c) (i) Cash price on laptop = \$4799.00				
Deposit = \$540.00				
Total Instalments = \$374.98 × 12				
= \$4499.76		1		
The total hire purchase price of the laptop				
= \$ 540.00 + \$ 4499.76			1	
= \$5039.76				
(ii) The amount saved by buying the laptop at the cash price				
= \$5039.76 − \$4799.00		1		
= \$ 240.76	**–**	**2**	**1**	**3**
TOTAL	**4**	**3**	**2**	**9**

MATHEMATICS

PAPER 02 – GENERAL PROFICIENCY

SPECIMEN

MARK SCHEME

QUESTION: 2

	PROFILES K	PROFILES C	PROFILES R	Total
(a) $p^3q^2 \times pq^5 = p^4q^7$	1			
	1	**–**	**–**	**1**
(b) $a * b = 2a - 5b$ (i) $3 * 4 = 2(3) - 5(4)$ $= 6 - 20$ $= -14$ (ii) $(3 * 4) * 1 = (-14) * 1$ $= 2(-14) - 5(1)$ $= -28 - 5$ $= -33$		1	1	
	–	**1**	**1**	**2**
(c) $3x + 6y - x^2 - 2xy$ $= 3(x + 2y) - x(x + 2y)$ $= (3 - x)(x + 2y)$	1	1		
	1	**1**	**–**	**2**
(d) (i) Length of 2nd piece $= \frac{1}{2}x + 5$ (ii) Sum of two lengths $= x + \frac{1}{2}x + 5$ $= \frac{3}{2}x + 5$ (iii) $= \frac{3}{2}x + 5 = 14$ $= \frac{3}{2}x = 9$ $x = 6$ Length of 1st piece is 6 cm	1	1 1	1	
	1	**2**	**1**	**4**
TOTAL	**3**	**4**	**2**	**9**

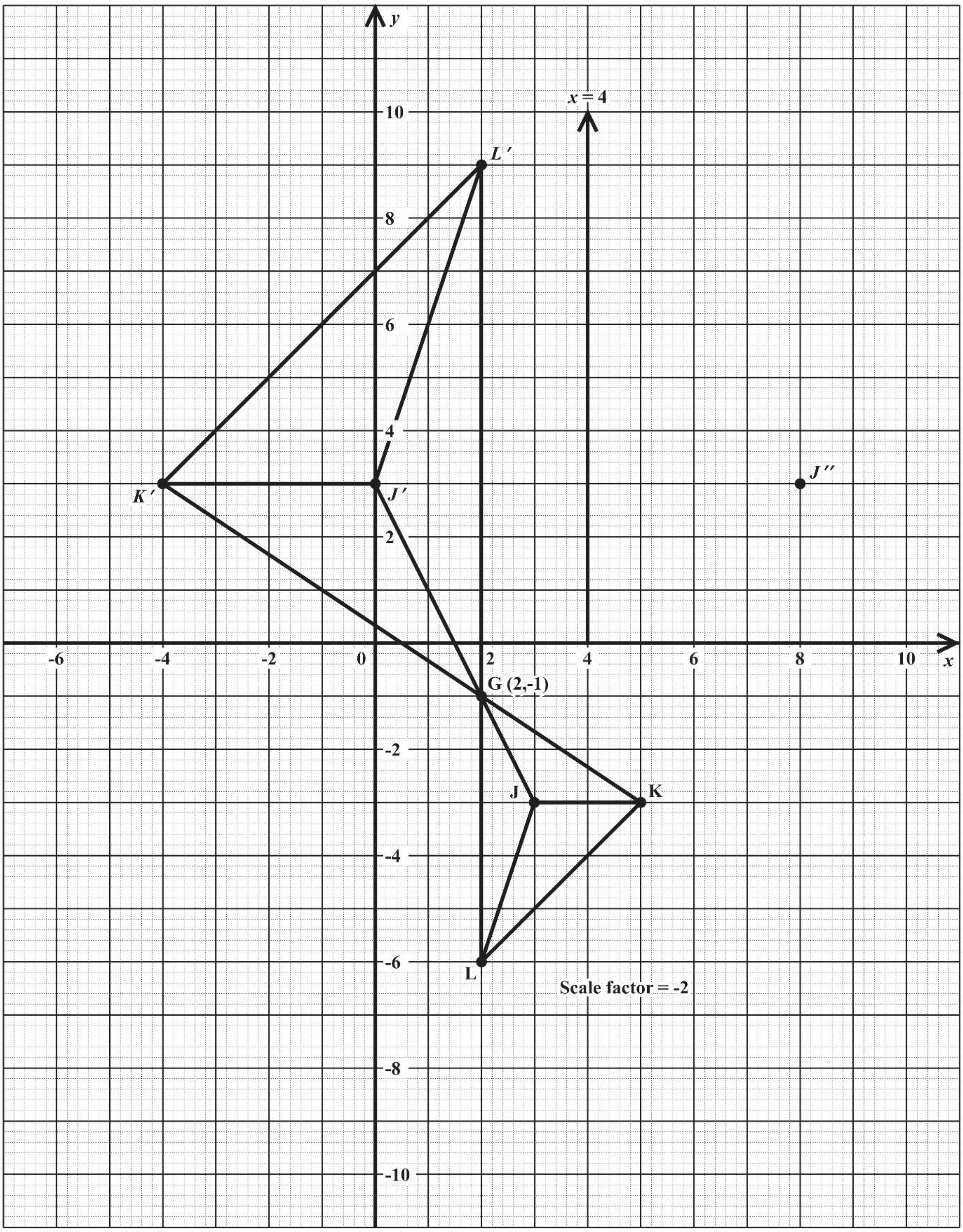
MATHEMATICS

PAPER 02 – GENERAL PROFICIENCY

SPECIMEN

MARK SCHEME

QUESTION: 3



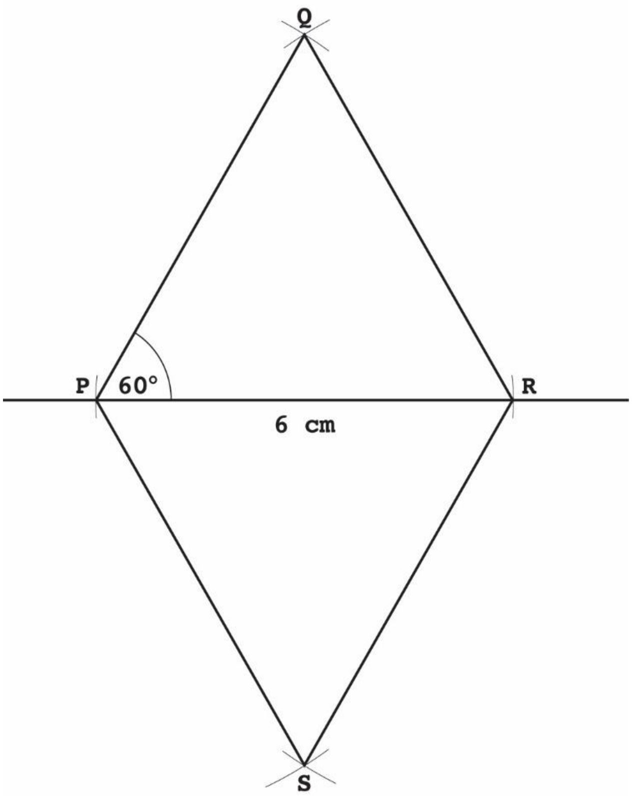
MATHEMATICS

PAPER 02 – GENERAL PROFICIENCY

SPECIMEN

MARK SCHEME

QUESTION: 3

	PROFILES K	PROFILES C	PROFILES R	Total
(a)				
PR = 6cm			1	
∠ RPQ = 60°		1		
Locating Q		1		
Locating S	1			
 [Not drawn to scale]				
	1	2	1	4

MATHEMATICS

PAPER 02 - GENERAL PROFICIENCY

SPECIMEN

MARK SCHEME

				PROFILES			Total
				K	C	R	
(b)	(i)	Any 2 of KK, JJ and LL	R1			**1**	
	(ii)	G (2, −1)	K1	**1**			
	(iii)	Scale factor = −2	R1			**1**	
	(iv)	a) $x = 4$ drawn (soi)	K1	**1**			
		b) J” at (8, 3)	C1		**1**		
				2	**1**	**2**	**5**
TOTAL				**3**	**3**	**3**	**9**

MATHEMATICS

PAPER 02 – GENERAL PROFICIENCY

SPECIMEN

MARK SCHEME

QUESTION: 4

	PROFILES K	C	R	Total
(a) A(−5, 3); m = $\frac{2}{5}$				
(i) Equation of line BC is $y - 3 = \frac{2}{5}(x + 5)$		**1**		
i.e. $y = \frac{2}{5}x + 5$		**1**		
(ii) Gradient of line perpendicular to BC is $-\frac{5}{2}$	**1**			
Equation of line through (0, 0) is $y = -\frac{5}{2}x$			**1**	
	1	**2**	**1**	**4**
(b) $f(x) = \frac{2x-1}{x+3}$; $g(x) = 4x - 5$				
(i) $g(3) = 12 - 5 = 7$	**1**			
$\therefore\ fg(3) = \frac{14 - 1}{7 + 3} = \frac{13}{10}$			**1**	
(ii) $\frac{2(f^{-1})-1}{(f^{-1})+3} = x$			**1**	
$2(f^{-1}) - 1 = xf^{-1} + 3x$		**1**		
$f^{-1}(2 - x) = 3x + 1$				
$f^{-1} = \frac{3x + 1}{2 - x}$	**1**			
	2	**1**	**2**	**5**
TOTAL	**3**	**3**	**3**	**9**

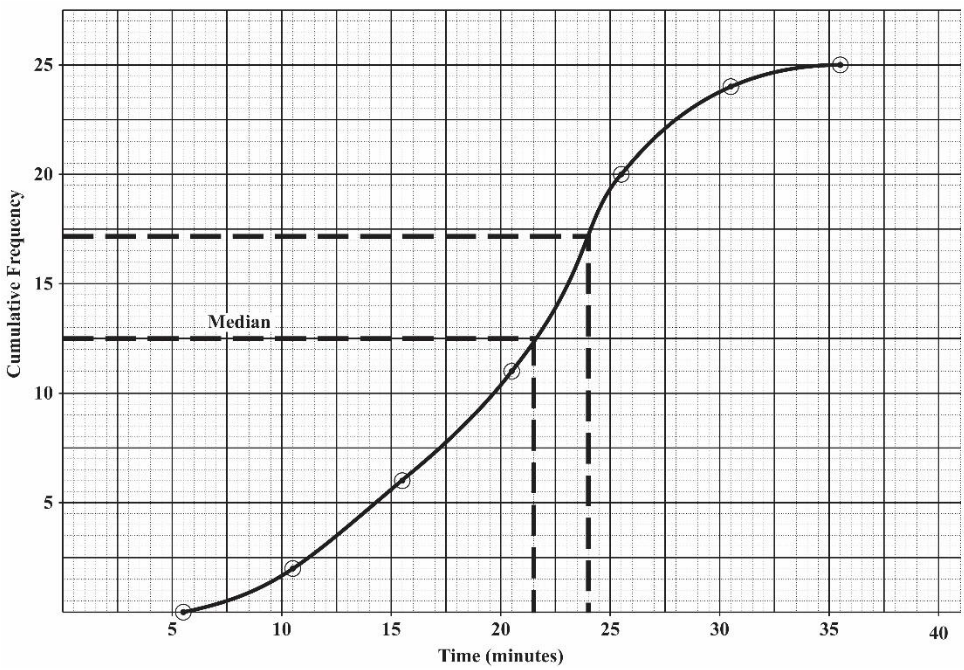
MATHEMATICS

PAPER 02 - GENERAL PROFICIENCY

SPECIMEN

MARK SCHEME

QUESTION: 5

	PROFILES K	PROFILES C	PROFILES R	Total
(a) Missing frequency = 9 Missing cumulative frequency = 24	1 1			
	2			**2**
(b)  Correct scale on both axes 3 or more points plotted correctly Plotting at class boundary Smooth ogive	1 	 1 1 	 1	
	1	**2**	**1**	**4**

MATHEMATICS

PAPER 02 – GENERAL PROFICIENCY

SPECIMEN

MARK SCHEME

	K	C	R	Total
(c) (i) Median time spent = 21.5 minutes			**1**	
(d) (ii) P (time spent < 24) = $\frac{17}{25}$		**1**	**1**	
		1	**2**	**3**
TOTAL	**3**	**3**	**3**	**9**

MATHEMATICS

PAPER 02 - GENERAL PROFICIENCY

SPECIMEN

MARK SCHEME

QUESTION: 6

	PROFILES K	C	R	Total
(a) (i) Area of square = 121 cm^2 a) ∴ length of side = $\sqrt{121}$ cm = 11 cm		1		
b) Perimeter of square = 4 × 11 cm = 44 cm	1			
(ii) Perimeter of circle = 44 cm i.e. $2\pi R = 44$ cm $R = \frac{44}{2\pi}$ cm = 7 cm		1	1	
	1	**2**	**1**	**4**
(b) (i) LM = 8 cm on map	1			
(ii) Actual distance in km $= \frac{8 x 50\,000}{100\,000}$ km = 4 km		1	1	
(iii) Area of forest reserve on map ≈ 15 cm^2 Actual area in km^2 $= \frac{15 x (50\,000)^2}{1 x 10^{10}}$ km^2 = 3.75 km^2		1	1	
	1	**2**	**2**	**5**
TOTAL	**2**	**4**	**3**	**9**

MATHEMATICS

PAPER 02 – GENERAL PROFICIENCY

SPECIMEN

MARK SCHEME

QUESTION: 7

<table>
<tr><th rowspan="2"></th><th colspan="3">PROFILES</th><th rowspan="2">Total</th></tr>
<tr><th>K</th><th>C</th><th>R</th></tr>
<tr><td>(a)

	n	Series	Sum	Formula
(i)	5	$1^3+2^3+3^3+4^3+5^3$	225	$\frac{5^2}{4}(1+5)^2$
(ii)	8		1296	$\frac{8^2}{4}(1+8)^2$
(iii)	n			$\frac{n^2}{4}(1+n)^2$

</td><td>1
1</td><td>1
1</td><td>1

1</td><td></td></tr>
<tr><td></td><td>2</td><td>2</td><td>2</td><td>6</td></tr>
<tr><td>(b) (i) $1 + 2 + 3 + 4 + 5 + 6 = \sqrt{x}$
$\Rightarrow x = (1 + 2 + 3 + 4 + 5 + 6) = 21$
$= 21^2$
$= 441$

(ii) $1 + 2 + 3 + 4 + \ldots + n = \sqrt{\frac{n^2}{4} + (1+n)^2}$
$= \frac{n}{2}(1 + n)$</td><td>1</td><td></td><td>1
1
1</td><td></td></tr>
<tr><td></td><td>1</td><td></td><td>3</td><td>4</td></tr>
<tr><td>TOTAL</td><td>3</td><td>2</td><td>5</td><td>10</td></tr>
</table>

MATHEMATICS

PAPER 02 - GENERAL PROFICIENCY

SPECIMEN

MARK SCHEME

QUESTION: 8

	PROFILES K	PROFILES C	PROFILES R	Total
(a) $x^2 = 4 - y \ldots (1) \quad x = y + 2 \ldots (2)$			1	
From (2): $y = x - 2$				
$\therefore x^2 = 4 - (x - 2)$				
$x^2 = 4 - x + 2$		1		
$\Rightarrow x^2 + x - 6 = 0$				
$(x + 3)(x - 2) = 0$		1		
$\Rightarrow x = -3, 2$				
when $x = -3$, $y = -5$				
when $x = 2$, $y = 0$	1			
	1	2	1	4
(b) (i) $3x^2 + 2x + 1 = 3\left(x^2 + \frac{2}{3}x\right) + 1$				
$= 3\left[\left(x + \frac{1}{3}\right)^2 - \frac{1}{9}\right] + 1$		1		
$= 3\left(x + \frac{1}{3}\right)^2 - \frac{1}{3} + 1$				
$= 3\left(x + \frac{1}{3}\right)^2 + \frac{2}{3}$	1			
(ii) For $f(x) = 3x^2 + 2x + 1$		1		
(a) $f(x)_{\min} = \frac{2}{3}$			1	
(b) Axis of symmetry is $x = -\frac{1}{3}$				
	1	2	1	4
(c) (i) Speed was 40 km/h at 10:45 am		1		
(ii) Distance travelled $= \frac{1}{2}(3 + 2.5)(80)$ km $= 220$ km	1		1	
(iii) Average speed $= \frac{220}{3}$ km/h $= 73\frac{1}{3}$ km/h		1		
	1	2	1	4
TOTAL	**3**	**6**	**3**	**12**

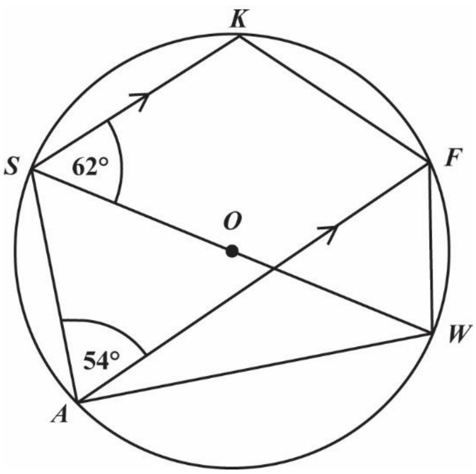
MATHEMATICS

PAPER 02 - GENERAL PROFICIENCY

SPECIMEN

MARK SCHEME

QUESTION: 9

	PROFILES K	PROFILES C	PROFILES R	Total
(a) (i) 				
$F\hat{A}W = 90° - 54° = 36°$	1			
Use of 90° (angle in a semicircle)			1	
(ii) $S\hat{K}F = 180° - 54° = 126°$		1		
Use of 180° (opposite angles of cyclic quad)		1		
(iii) $A\hat{S}W = 180° - (62° + 54°) = 64°$		1		
(co-interior angles)			1	
	1	3	2	6

MATHEMATICS

PAPER 02 - GENERAL PROFICIENCY

SPECIMEN

MARK SCHEME

<table>
<tr><th rowspan="2"></th><th colspan="3">PROFILES</th><th rowspan="2">Total</th></tr>
<tr><th>K</th><th>C</th><th>R</th></tr>
<tr><td>(b) (i) $6^2 = 5^2 + 5^2 - 2 \times 5 \times 5 \cos \theta$
$\cos Q = \frac{14}{50}$
$\therefore \quad Q = 73.7°$

(ii) Area of $\Delta AOB = \frac{1}{2}(5 \times 5) \sin 73.7°$
$= 12 \text{ cm}^2$

(iii) Area of sector $AOB = \frac{73.7}{360} \pi (5^2)$
$= 16.1 \text{ cm}^2$
$\therefore$ Area of segment removed
$= 16.1 \text{ cm}^2 - 12 \text{ cm}^2 = 4.1 \text{ cm}^2$</td><td>1
1</td><td>1
1
1</td><td>1</td><td></td></tr>
<tr><td></td><td>2</td><td>3</td><td>1</td><td>6</td></tr>
<tr><td>TOTAL</td><td>3</td><td>6</td><td>3</td><td>12</td></tr>
</table>

MATHEMATICS

PAPER 02 – GENERAL PROFICIENCY

SPECIMEN

MARK SCHEME

QUESTION: 10

	PROFILES K	C	R	Total
(a) (i) OP = $\begin{pmatrix}4\\2\end{pmatrix}$	1			
RQ = $\begin{pmatrix}6\\10\end{pmatrix} - \begin{pmatrix}2\\8\end{pmatrix} = \begin{pmatrix}4\\2\end{pmatrix}$		1		
(ii) $\lvert OP \rvert = \sqrt{4^2+2^2} = \sqrt{20}$	1	1		
(iii) *OP* and *RQ* either – have the same length or – are parallel			1	
	2	2	1	5
(b) (i) $\begin{pmatrix}a & b\\c & d\end{pmatrix}\begin{pmatrix}1 & 3\\4 & 5\end{pmatrix} = \begin{pmatrix}-4 & -5\\-1 & -3\end{pmatrix}$	1		1	
(ii) det of $\begin{pmatrix}1 & 3\\4 & 5\end{pmatrix}$ = 5 – 12 = -7		1		
Inverse of $\begin{pmatrix}1 & 3\\4 & 5\end{pmatrix} = \frac{1}{7}\begin{pmatrix}5 & -3\\-4 & 1\end{pmatrix}$				
$\therefore \begin{pmatrix}a & b\\c & d\end{pmatrix} = -\frac{1}{7}\begin{pmatrix}-4 & -5\\-1 & -3\end{pmatrix}\begin{pmatrix}5 & -3\\-4 & 1\end{pmatrix}$				
$= -\frac{1}{7}\begin{pmatrix}0 & 7\\7 & 0\end{pmatrix}$		1	1	
$= \begin{pmatrix}0 & -1\\-1 & 0\end{pmatrix}$				
[Complete matrix C2 R1 *b* or *c* only C1 each]				
(iii) The matrix $K = \begin{pmatrix}0 & -1\\-1 & 0\end{pmatrix}$				
Representing a reflection in the line		1		
$y = -x$		1		
	1	4	2	7
TOTAL	**3**	**6**	**3**	**12**

TEST CODE **01234032/SPEC**

FORM TP 01234032/SPEC

C A R I B B E A N   E X A M I N A T I O N S   C O U N C I L

CARIBBEAN SECONDARY EDUCATION CERTIFICATE® EXAMINATION

MATHEMATICS

SPECIMEN PAPER

Paper 032 – General Proficiency

2 hours 40 minutes

READ THE FOLLOWING INSTRUCTIONS CAREFULLY.

1. This paper consists of TWO questions.
2. Answer ALL questions.
3. Write your answers in the booklet provided.
4. Do NOT write in the margins.
5. All working MUST be clearly shown.
6. **A list of formulae is provided on page 2 of this booklet.**
7. If you need to rewrite any answer and there is not enough space to do so on the original page, you must use the extra page(s) provided at the back of this booklet. **Remember to draw a line through your original answer**.
8. **If you use the extra page(s) you MUST write the question number clearly in the box provided at the top of the extra page(s) and, where relevant, include the question part beside the answer.**

Examination Materials Permitted

Silent, non-programmable electronic calculator
Mathematical instruments

DO NOT TURN THIS PAGE UNTIL YOU ARE TOLD TO DO SO.

LIST OF FORMULAE

Volume of a prism — $V = Ah$ where A is the area of a cross-section and h is the perpendicular length.

Volume of cylinder — $V = \pi r^2 h$ where r is the radius of the base and h is the perpendicular height.

Volume of a right pyramid — $V = \frac{1}{3}Ah$ where A is the area of the base and h is the perpendicular height.

Circumference — $C = 2\pi r$ where r is the radius of the circle.

Arc length — $S = \frac{\theta}{360} \times 2\pi r$ where θ is the angle subtended by the arc, measured in degrees.

Area of a circle — $A = \pi r^2$ where r is the radius of the circle.

Area of a sector — $A = \frac{\theta}{360} \times \pi r^2$ where θ is the angle of the sector, measured in degrees.

Area of trapezium — $A = \frac{1}{2}(a + b)\, h$ where a and b are the lengths of the parallel sides and h is the perpendicular distance between the parallel sides.

Roots of quadratic equations — If $ax^2 + bx + c = 0$,

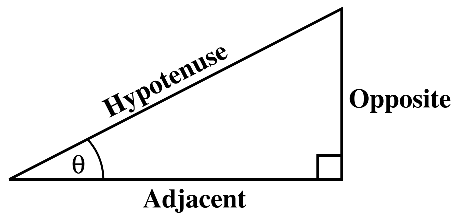
then $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Trigonometric ratios

$\sin\theta = \dfrac{\text{opposite side}}{\text{hypotenuse}}$

$\cos\theta = \dfrac{\text{adjacent side}}{\text{hypotenuse}}$

$\tan\theta = \dfrac{\text{opposite side}}{\text{adjacent side}}$

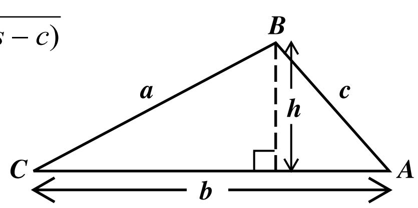


Area of triangle — Area of $\Delta = \frac{1}{2}bh$ where b is the length of the base and h is the perpendicular height.

Area of $\Delta\, ABC = \frac{1}{2}ab\sin C$

Area of $\Delta\, ABC = \sqrt{s(s-a)(s-b)(s-c)}$

where $s = \dfrac{a+b+c}{2}$

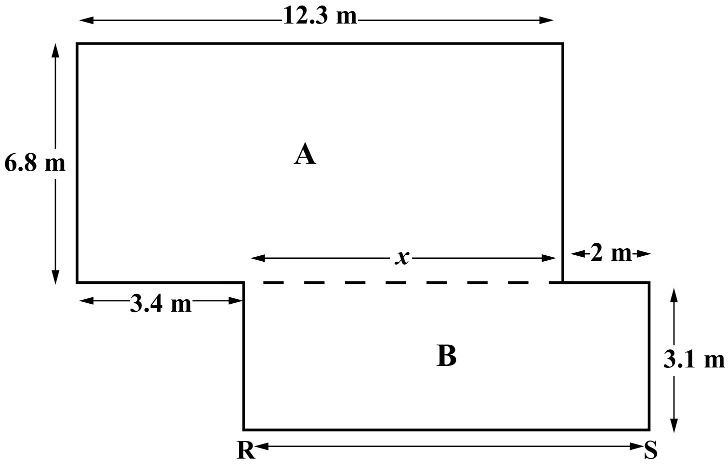


Sine rule — $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

Cosine rule — $a^2 = b^2 + c^2 - 2bc\cos A$

GO ON TO THE NEXT PAGE

1. The diagram below, **not drawn to scale**, represents the plan of a floor.
The broken line *RS* divides the floor into two rectangles, **A** and **B**.



(a) (i) Calculate the value of x.

(1 mark)

(ii) Hence, determine the length of RS.

(1 mark)

GO ON TO THE NEXT PAGE

(b) Calculate the area of the entire floor.

(4 marks)

(c) Section A of the floor is to be covered with flooring boards measuring 1 metre by 30 centimetres.

What is the MINIMUM number of flooring boards that would be needed to completely cover Section A?

(4 marks)

Total 10 marks

2. A graph sheet is provided for this question.

A company manufactures gold and silver stars to be used as party decorations. The stars are placed in packets so that each packet contains x gold stars and y silver stars.

The table below shows some of the conditions for packaging the stars.

	Condition	**Inequality**
(1)	Each packet must have at least 20 gold stars	$x \geq 20$
(2)	Each packet must have at least 15 silver stars	
(3)	The total number of stars in each packet must be no more than 60	
(4)		$x < 2y$

(a) Complete the table above by

(i) writing the inequalities to represent conditions (2) and (3) **(2 marks)**

(ii) describing in words, the condition represented by the inequality $x < 2y$. **(2 marks)**

(b) Complete the graph on page 5, to show the common region represented by ALL FOUR inequalities in the table above. **(3 marks)**

(c) Three packets of stars (A, B, and C) were selected for inspection. Their contents are shown in the table below.

Packet	**No. of gold stars (x)**	**No. of silver stars (y)**
A	25	20
B	35	15
C	30	25

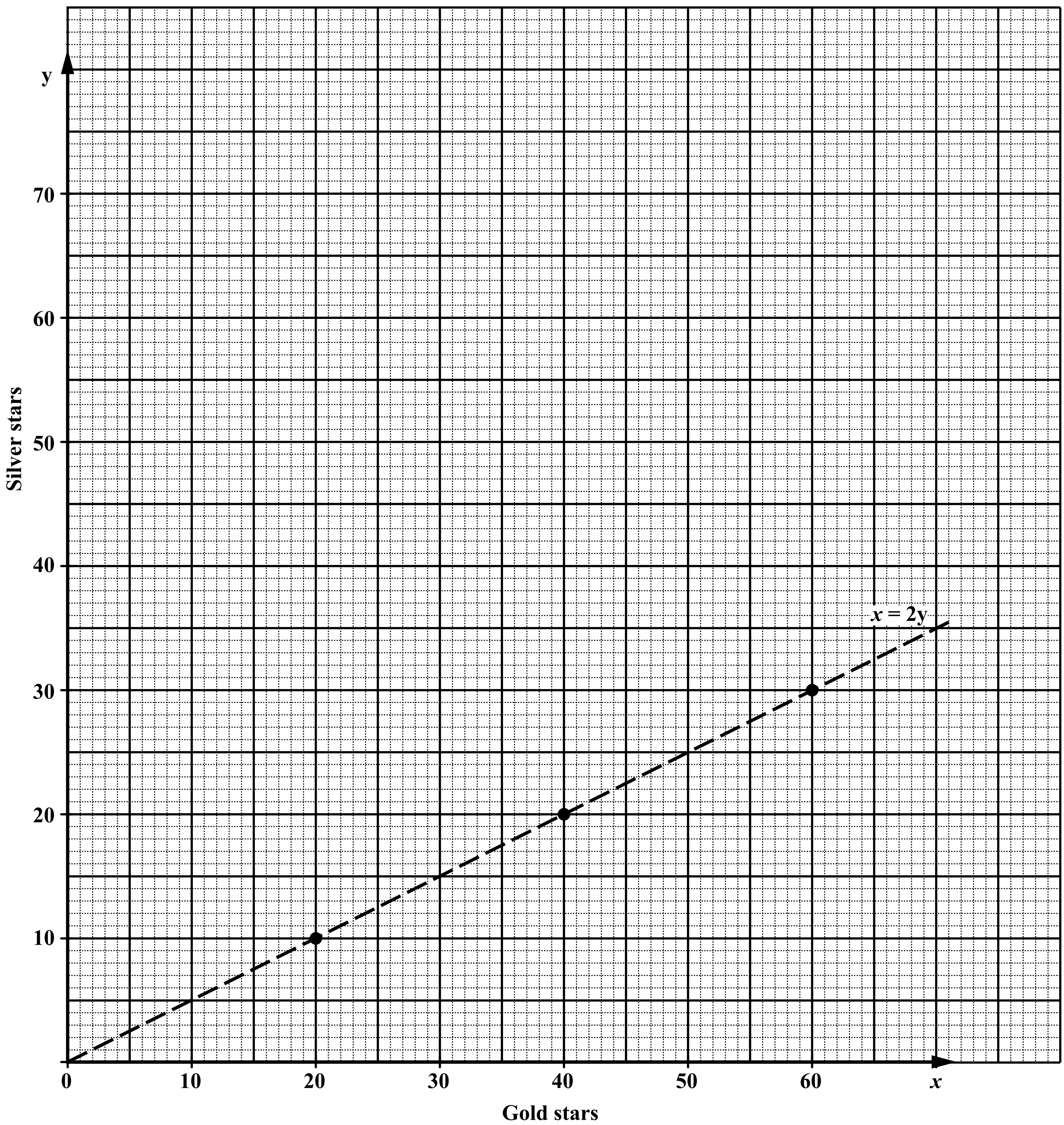
(i) Plot the points representing A, B and C on the graph drawn at (b). **(1 mark)**

(ii) Hence, state which of the three packets satisfy ALL the conditions for packaging. Justify your response.

....

....

(2 marks)



Total 10 marks

END OF TEST

IF YOU FINISH BEFORE TIME IS CALLED, CHECK YOUR WORK ON THIS TEST.

C A R I B B E A N   E X A M I N A T I O N S   C O U N C I L

CARIBBEAN SECONDARY EDUCATION CERTIFICATE®

EXAMINATION

MATHEMATICS

PAPER 032 – GENERAL PROFICIENCY

MARK SCHEME

SPECIMEN PAPER 2015

MATHEMATICS

PAPER 032 – GENERAL PROFICIENCY

SPECIMEN

MARK SCHEME

QUESTION: 1

	PROFILES K	PROFILES C	PROFILES R	Total
(a) (i) $x = (12.3 - 3.4)\text{m} = 8.9$ m		1		
(ii) RS = (8.9 + 2.0)m = 10.9 m			1	
	-	**1**	**1**	**2**
(b) Area of **A**: $12.3 \text{ m} \times 6.8 \text{ m} = 83.64 \text{ m}^2$	1	1		
Area of **B**: $10.9 \text{ m} \times 3.1 \text{ m} = 33.79 \text{ m}^2$				
TOTAL area = 117.43 m^2	1		1	
Correct method for finding area: C1 Either Area of A or Area of B correct: K1 Adding to find the total area: R1 Correct total: K1				
	2	**1**	**1**	**4**
(c) Area of board 0.3 m^2 (seen or implied)		1		
Number of boards = $\dfrac{83.64}{0.3}$ (Division:C1) = 278.8 (Correct answer K1)	1	1		
Number of boards needed is 279			1	
	1	**2**	**1**	**4**
TOTAL	**3**	**4**	**3**	**10**

MATHEMATICS

PAPER 032 – GENERAL PROFICIENCY

SPECIMEN

MARK SCHEME

QUESTION: 2

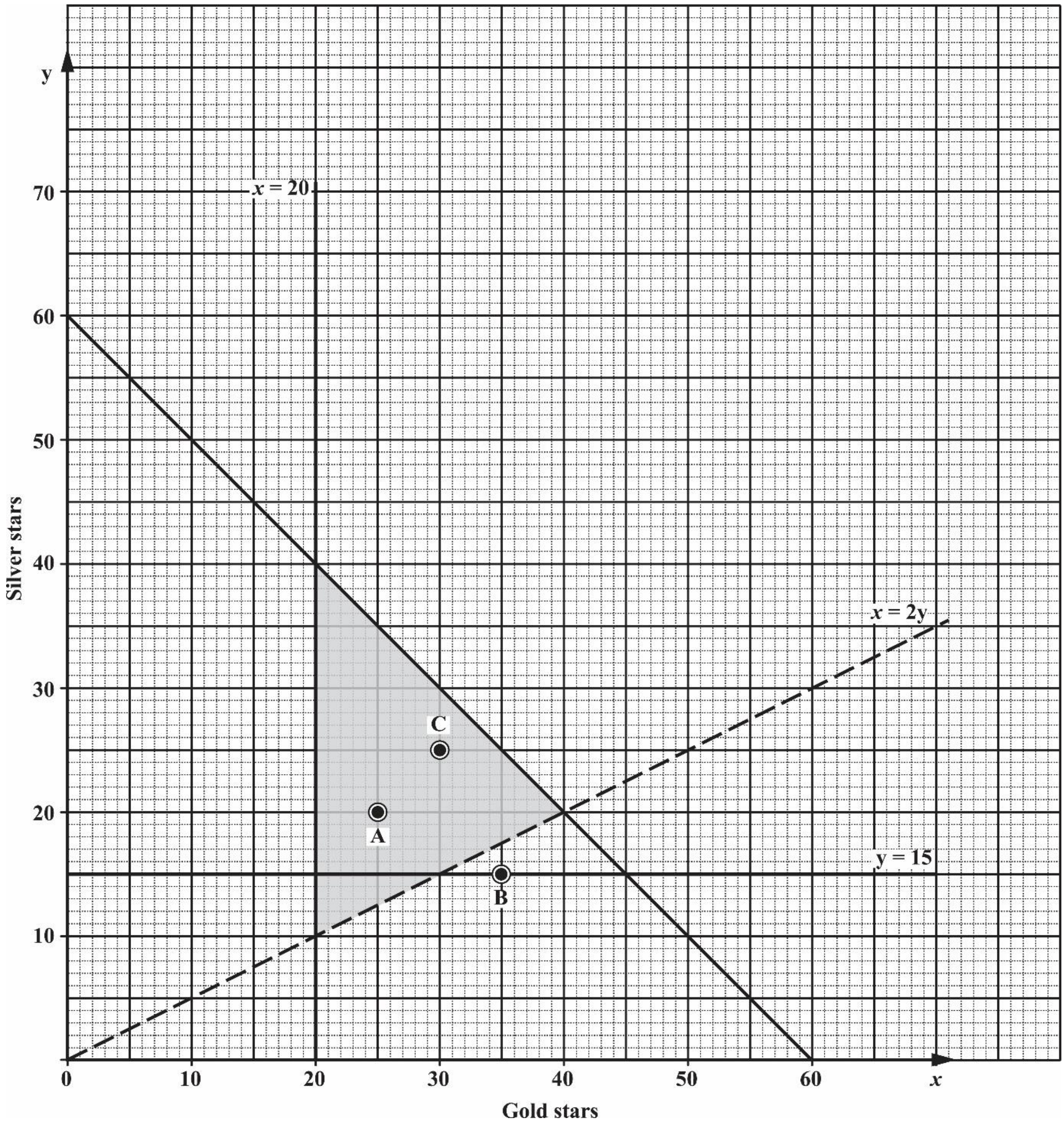
	PROFILES K	PROFILES C	PROFILES R	Total
(a) (i) [2] $y \geq 15$	1			
[3] $x + y \leq 60$		1		
(ii) The number of gold stars must be LESS than TWICE the number of silver stars. - Less than (C1); Twice the number of silver(R1)		1	1	
	1	**2**	**1**	**4**
(b) **See graph on next page**				
Line $x = 20$ **OR** line $y = 15$	1			
Line $x + y = 60$	1			
Correct region seen or implied		1		
	2	**1**	**-**	**3**
(c) (i) Plotting any two points correctly		1		
(ii) Packets A and C satisfy the conditions but Packet B does not satisfy condition [4] - Response must be supported by points seen on the graph			1 1	
	-	**1**	**2**	**3**
TOTAL	**3**	**4**	**3**	**10**

MATHEMATICS

PAPER 032 – GENERAL PROFICIENCY

SPECIMEN

MARK SCHEME



CARIBBEAN EXAMINATIONS COUNCIL

REPORT ON CANDIDATES' WORK IN THE CARIBBEAN SECONDARY EDUCATION CERTIFICATE

JANUARY 2004

MATHEMATICS

MATHEMATICS

CARIBBEAN SECONDARY EXAMINATIONS CERTIFICATE

JANUARY 2004

GENERAL COMMENTS

The General Proficiency Examination is offered in January and June each year. The Basic Proficiency is offered in June only.

There was a candidate entry of approximately 11,672 in January 2004. There is an overall improvement in this year's examination over 2003. The mean mark was 48.21. Fifty-seven per cent of the candidates achieved Grades I -III.

Candidates performed satisfactorily on Number Theory, Algebra and Sets. However, they performed poorly on Measurement, Geometry and Graphs.

DETAILED COMMENTS

The examination consists of two papers.

PAPER 01 - Multiple Choice

Paper 1 consists of 60 multiple choice items. Seven candidates scored full marks, and approximately 85 per cent of the candidates scored more than half the marks.

PAPER 02 - Essay

Paper 2 consists of eight compulsory questions in Section I and six optional questions in Section II, from which candidates are required to answer two. One candidate scored full marks and approximately 26 per cent scored more than half the marks.

<u>Question 1</u>

The question tested the candidates' ability to:

- perform the basic operations with rational numbers
- solve problems involving payments by instalments as in the case of hire purchase

The question was attempted by all of the candidates and the performance was generally good, with almost 28 per cent of the candidates scoring the full 10 marks. The mean mark was 7.6. Most candidates were able to calculate the cash price of the dining room suite and its total hire purchase price. However, candidates experienced difficulty calculating the extra cost of buying on hire purchase as a percentage of the cash price. It appears that many of them did not recognise that $880 was given as the cash price of the suite. Several candidates in attempting to divide by a fraction did not invert the second fraction.

In addition, a number of candidates wrote $1\frac{3 - 10}{15}$ with no further attempt at simplifying.

Solutions:

(a) $\mathbf{\frac{14}{15}}$

(b) **$105**

(c) **(i) $1 056** **(ii) 20%**

Question 2

The question tested the candidates' ability to:

- substitute numbers for algebraic symbols in simple algebraic expres sions
- change the subject of formulae
- solve simultaneous linear equations in two unknowns algebraically
- apply the distributive law to remove brackets in algebraic expressions

The question was attempted by 99.9 per cent of the candidates, about 12 per cent of these scored the full 12 marks. The mean mark was 6.81. Most candidates knew how to substitute numbers into algebraic expressions and how to solve a pair of simultaneous equations in two unknowns. They however experienced difficulties in changing the subject of the formula, multiplying by zero, and in computing $(-3)^3$. Some common occurrences were:

$2(-3)^3 = (2 \text{ x } -3) \text{ x } (2 \text{ x } -3) \text{ x } (2 \text{ x } -3)$ **OR** $2(-3)^3 = 2 \text{ x } 27$

$C = \frac{5}{9}(F - 32) \quad = \quad C = \frac{5}{9}F - 32$ **OR** $C + 32 = \frac{5}{9}F$

Solutions:

(a) (i) 20 (ii) -54

(b) (i) F = $\frac{9}{5}$ C + 32 (ii) 59°C

(c) *x* = 3; *y* = 4

Question 3

The question tested the candidates' ability to:

- construct Venn diagrams to show subsets, intersection and union of two sets
- list the members of a set from a given description
- solve geometric problems using the properties of similar figures

Approximately 99 per cent of candidates attempted this question. The mean mark was 5.19 out of a maximum of 10 marks. Two percent of the candidates scored full marks. Most candidates were able to complete correctly labelled Venn diagrams to illustrate the information. Some candidates experienced difficulty in determining the complement of the union of the sets P and Q. Many candidates did not attempt Part (b) of the question. Most of those who attempted Part (b) could not calculate the scale factor and most of those who did could not use it in calculating the area of the enlarged figure.

Solutions:

(a) (ii) a) = {2, 5} b) = {4, 6, 7}

(b) (i) $\frac{5}{3}$ (ii) 50 cm^2

Question 4

The question tested the candidates' ability to:

- convert units of time within the SI system
- solve simple problems involving time, distance and speed
- calculate the area of sectors of circles
- calculate the volume of a right prism

The question was attempted by 98 per cent of the candidates of whom nearly two per cent scored the maximum of 11 marks. The mean mark was 3.71. The candidates demonstrated competence in calculating speed, area and volume. The areas of weak performance included extracting the correct information from the given table, manipulating hours and minutes using consistent units.

Solutions:

(a) (i) 1 hour 36 minutes (ii) 50 km/h

(b) (i) 353 cm^2 (ii) 42 400 cm^3

Question 5

The question tested the candidates' ability to:

- interpret and make use of functional notation and their combinations
- draw, read and interpret graphs of functions
- draw and use graphs of a given quadratic function to determine the roots of the given function

This question was attempted by approximately 98 per cent of the candidates of whom 5 per cent earned the maximum of 13 marks. The mean mark was 7.79. The areas of good performance included the use of correct scales and plotting points on the graph. The areas of weak performance included the inability of several candidates to determine the composite function, drawing a smooth curve to represent the quadratic function and plotting points to draw the line y = x. Many candidates did not recognise that the solutions to the quadratic equation were at the points of intersection of the two graphs.

Solutions:

(a) (i) 5 (ii) 7

(b) (i) 10, -2
(iv) $x = 0$ and $x = 4$

Question 6

The question tested the candidates' ability to:

- use Pythagoras' theorem to solve simple problems
- use the sine, cosine and tangent ratios in the solution of right-angled triangles
- solve problems involving bearings
- find by drawing and/or calculation the gradients and intercepts of graphs of linear functions
- determine the equation of a given line
- state the relationship between an object and its mage in a plane when reflected in a line in that plane

The question was attempted by 92 per cent of the candidates. Of these 3 per cent scored the maximum 12 marks. The mean mark was 3.81. The areas of good performance included the use of Pythagoras Theorem and determining the y intercept of the mirror line drawn. The areas of weak performance included finding the bearing, correctly determining the position of the mirror line and finding the equation of the mirror line.

Solutions:

(a) (i) 32.2 km (ii) 120°
(b) (ii) (0,4) (iii) $y = -x + 4$

<u>Question 7</u>

The question tested the candidates' ability to:

- use the mid-point of a class interval to estimate the mean of data presented in grouped frequency tables
- construct a cumulative frequency table for a given set of data
- draw and use a cumulative frequency curve
- estimate the median of a set of grouped data
- use theorectical probability to predict the expected value of a given set of outcomes

The question was attempted by 94 per cent of the candidates. Of these 2 per cent scored the maximum 12 marks. The mean mark was 3.57. The areas of good performance were finding the mid-interval values and completing the cumulative frequency column. The areas of weak performance included using the ogive to determine the median score (many candidates used the cumulative frequency value of 25 instead of 20 for this purpose), reading the scales used on the graph and computing the mean of the group distribution.

Solutions:

(a) **15.5**

(b) **18, 32, 37, 40**

(c) (i) 20 (ii) 11 (iii) 20 or 21 (iv) $\frac{9}{40}$

Question 8

The question tested the candidates' ability to:

- use instruments to draw and measure angles and line segments
- use instruments to construct angles and triangles
- calculate the area of the region enclosed by a triangle

The question was attempted by 84 per cent of the candidates. Of these, nearly 3 per cent scored the maximum 10 marks. The mean mark was 4.20. Candidates performed well on the construction of the equilateral triangle and in sub-dividing the larger triangle into nine other triangles. They performed poorly on completing the table and on determining the area of the basic triangle.

Solutions:

(c)	**3**	**8**
	9	**64**

(d) $\sqrt{3}\ \text{cm}^2$

Question 9

The question tested the candidates' ability to:

- draw and use distance-time graphs
- use the gradient of a graph of a linear function to determine the rate of change of one variable with respect to another
- determine the maximum and minimum values of quadratic functions by the method of completing the square

The question was attempted by 55 per cent of the candidates. Of these, less than 1per cent scored the maximum 15 marks. The mean mark was 4.06. The areas of good performance included reading and interpreting information from the distance-time graph and determining the speed. However, in part (b) few candidates correctly determined the time that the two persons would meet. In part (c), although most candidates identified a method for completing the square, they were not able to complete the process due to errors in working with fractions or collecting like terms. Some candidates expressed the quadratic in the correct form, but were unable to transfer the results to state the minimum value of the of f(x) or the value of x at which it occurred.

Solutions:

(a) (i) 04:15 (ii) 9 km (iii) 30 mins (iv) 6 km/h

(b) (i) 06:42 (ii) 2 km

(c) (i) $4\left(x-\frac{7}{8}\right)^2-\frac{1}{16}$.

(ii) a) $-\frac{1}{16}$ **b)** $\frac{7}{8}$

Question 10

The question tested the candidates' ability to use linear programming techniques to solve problems involving two variables.

The question was attempted by 30 per cent of the candidates. Of these, 5 per cent scored the maximum 15 marks. The mean mark was 4.53. Good performance were noted in candidates' attempts at interpreting and using the given scales, writing the profit function and substituting values into this function to find the maximum profit. The areas of weak performance were in writing the inequations (many candidates used the equal sign instead of the inequality sign) and in drawing the lines representing the boundaries of the inequations.

Solutions:

(a) (i) $x \geq 15$; $y \geq 20$ **(ii)** $40x + 30y \leq 2400$
(c) (i) $P = 25x + 6y$ **(ii) 45 dresses and 20 shirts**
(iii) $1 245

Question 11

The question tested the candidates' ability to:

- calculate the distance between two points on the earth, treated as a sphere, measured along the parallels of latitude or meridians
- calculate the area of a triangle given two sides and the included angle by means of the formula:
  Area of $\Delta ABC = \frac{1}{2} ab \sin C$
- calculate the area of a sector of a circle
- calculate the area of a segment of a circle.

The question was attempted by 17 per cent of the candidates. Of these, 3 per cent scored the maximum 15 marks. The mean mark was 3.22. The areas of good performance included knowing how to calculate the distance between two points on the Earth's surface and determining the area of the segment of the circle. The areas of weak performance were candidates' inability to determine the angle subtended at the centre by the arc joining the two points on the Earth's surface and in calculating the radius of the given circle in part (b).

Solutions:

(a) **4 331 km**

(b) **(i) 9.33 cm** **(ii) 60.74 cm^2** **(iii) 17.88 cm^2**

Question 12

The question tested the candidates' ability to:

- solve problems using the theorems related to properties of a circle
- use simple trigonometric ratios to solve the problems based on measures in the physical world: heights and distances

The question was attempted by 18 per cent of the candidates. Of these less than 4 per cent scored the maximum 15 marks. The mean mark was 4.59. The areas of good performance included candidates' knowledge of the circle theorems and their ability to use them in calculating the size of the unknown angles. There was also satisfactory performance in part (b) in recognising that angle MTN was 23^o. An area of weak performance included not writing the reasons to support the calculations in part (a).

Solutions:

(a) **(i) 52°** **(ii) 38°** **(iii) 142°** **(iv) 76°**

(b) **(ii) 23.6 m**

Question 13

The question tested the candidates' ability to:

- associate a position vector with a given point P(a, b) where O is the origin (0,0)
- determine the magnitude of a vector
- use vectors to represent and solve problems in Geometry.

The question was attempted by 36 per cent of the candidates. Of these less than 2 per cent scored the maximum 15 marks. The mean mark was 3.74. The areas of good performance include writing the column vectors representing the given position vectors and in determining the position vector of the point D by a graphical method. The area of weak performance included the inability to add vectors and in recognising that they were required to find the lengths of at least two sides of the triangle to prove that it was isosceles.

Solutions:

(a) **Position Vector of:**

A: $\begin{pmatrix} 2 \\ 2 \end{pmatrix}$ **B:** $\begin{pmatrix} 6 \\ 2 \end{pmatrix}$ **C:** $\begin{pmatrix} 4 \\ 6 \end{pmatrix}$

(b) **(i)** $\begin{pmatrix} 4 \\ 0 \end{pmatrix}$ **(ii)** $\begin{pmatrix} 2 \\ 4 \end{pmatrix}$ **(iii)** $\begin{pmatrix} -2 \\ 4 \end{pmatrix}$

(c) **The magnitude of vectors AB, AC and BC are $\sqrt{16}$, $\sqrt{20}$ and $\sqrt{20}$ respectively.**

(d) **Position vector of D is**

Question 14

The question tested the candidates' ability to:

- identify a 2 x 2 singular matrix
- obtain the inverse of a non-singular 2 x 2 matrix
- use matrices to solve simple problems in Algebra
- determine the 2 x 2 matrices associated with the following transformations: enlargements, rotations, reflections
- use matrices to solve simple problems in Geometry.

The question was attempted by 27 per cent of the candidates. Of these two per cent scored the maximum 15 marks. The mean mark was 5.88. The areas of good performance included understanding and using the definition of a singular matrix, expressing the simultaneous equations in matrix form, finding the inverse of the matrix and multiplying two matrices.

The areas of weak performance included not being able to describe the transformation represented by the matrix W and in correctly ordering the matrices to determine the single matrix for the combined transformation.

Solutions:

(a) $\mathbf{\frac{5}{3}}$

(b) **(i)** $\begin{pmatrix} 3 & 5 \\ 2 & 4 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 19 \\ 14 \end{pmatrix}$

(ii) $\frac{1}{2} \begin{pmatrix} 4 & -5 \\ -2 & 3 \end{pmatrix}$

(iii) $\begin{pmatrix} 3 \\ 2 \end{pmatrix}$

(c) (i) **a reflection in the *x* axis or y = 0**

(ii) **a rotation of 180^{o} about (0,0) or an enlargement by scale factor – 1, centre (0,0)**

(iii) $\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$

(iv) $\begin{pmatrix} -6 \\ -4 \end{pmatrix}$

CARIBBEAN EXAMINATIONS COUNCIL

REPORT ON CANDIDATES' WORK IN THE SECONDARY EDUCATION CERTIFICATE EXAMINATION MAY/JUNE 2004

MATHEMATICS

MATHEMATICS

MAY/JUNE 2004

GENERAL COMMENTS

The General Proficiency Mathematics examination is offered in January and June each year, while the Basic Proficiency examination is offered in June only.

In June 2004, approximately 84 786 candidates registered for the General Proficiency examination, an increase of 1 340 over 2003. Candidate entry for the Basic Proficiency examination decreased from 11 208 in 2003 to 7 861 in 2004.

At the General Proficiency level, approximately 35 per cent of the candidates achieved Grades I – III. This represents a 5 per cent decrease over 2003. Twenty per cent of the candidates at the Basic Proficiency level achieved Grades I – III compared with 29 per cent in 2003.

DETAILED COMMENTS

General Proficiency

In general, candidates' work revealed lack of knowledge of certain areas of the syllabus. Poor performance was noted for algebra, graphs, geometry and trigonometry, vectors and matrices. Candidates continue to experience difficulty with questions which require the application of mathematical concepts. The general areas of strength were in computation and set theory.

Six candidates scored full marks on the overall examination compared with one candidate in 2003. In addition 115 candidates scored more than 95 per cent of the total marks.

Paper 01 – Multiple Choice

Paper 01 consisted of 60 multiple-choice items. This year, 145 candidates earned full marks compared with 58 in 2003. Approximately 71 per cent of the candidates scored at least half the total marks for this paper.

Paper 02 – Essay

Paper 02 consisted of two sections. Section I comprised eight compulsory questions that totalling 90 marks. Section II comprised six optional questions; two each in Relations, Functions and Graphs; Trigonometry and Geometry, Vectors and Matrices. Candidates were required to choose any two questions. Each question in this section was worth 15 marks.

This year 16 candidates earned full marks on Paper 02 compared with 9 in 2003. Approximately 16 per cent of the candidates earned at least half the total marks on this paper.

Compulsory Section

<u>Question 1</u>

Part (a) of the question was designed to test candidates' ability to perform the following operations

- addition of the squares of decimal numbers
- division and subtraction of decimal numbers, and
- subtraction and division of mixed fractions

Part (b) of the question tested the candidates' ability to

- approximate a value to a given number of significant figures
- write a rational number in standard form.

Part (c) of the question tested the candidate's ability to

- calculate simple interest and the amount due after a given period
- solve a problem involving compound interest.

This question was attempted by all candidates. The mean score was 6.44 out of 12.

Part (a) of the question was fairly well done, particularly the calculation of decimals and fractions. However, some candidates had problems working with mixed numbers.

Many candidates did not follow through to calculate the amount received by adding principal to interest in Section c (i). Many seemed not to have understood the term appreciation and used the depreciation formula in c (ii). Candidates were aware that they were required to invert when they were dividing the fraction in a(iii); however, they were unsure about which fraction was the divisor and which was the dividend.

In Section (b), many candidates had difficulty with approximation in standard form and significant figures. Answers such as 20.0 and 2 were given. In addition, a number of students made the mistake of writing $\frac{11}{15} \infty \frac{5}{11}$ as 3, instead of $\frac{1}{3}$.

<u>Recommendation</u>

The difference between depreciation and appreciation should be emphasised. Additionally, more practice should be given in approximation. Students must understand which fraction is the divisor and which is the dividend in the division of fractions.

Answer: (a) 22.1, 0.297, $-\frac{1}{3}$ (b) 20, 2.97 x 10^{-1} (c) \$45 600, \$45 796

<u>Question 2</u>

This question tested candidates' ability to

- simplify algebraic fractions
- solve quadratic equations
- factorize algebraic expression

The question was attempted by 98 per cent of the candidates. The mean score was 2.78 out of 10.

In Part (a) (i) many candidates did not recognise the difference of 2 squares. The factors given for $x^2 - 1$ were $x(x - 1)$ and $(x - 1)^2$.

More over, some candidates did not recognise that they had to factorise and attempted to cancel the variables (x) and the numbers (–1) in the numerator and denominator.

In 2a(ii) candidates had difficulty in factorising the numerator. Most candidates who attempted this question had difficulty with the fraction in which the numerator consisted of two terms. They cancelled "ad hoc" any term in the numerator with that of the denominator.

e.g. $\dfrac{4ab^2 + 2a^2 b}{ab} \Rightarrow 4b + 2a^2b$

$\dfrac{4ab^2 + 2a^2 b}{ab} \Rightarrow 4ab^2 + 2a$

In 2(b), candidates treated the expression as an equation and "cross multiplied" the numerator and denominator of the given fractions. Candidates also tried to further simplify their results.

e.g. $\dfrac{3p^2 + 2q}{2p} \Rightarrow \dfrac{5p^2q}{2p}$

$\dfrac{3p^2 + 2q}{2p} \Rightarrow 3p + q$

In 2(c), candidates had difficulty in factorising the *LHS* of the equation. Some who used the formula encountered problems in stating it properly. $x = -b \pm \dfrac{\sqrt{b^2 - 4ac}}{2a}$ was stated by many of the candidates who chose this method of solution. Substitution of values for *a*, *b* and *c* into the equation was not well done. This may be attributed to a lack of exposure to the formula or the candidates' inability to deal with 'directed numbers'. Many students attempted to find the square root of negative numbers. A few candidates attempted to use the completion of squares method; but most of them were unsuccessful.

Areas of good performance:

- Some candidates were able to combine the skills of factorisation (choosing common factor and difference of 2 squares) with those of reduction (cancelling the common factor) in numerator and denominator.
- Candidates were able to find the LCM of two terms (one of them non numeric) and use this result in combining two fractions.
- Candidates were able to solve quadratic equations using factorisation techniques and the formula method.

Recommendations to Teachers

Teachers need to expose students to a variety of problems, particularly problems that combine skills e.g. factorisation and reduction of terms, LCM of terms that are numeric and non-numeric. A distinction also needs to be made between an expression and an equation.

Answer: (a) (i) x + 1

(ii) $4b + 2a$

(b) $\dfrac{3p^2 + 2q}{2p}$

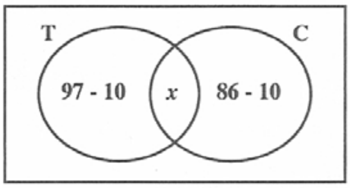
(c) $\dfrac{1}{3}$, 2

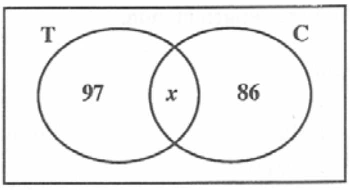
Question 3

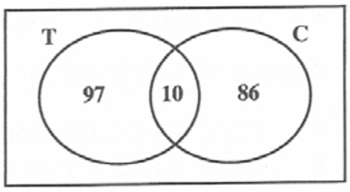
The question tested the candidate's ability to

- construct and use Venn diagrams to show subsets, complements and the intersection and union of sets
- determine and count the elements in the intersection and union of sets
- determine the mean, median and mode for a set of data

Approximately 99 per cent of candidates attempted this question. The mean score was 5.73 out of 11. Most candidates who attempted part (a) were able to draw two intersecting sets although many were unabe to correctly insert the given information. Common variations were:





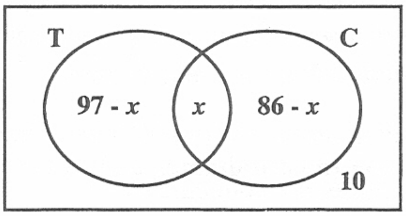


The concept of using an equation to find the intersection did not appear to be well known and many candidates who attempted to use the formula did not set it out correctly (e.g. $x + 97 + 86 + 10 = 160$).

Many candidates were able to score at least five of the six marks awarded in part (b). However, some candidates did not know the difference between mean, median and mode. The median was often incorrect because of candidates' poor use of the calculator. It was common to see $\dfrac{6.5 + 6.7}{2} = 9.85$.

Answer:

(a) (i)



(ii) 33 members play both tennis and cricket

(b) (i) (a) 6.65
(b) 6.6
(c) 6.7

(ii) Omit one from {5.9, 6.1, 6.3}
and one from {6.7, 6.8, 8.2}

Question 4

The question tested

- substitution and operations of real numbers
- transposition
- knowledge and application of trigonometrical ratios
- properties of rectangle and right-angled triangle
- use of Pythagoras' theorem.

The question was attempted by approximately 96 per cent of the candidates. The mean score was 3.69 out of 12.

The students demonstrated competence in

- substitution
- recognizing property of a rectangle (i.e. opposite sides are equal)
- use of trigonometry ratio to find the length of side.

The areas of weak performance included

- changing the subject of the formula
- giving answers to correct decimal places.

Answer: (a) (i) $\frac{5}{12}$

(ii) $M = \frac{12nt^2}{5}$

(b) (i) 11.2 cm

(ii) 4.3 cm

(iii) 69.0·

Question 5

The question tested the candidates' ability to

- determine the co-ordinates of a given point
- recognize the gradient of a line as the ratio of the vertical rise to the horizontal shift
- identify and describe the single transformation which results from a combination of transformations
- locate the image of a quadrilateral under a reflection in a given mirror line.

This question was attempted by approximately 95 per cent of the candidates. The mean mark was 3.39 out of 10. The areas of good performance was in determining the coordinates and in performing the first reflection.

The areas of weak performance included the inability to

– plot a straight line given the point and a gradient

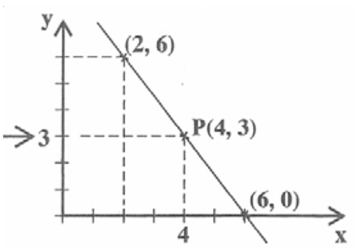
– reflect the first image in relation to the second mirror line

– describe fully the transformation (translation).

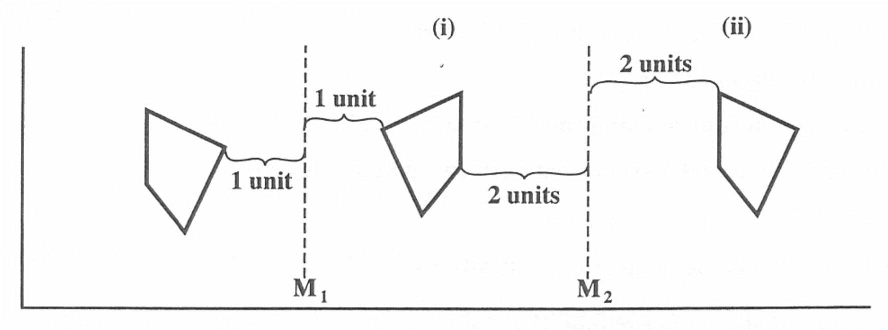
Answer:

(a) (i) $P(4,3)$

(ii) $y = \frac{3}{2}x + c$



(b)



(c) Translation by vector $\binom{10}{0}$

Question 6

The question tested the candidates' ability to

- interpret data presented in line graph form
- recognize the gradient of the line as the ratio of the vertical rise to horizontal shift
- find by calculation, the gradient and intercept of the graph of linear functions
- determine the equation of a line given
  - (i) the graph of the line
  - (ii) the coordinates of two points on the line
- solve linear equation in one unknown
- substitute numbers for algebraic symbols in simple algebraic expressions.

The question was attempted by 97 per cent of the candidates. The mean mark was 3.43 out of 11.

The areas of good performance include students' ability to read and interpret graphs.

Most candidates were able to correctly calculate the gradient of the line. The areas of weak performance include candidates' inability to write down the equation of the line, change the subject of the formula, and perform operations with fractions.

Answer:

(a) \$30.00

(b) (i) 35 minutes
(ii) 0 minutes

(c) \$20

(d) gradient = \$0.50 per minute

(e) $d = 0.50t + 20$

(f) $t = 116$ minutes

Question 7

The question tested candidates' ability to

- find the radius of a circle given the area of the circle
- calculate the circumference of a circle
- calculate the area of a square using the circumference of the circle as the perimeter of the square
- use scales to find the actual distance between two points, on a grid
- perform conversions using a given scale
- measure or calculate the bearing of given positions.

The question was attempted by approximately 87 per cent of the candidates. The mean mark was 2.81 out of 12.

The areas of good performance were

– finding the circumference of the circle

– finding the distance between the two points, Rose Hall and South Port.

The areas of weak performances were

– transposing $154 = \pi r^2$ to find the radius of the circle

– recognising the circumference of the circle was equal to the perimeter of the square

– inaccuate use of the given scales

(iv) calculating the bearings required or measuring using the protractor to find the bearing.

Answer:	(a)	(i)	a) 7 cm		(b)	(i)	5 km
			b) 44 cm			(ii)	166°
		(ii)	121 cm^2				

Question 8

The question tested candidates' ability to apply the concept of profit and loss to solve problems involving business transactions.

Approximately 89 per cent of the candidates attempted this question. The mean score was 4.52 out of 12.

In part (a) many candidates correctly calculated the percentage of chocolate in recipe A.

Part (b) was poorly done by most candidates. In part (c), the majority of candidates converted the required ratio to a proper fraction and calculated the percentage. Some candidates, however, were unable to find the correct ratio for the new mixture in part (c).

In part (d), most candidates found the cost of six bottles for $4.40 but were unable to find the cost of 150 bottles.

Answer:	(a)	40%	(c)	$37\frac{1}{2}$ %	
	(b)	A has 40%	(d)	(i)	$110
		B has $33\frac{1}{3}$%		(ii)	$0.88
		A richer			

OPTIONAL SECTION

Question 9

The question tested the candidates' ability to

- interpret and use the concept of variation
- find the area of a rectangle
- expand and simplify an algebraic expression
- use factors to solve a quadratic equation
- find the value of an algebraic expression by substituting numerical values.

The question was attempted by about 26 per cent of the candidates. The mean mark was 2.57 out of 15.

The areas of good performance included finding the area of a rectangle, and the factors of a quadratic expression. In part (a), many candidates had difficulty with the variation; they omitted the constant. In part (ii) candidates produced the answer for part (iii) and could not show that $2x^2 + 7x - 15 = 0$, given that the area was 60.

In part (iii), though the candidates got the factors for the expression, they did not solve the equation. Some candidates tried to use the formula or the method of completing the square but did so incorrectly in many instances. In some cases, the values of x were found, but candidates were unable to transfer the results to obtain the values for *AK* and *AM*.

Answers:

(a) $M = 0.004$ $\quad n = 5$

(b) (i) Area = $(3x + 3)(5 + 2x)$

(ii) $6x^2 + 6x + 15x + 15 = 60$
$6x^2 + 21x - 45 = 0$
$2x^2 + 7x - 15 = 0$

(iii) $x = 1.5$ or -5
$AK = 7.5$ cm $\quad AM = 8$ cm

Question 10

The question tested candidates ability to:

- Translate algebraic statements into inequalities
- Draw the graphs of their functions
- Identify the region defined by their stated inequalities
- Determine conditions for the maximum profit

Approximately 24 per cent of candidates attempted this question. The mean score was 3.42 out of 15. Most candidates were able to translate the algebraic statements into the inequalities $x \geq 10$ and $y \geq 5$. They knew the relationship between $4x$, $8y$ and 200 but were unable to show the relationship as an inequality.

Most candidates were able to use the correct scale for the graph and more than 90 per cent of those who attempted the question were able to draw the lines $x = 10$ and $y = 5$ and $x + y = 60$. However, about 50 per cent of them encountered difficulty with plotting the line $4x + 5y = 200$.

Although most candidates were proficient in identifying the required region and writing the statements for the profit expressions, many did not show the test to select the maximum profit.

Answer:

(a) (i) $x \geq 10$ (ii) $x + y \leq 60$
$y \geq 5$ $4x + 8y \geq 200$

Profit statement

(c) (i) For Max. profit he must sell 10 kg of peanuts and 50 kg of cashew nuts
(ii) $2x + 5y$
Max profit $270

Question 11

This question consisted of two parts.

Part (a) required the candidates to calculate the unknown angles in a given diagram using the angle properties of circles.

Part (b) required candidates to solve a problem on bearing by drawing and calculation.

About 24 per cent of candidates attempted this question. The mean score was 3.33 out of 15.

It was obvious that students need to discover the properties of circles by practice in their classrooms as most students failed to score marks for this part of the question.

The majority of the candidates who attempted the question were able to score marks for drawing the diagram using the given information in part (b). However, quite a number of candidates displayed a lack of knowledge of direction; hence they were unable to correctly identify the bearing.

A large number of candidates were aware that they needed to use the Cosine rule to find the distance GH. A few attempted to use Pythagoras' Theorem to calculate the same distance, but were unsuccessful.

Answer:

(a) (i) a) $V\hat{Z}W = 51°$ ... (angles in the alternate segment are equal)

b) $XYZ = 78°$ ... (exterior angle of a cyclic quadrilateral = interior opposite angle)

(b) $GH = 104.4$ km
Bearing of H from $G = 62°$

Question 12

This question tested the candidates' ability to

- express a trigonomic ratio in fractional or 'surd form'
- use a trigonomic ratio to express the area of a triangle in surd form
- use a trigonomic ratio to find the length of a side of a triangle
- recognise positions on the globe
- calculate distance between two points on the globe.

The question was attempted by about 8 per cent of the candidates . The mean score was 1.68 out of 15.

The area of good performance was the candidates' ability to recognise the position of A east of B on the globe. Candidates chose the correct formula to calculate the radius as well as the length of the arc, but unfortunately, did not know which angle to use. An area of very weak performance was the candidates' inability to express the trigonomic ratio in fractional or surd form.

Answer:

(a) (i) $\cos\theta = \frac{1}{2}$

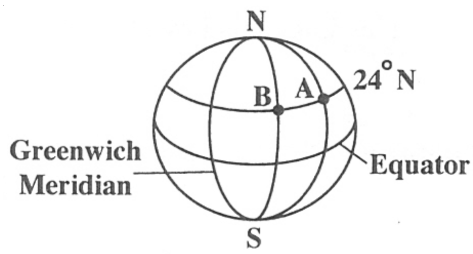
(ii) Area of Ð $CDE = \frac{1}{2}$ x 20 x 30 sin θ

$= \frac{1}{2}$ x 600 x $\frac{\sqrt{3}}{2}$

$= 150\sqrt{3}$ square units

(iii) (a) EC = 26.5 to 3 s.f.

(b) (i)



(ii) (a) r = 5819 km
(b) arc AB = 3350 km

Question 13

The question tested the candidates ability to

- add vectors using the triangle law
- combine vectors written as 2 x 1 column matrices
- associate a position vector with a given point P(a, b) where O is the origin (0, 0)
- determine the magnitude of a vector
- use vectors to represent and solve problems in Geometry.

Approximately 32 per cent of the candidates attempted this question. The mean mark was 2.77 out of 15.

The areas of good performance included writing the column vector representing the given position vector A(4, 2) and determining the magnitude of the column Vector $\overrightarrow{OA}$.

The area of weak performance was the candidates' inability to use a vector approach to find the position vector of N. Coordinate geometry was frequently used. Others failed to identify the vector $\overrightarrow{ON}$ as $\frac{1}{2}\overrightarrow{OA}$ + – $\overrightarrow{AC}$ but instead wrote $\frac{1}{2}\overrightarrow{ON}$ = – $\overrightarrow{AC}$.

Answer: (a) (i) $\overrightarrow{OA} = \begin{pmatrix} 4 \\ 2 \end{pmatrix}$

(ii) $\overrightarrow{CB} = \begin{pmatrix} 4 \\ 2 \end{pmatrix}$

(b) $|\overrightarrow{OA}| = \sqrt{20} = 4.47$

(c) (i) $\overrightarrow{OA}$ is parallel to $\overrightarrow{CB}$

and $|\overrightarrow{OA}| = |\overrightarrow{CB}|$

(ii) *OABC* is a parallelogram because one pair of opposite sides is equal and parallel.

(d) (i) $OM = \begin{pmatrix} 3 \\ 5 \end{pmatrix}$

(ii) $\text{ON} = \begin{pmatrix} 3 \\ 5 \end{pmatrix}$

M and N coincide <u>OR</u> the diagonals of the parallelogram bisect each other

OR

The diagonals have the same midpoint.

<u>Question 14</u>

The question tested candidates' ability to

- reflect a given object in the y–axis
- draw the line $y = x$
- reflect an object in the line $y = x$
- describe the result of 2 transformations as a single transformation
- reflect an object in the x-axis
- write the matrix of a transformation and
- write a matrix as the result of 2 transformations.

The question was attempted by 52 per cent of the candidates. The mean mark was 2.98 out of 15.

The majority of the candidates who scored marks were able to score for the reflection in the y-axis and the reflection in the x-axis. Many were able to write down the matrices for reflection in the y-axis and reflection in the line y = x. The responses to section b (i) and (ii) were written in response to section a (i) and (ii) although it was not required in that section. Only a few candidates were able to obtain a single matrix which represented the result of two transformations. The others did not write down the correct order for the multiplication.

Candidates' poor responses did not suggest misconceptions. They indicated a lack of knowledge of the concepts.

Answer: (a) (iii) Rotation about (0, 0) 90° clockwise

(v) $\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$

(b) (ii) $\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$

DETAILED COMMENTS

Basic Proficiency

The Basic examination is designed to provide the average citizen with a working knowledge of the subject area. The range of topics tested at the Basic level is narrower than that tested at the General Proficiency level.

Candidates continue to demonstrate lack of knowledge of the fundamental concepts being tested at this level. The extent of their weakness is evident in their lack of ability to cope with questions requiring higher order skills.

Approximately 20 per cent of the candidates achieved Grades I - III. This reflected a 9 per cent decline in performance compared with 2003 where 29 per cent of the candidates achieved Grades I - III.

Paper 01 – Multiple Choice

Paper 01 consisted of 60 multiple-choice items. No candidate earned full marks on this paper. The highest mark was 57 out of 60, this was earned by one candidate. Approximately 35 per cent of the candidates scored at least half the total marks for this paper.

Paper 02 – Essay

Paper 02 consisted of 10 compulsory questions. Each question was worth 10 marks. The highest score earned was 98 out of 100. This was earned by one candidate. Seven candidates earned at least half the total marks on this paper.

Question 1

The question tested the candidates' ability to:

- Perform basic operations with rational numbers
- Approximate a value to a given number of significant figures and express any decimal to a given number of decimal places
- Divide a given quantity in a given ratio

The question was attempted by 98.5 per cent of the candidates, with 1.5 per cent scoring the full marks. The mean mark was 3.35.

Most candidates were able to complete the computation and round off as specified, although some candidates interpreted "exactly" to mean the the nearest whole number and "one significant figure" to be in standard form. In 1(b) although some candidates were able to determine the amount of one share, they did not use this information to calculate the required shares. Part 1 (c) was not well done since the candidates could not link the 30% to the 150 fish. Instead, many of them calculated 70% of 150. In part (d), candidates attempted to complete the division first before removing the brackets. Then, to perform the division, the candidates inverted the wrong fraction.

Answer:
(a) (i) 4.93
(ii) 4.9
(iii) 5

(b) \$120: \$80

(c) 500 fish

(d) $1\frac{1}{2}$

Question 2

The question tested the candidates' ability to

- perform the four basic operations with algebraic expressions
- perform operations involving directed numbers
- substitute numbers for algebraic symbols
- apply the distributive law to remove brackets
- simplify algebraic fractions
- solve a simple linear inequality.

The question was attempted by 96.5 per cent of the candidates, 56 of whom scored full marks. The mean mark was 2.59.

Most candidates knew how to substitute numbers for the variables but could not simplify the directed numbers. In part (b), candidates were able to find the lowest common multiple but could not determine the equivalent fractions required. In part (c), candidates had difficulty transposing and using the distributive law correctly. In part (d), candidates were able to simplify at least one bracket and collect like terms, although the distributive law was used incorrectly in some cases.

Answer: (a) 2 (b) $\frac{5x-2}{6}$ (c) $x \geq -2$ (d) $5x - 5y$

Question 3

The question tested the candidates' ability to

- solve problems involving payments by instalments as in the case of hire purchase and mortgages
- solve problems involving salaries and wages

The question was attempted by 95.4 per cent of the candidates with 586 candidates scoring full marks. The mean mark was 3.81. Most candidates were able to find the interest, in dollars, in part (a), but could not express this interest as a percentage of the loan amount. Most candidates experiened difficulty calculating the hire purchase price in part (c) since they used 10% of the instalment as the deposit instead of 10% of the cash price. Part (c) was fairly well done with candidates correctly determining the amount earned at the basic rate and number of hours worked.

Answer: (a) (i) \$300 (c) (i) \$140
(ii) 5% (ii) 25 hours

(b) \$48.60

Question 4

The question tested the candidates' ability to

- use instruments to draw and measure angles and line segments
- use instruments (not necessarily ruler and compasses) to construct angles
- solve geometric problems using the properties of polygons, lines and angles and using the properties of circles.

The question was attempted by 83.5 of the candidates, 142 of whom scored full marks. The mean mark was 3.09.

Most candidates were able to determine the values of the angles in part (a). In part (b), candidates were unable to use the properties of the triangles to determine the unknown angle, including the fact that angle ABC was a right angle. Furthermore, candidates experienced difficulty identifying the angles correctly; hence they attempted to find the values of the wrong angles. In part (c), most candidates attempted to construct a full-scale diagram but there were some inaccuracies.

Answer: (a) (i) 56° (b) 55°
(ii) 124°
(c) (i) 4 cm

Question 5

The question tested the candidates' ability to

- solve problems involving measures and money (including exchange rates)
- calculate compound interest as appreciation, depreciation and amount (for not more than three periods).

The question was attempted by approximately 85 per cent of the candidates, 53 of whom earned full marks. The mean mark was 1.87.

Most candidates were able to convert the US dollars to Eastern Caribbean dollars, but did not understand the concept of bank charges either as a fixed amount or as a percentage. In part (b), although a few candidates were able to find the value of the vase after one year, they were unable to complete the task for two or three years.

Answer: (a) (i) EC $19 190 (b) (i) $441
(ii) Loss of EC $1 890

Question 6

The question tested the candidates' ability to:

- Calculate the length of an arc of a circle using angles at the centre whose measures are factors of 360°
- Calculate the area of the region enclosed by a rectangle, a triangle, a parallelogram, a trapezium, a circle or any combination of them
- Calculate the areas of sectors of circles
- Convert from one set of units to another, given a conversion scale.

The question was attempted by 70 per cent of the candidates. Of these, 13 scored full marks. The mean mark was 1.18. Most candidates were able to write down the length of the line segment AD and to determine the perimeter of ABCDE. The areas of weak performance included finding the length of the sector CD, where at times the length of the arc was confused with the area of the sector. They also had difficulty recognising that the sector represented one-quarter of a circle. In addition, candidates failed to add the two correct measurements to find the area of the composite figure. Very few candidates were able to answer part (c) where they were required to find the actual area given the scale.

Answer: (a) 12 cm (b) (i) 11 cm (c) 35m^2
(ii) 35 cm
(iii) 73.5 cm^2

Question 7

The question tested the candidates' ability to

- translate verbal phrases into algebraic symbols and vice versa
- solve linear equations in one unknown
- solve simultaneous linear equations in two unknowns algebraically.

The question was attempted by 81 per cent of the candidates. Of these, 44 scored full marks. The mean mark was 1.64. In 7 (a), the candidates selected and attempted to use appropriate methods to solve the simultaneous equations. However, the candidates tried to eliminate variables with different coefficients. In addition, candidates recognised that two values were required; hence they attempted to substitute the first value found to find a second value. In 7(b), parts (i) and (ii) were fairly well done. Candidates showed some understanding of writing algebraic expressions, however, they did not use the expressions obtained to find the equation needed in parts (iii) to (v).

Answer: (a) $x = -2$ and $y = 3$ (b) (i) \$(12 + x)
(ii) \$(2 + x)
(iii) $12 + x = 3(2 + x)$
(iv) $x = 3$
(v) \$3.00

Question 8

The question tested the candidates' ability to

- use Pythagoras' Theorem to solve simple problems
- use simple trigonometrical ratios to solve problems based on measures in the physical world related to heights and distances.

The question was attempted by 62 per cent of the candidates. Of these, 55 scored full marks. The mean mark was 1.40. A few candidates were able to draw the position of the ladder and show the horizontal angle. Generally, students were unable to complete a number of processes including selecting the correct ratio to calculate the height and distance. In cases where the correct ratio was selected, candidates had difficulty evaluating the expression to find the unknown angle as in part (c). In fact, many candidates did not attempt part (c).

Answer: (b) (i) 2.6 m
(ii) 2.3 m

(c) 30°

Question 9

The question tested the candidates' ability to

- construct a simple frequency table for a given set of data
- draw and use a histogram
- determine the range for a set of data
- determine experimental probability of simple events

The question was attempted by 91 per cent of the candidates. Of these, 7 scored full marks. The mean mark was 3.53.

The candidates were able to complete the frequency table, identify the modal mark and draw the correct heights for the bars on the histogram, however, the histogram was sometimes drawn as a bar graph or a line graph and the boundaries were not used in constructing the histogram. Candidates also experienced difficulty calculating the median mark and the range, and did not express the probability as a fraction.

Answers: (b) (i) 5 marks
(ii) 4 marks
(iii) (7 – 1) or 6 marks

(d) $\frac{4}{25}$

Question 10

The question tested the candidates' ability to

- interpret data presented as in a line graph
- recognise the gradient of a line as the ratio of the vertical rise to the horizontal shift
- determine the equation of a line given the gradient and one point on the line

The question was attempted by 85 per cent of the candidates. Of these, 5 scored full marks. The mean mark was 2.47.

Candidates failed to read the values from the graph to determine the solutions to parts (a), (b) and (c), although a few candidates were able to answer (a)(i) and (b) correctly. Some candidates knew the basic form of an equation and substituted their values of the gradient and y intercept. Candidates expressed the gradient as a ratio of two numbers but without reference to the given scale. Candidates did not see the link between the equation and determining the cost in part (f). Instead, they used a variety of incorrect approaches.

Answer: (a) (i) \$70.00
(ii) \$51.00

(b) 100 km

(c) \$20.00

(d) $\frac{\$0.20}{\text{km}}$

(e) $y = 0.20x + 20$

(f) \$86.00

C A R I B B E A N E X A M I N A T I O N S C O U N C I L

REPORT ON CANDIDATES' WORK IN THE
SECONDARY EDUCATION CERTIFICATE EXAMINATION
MAY/JUNE 2005

MATHEMATICS

MATHEMATICS

MAY/JUNE 2005

GENERAL COMMENTS

The General Proficiency Mathematics examination is offered in January and May/June each year, while the Basic Proficiency examination is offered in May/June only.

In May/June 2005 approximately 88 559 candidates registered for the General Proficiency examination, an increase of 3 773 over 2004. Candidate entry for the Basic Proficiency examination decreased from 7 861 in 2004 to 6964 in 2005.

At the General Proficiency level, approximately 39 per cent of the candidates achieved Grades I – III. This represents a 4 per cent increase over 2004. Seventeen per cent of the candidates at the Basic Proficiency level achieved Grades I – III compared with 20 per cent in 2004.

DETAILED COMMENTS

General Proficiency

In general, candidates continue to show lack of knowledge of basic mathematical concepts. The optional section of Paper 02 seemed to have posed the greatest challenge to candidates, particularly the areas of Relation, Function and Graphs; and Geometry and Trigonometry. Candidates did not access most of the marks awarded for higher order thinking. This indicates the inability to apply mathematical skills in novel situations.

Eleven candidates scored the maximum mark on the overall examination compared with six candidates in 2004. In addition, 351 candidates scored more than 95 per cent of the total marks.

Paper 01 – Multiple Choice

Paper 01 consisted of 60 multiple-choice items. This year, 167 candidates earned the maximum mark compared with 145 in 2004. Approximately 71 per cent of the candidates scored at least half the total marks for this paper.

Paper 02 – Essay

Paper 02 consisted of two sections. Section I comprised eight compulsory questions that totalled 90 marks. Section II comprised six optional questions: Two each in Relations, Functions and Graphs; Trigonometry and Geometry; Vectors and Matrices. Candidates were required to choose any two questions. Each question in this section was worth 15 marks.

This year, 14 candidates earned the maximum mark on Paper 02 compared with 16 in 2004. Approximately 20 per cent of the candidates earned at least half the maximum mark on this paper.

Compulsory Section

Question 1

This question tested candidates' ability to

- perform the basic operations on fractions
- solve problems involving invoices and shopping bills
- solve problems involving selling price, cost price and profit or loss.

The question was attempted by 99% of the candidates. The mean score was 7.82 out of 11.

Candidates were generally able to deduce unit price and quantity; calculate the difference between cost price and selling price to obtain profit or loss; obtain an appropriate LCM for the subtraction of two fractions and calculate 15% of a quantity.

However, some candidates experienced difficulty with writing a mixed number as an improper fraction, e.g. $1\frac{1}{9}$ was written as $\frac{11}{9}$; and applying the correct use of decimals to currency, e.g. \$16.242 was written as \$16.2 and \$37.50 to the nearest cent was written as \$37.5.

Recommendations:

Teachers need to

- Continue to reinforce operations on fractions at all levels.
- Encourage candidates to reflect on the stages in computations involving fractions when calculators are used.

Answers:

(a) $\frac{13}{15}$ (b) (i) A = \$37.50, B = \$16.95, C = 5 (ii) Profit

Question 2

The question tested candidates' ability to

- factorize using the distributive law, the difference of squares, and the method of grouping
- translate verbal phrases into algebraic symbols in forming expressions and equations
- use the distributive property to expand the product of two binomials.

The question was attempted by 99% of the candidates. The mean score was 4.63 out of 12.

Candidates were able to

collect terms in x and apply the distributive property in Part (b). They were also able to use the method of splitting and grouping to factorize the quadratic expression in (a) (iii); sum their three sets of points to 39 in Part (c) (ii).

Candidates experienced difficulty with differentiating between an expression and an equation; identifying the difference between two squares; differentiating between "twice" and "square": e.g. $2\,(x - 3)$ was written as $(x - 3)^2$; and translating verbal phrases into algebraic expressions: e.g. 3 points fewer than x points was written as $3 - x$ or as $3 < x$.

Recommendations:

- Help candidates to differentiate between expressions and equations.
- Remind candidates that marks allotted are usually in proportion to depth of response expected.

Answers:

(a) (i) $ab\,(5a + b)$ (ii) $(3k - 1)\,(3k + 1)$ (iii) $(2y - 1)\,(y - 2)$
(b) $bx^2 + 7x - 20$
(c) (i) $2\,(x - 3)$ (ii) $x + x - 3 + 2\,(x - 3) = 39$

Question 3

The question tested candidates' ability to

- identify the regions in a Venn diagram
- associate algebraic expressions with regions
- formulate simple linear algebraic equations
- solve simple linear equations in one unknown
- find the equation of a straight line given the gradient and a pair of coordinates
- show that two lines are parallel.

Approximately 93% of the candidates attempted this question. The mean score was 3.46 out of 11.

Most candidates were able to construct an equation in Part (a), in an attempt to find the value of the variable x. However, many were unable to correctly solve the equation.

Some candidates were not familiar with the use of the word "only", as such they were unable to correctly identify the region "drama only" represented by $7 + x$.

The majority of the candidates were able to substitute the values y, m and x into the equation of the straight line $y = mx + c$ in Part (b) (i).

Most candidates had the knowledge that parallel lines have the same gradient. However, most of them were unable to use the skills to show that gradients of the two lines were equal.

Candidates demonstrated a lack of basic computational skills and were unable to correctly manipulate the fractional gradient to find the value of c.

Transposing the equation $2x - 3y = 0$ into the form $y = mx + c$ posed a major challenge for the majority of the candidates.

Some candidates attempted to prove parallelism by plotting the graphs of the two lines. However, they were rarely able to plot correctly the points on the plane.

The following errors were identified in candidates' work.

(b) (ii) Finding the gradient using two points on the line:

When $x = 1$

$$2(1) - 3y = 0$$
$$2 - 3y = 0$$
$$2 = 3y$$
$$y = \frac{2}{3}$$

When $x = 0$

$$2(0) - 3y = 0$$
$$0 = 3y$$
$$0 = y$$

$$\text{Gradient} = \frac{y_2 - y_2}{x_2 - x_2}$$

$$= \frac{{}^2/_3}{1}$$

$$= \frac{2}{3}$$

Hence lines are parallel.

Recommendations:

- Teachers should use exercises that involve not only Universal set but the different regions and complements.
- Teachers should insist that candidates write the equation of the line after finding the values of the intercept and/or gradient.

Answers:

(a) (i) 6 (ii) 13

(b) (i) $y = \frac{2}{3}x + 7$

Question 4

The question tested candidates' ability to

- use scale factors in relation to the area of similar shapes
- compare the price of equal areas of pizzas or the areas that could be bought at the same cost
- use critical thinking in making judgements related to shopping.

The question was attempted by 94% of the candidates. The mean score was 4.56 out of 10.

Candidates generally knew how to calculate the cost of a whole medium pizza. They made reasonable conclusions based on the approaches taken to compare the sizes of the pizza.

Most candidates used either circumference or diameter to compare the sizes of the two circles. Some correctly used the areas but frequently arrived at the wrong conclusion even after calculating the area of the medium pizza to be 4 times the area of the small pizza.

In Part (b) of the question, the majority of candidates compared either areas only or prices only.

Recommendation:

- Candidates need to use the list of formulae on the question papers. The majority of candidates used p^2r, $2pr^2$, pd^2, $2pr$ as the formula for area of a circle.

Answers:

(a) The medium pizza is 4 times as large as the small one.
(b) The medium slice is the better buy.

Question 5

This question tested candidates' ability to

- locate the points on a grid to form a triangle
- locate the image of an object under a given transformation
- state a single transformation that maps an object to an image
- use and apply an appropriate trigonometric ratio in finding the angle of elevation in a given right-angle triangle.

Approximately 93% of the candidates answered this question. The mean score was 5.22 out of 12.

Some candidates were unable to plot a triangle accurately using the coordinate plane. A few had difficulty interpreting the scale of 1 cm to 1 unit and some inverted the axes.

A significant number of the candidates experienced difficulty reflecting in the line $x = 4$. Some used the line $y = 4$ for $x = 4$; some reflected in the x-axis, and others in the y-axis.

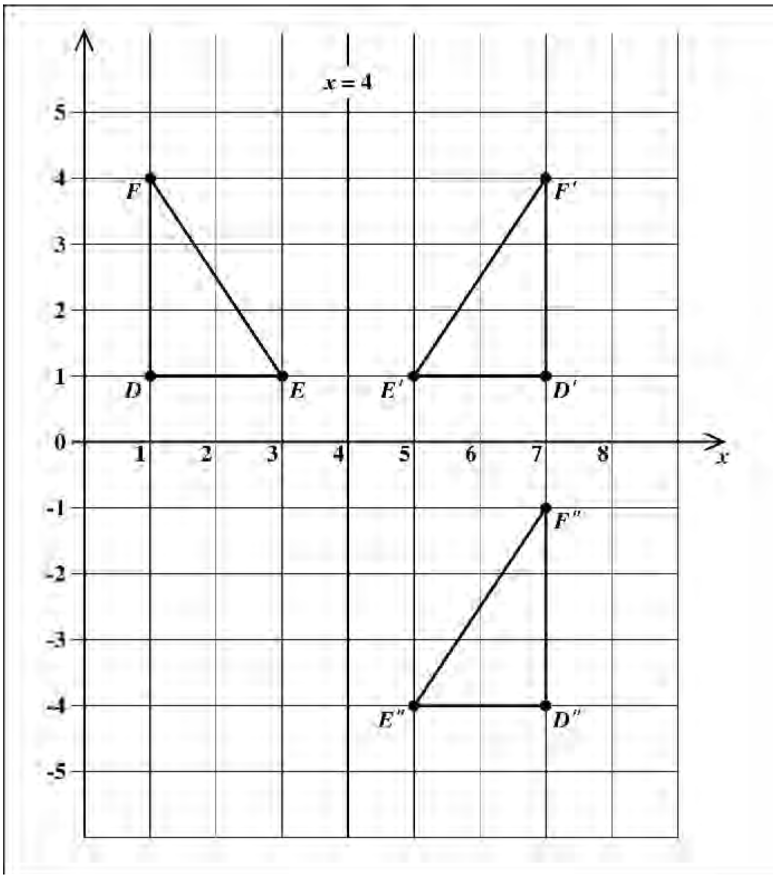
A few candidates preferred to use algebraic methods to locate the image of an object under the given translation. However, they multiplied the matrices instead of adding the column vectors. In general, the candidates showed mastery of translation, however, some had great difficulty interpreting the zero component of the vector and hence used their own vector to translate the figure.

In Part (b), most candidates were not familiar with the glide reflection and hence ended up describing the transformations instead of giving a single transformation that would map the object onto the image.

Part (c) was very challenging for many of the candidates. Identifying the angle of elevation was the major problem. Some candidates made their solutions much more complex than was necessary and attempted various methods in arriving at their answers. The sine and cosine rules were quite popular in calculating the required angle even though the triangles were right-angled.

Answers:

(a) (b) (c) angle of elevation of the sun = 42°



Question 6

This question tested candidates' ability to

- calculate unknown angles in given polygons by recognizing the relationship between angles formed by parallel lines
- find the values for simple, composite and inverse functions given values in their respective domains.

The question was attempted by 92% of the candidates. The mean score was 4.17 out of 12.

Part (a) of this question was poorly done. The candidates seem to lack knowledge of the relationship between parallel lines and the angles formed by them. They also had difficulty distinguishing the various shapes within the figure. A significant number of candidates indicated that the 108° was included in the quadrilateral ABCD.

It was evident, however, that candidates had no difficulty in determining the sum of angles in the triangle, quadrilateral or the polygon.

The performance in Part (b) was far better. For the most part, the candidates were comfortable with the concept of finding the value of the function given values in the domain. They knew substitution was required in determining the function $g(x)$. However, the response was less than clear as many included the g in their answers.

E.g. $g(x) = x^2$ implies that
$g(3) = g(9)$
$g(-3) = -3g$
$(-3)^2 = -9$

Finding inverses and computing composite functions also posed problems for candidates. A significant number of candidates determined the product, $[(fg(x) = (\frac{1}{2}x + 5) * (x^2)]$ or the sum of both functions, $[(fg(X) = (\frac{1}{2}X + 5) + (x^2)]$, instead of finding the function of a function, $(fg(x) = \frac{1}{2}[x^2] + 5)$. While most candidates realized that they had to interchange the variables in order to establish the inverse, most had difficulty with transposition. This underscores the difficulty with Algebra that is demonstrated generally.

Recommendations:

- Regular revision of the Euclidean Geometry should help as this was previously done in the lower forms. There is no alternative but to ensure that the students clearly understand functions. They also need to practise finding inverses and composite functions. The need to practise algebra and computation on an ongoing basis cannot be overstated.

Answers:

(a) (i) $x = 43°$ (ii) $y = 162°$
(b) (i) $g(3) + g(-3) = 18$ (ii) $f^{-1}(6) = 2$ (iii) $fg(2) = 7$

Question 7

This question tested candidates' ability to

- draw the cumulative frequency curve to represent data given the cumulative frequency table and a scale for each axis
- use the ogive to estimate
  – the number of persons in the sample having heights below a certain value
  – the median height
  – the height that 25% of the persons are less than
  – the probability that a person chosen at random had a height less than a given value.

About 77% of the candidates attempted the question. The mean score was 3.62 out of 12.

Part (a)

Most candidates showed knowledge of the axes although several of them did not pay attention to the given scales for each axis. They also failed to recognise the emphasis placed on the condition that the horizontal scale should start at 150 cm.

Candidates were not competent at assigning numbers to the axes. Some used the given class intervals; others, the class midpoint, the lower or upper class limit and the lower or upper class boundaries. Even though candidates would have chosen the incorrect class intervals, they were able to plot a cumulative frequency using their interval.

Part (b) presented the greatest difficulty.

In (b) (i) and (ii), depending on the numbering on the axes used in Part (a), candidates were unable to read off correctly from their graph. Although the appropriate lines were drawn, they did not read the values indicated correctly.

Candidates were aware that the median had something to do with 'the middle'. Some simply worked out ½ x 400 = 200.

Other responses included the average of any two values, the calculation of the mean, and the middle value of the highest value on <u>their</u> graph.

About 25% of the candidates drew appropriate lines at 170 cm and at 200 cm but were not successful in reading off the corresponding values asked for. A few drew lines at 50 to find the median.

For (b) (iii), candidates worked out as 25% x 400 = 100. Difficulty arose in reading off horizontally at 100.

Almost all the candidates who attempted (b) (iv) knew the answer had to be out of 400. Only about 10% obtained the correct answer. Many used the given height 162 and gave their answers as $\frac{162}{400}$. A few wrote answers that were greater than 1.

<u>Answers:</u>

(b) (i) 270 ± 5 (ii) 167 ± 1 cm (iii) 162.5 cm (iv) $\frac{95}{400}$ or $\frac{19}{80}$

<u>Question 8</u>

This was the *Critical Thinking* question which was designed to test candidates' ability to:

- identify number pattern from data given
- expand binomial expression
- simplify algebraic expression
- recognise an identity and prove same.

The question was attempted by approximately 90% of the candidates. The mean score was 5.45 out of 10.

Part (a) of the question was well done by most candidates. Candidates were able to successfully identifying the number pattern for 6^3 and 10^3 (Parts (i) and (ii) respectively), but had difficulty in obtaining the solution for the abstract n^3 (Part (iii).

In some cases, candidates calculated the value of 6^3 and 10^3 without using the pattern. They also assumed that since there were four spaces between 10^3 and n^3 and therefore substituted n as 14 and arrived at the correct solution.

Part (b) of the question was poorly done by candidates. The majority of candidates who attempted Part (b) had difficulty in expanding $(a - b)^2$ but were successful in expanding $ab\,(a + b)$. For the expansion of $(a - b)^2$ most candidates gave $a^2 - b^2$ or $(a - b)\,(a + b)$ as their solution.

There were instances where candidates gave numerical values for a and b and then substituted their values into the identity to prove the identity.

Recommendations:

- Teachers need to reinforce the differences between $a^2 - b^2$ (the difference of two squares) and $(a - b)^2$ (the square of the difference of two terms).
- Encourage more practice in multiplying a binomial expression by a linear expression.
- Encourage more practice in proving algebraic identities.

Answers:

(a) (i) $4 \times 7^2 + 3 \times 6 + 2 = 216$
(ii) $8 \times 11^2 + 3 \times 10 + 2 = 1000$
(iii) $(n - 2) \times (n + 1)^2 + (3 \times n) + 2$

Optional Section

Question 9

The question tested candidates' ability to

- write a quadratic expression in the form $a\,(x + h)^2 + k$
- deduce the minimum value of a quadratic function and the value of the domain for which this minimum value occurs
- solve a quadratic equation
- sketch the graph of a quadratic function.

Approximately 32% of the candidates attempted the question. The mean score was 4.13 out of 15.

Most candidates recognized the given equation as quadratic and set out to solve it by an acceptable method. Candidates, however, faltered in the following areas:

– The coefficients 5, 2 and –7 of $5x^2 + 2x - 7$ were substituted for a, b and c in $a(x + b)^2 + c$.

– When using the formula to solve $5x^2 + 2x - 7 = 0$, candidates used $x = -2 \pm \sqrt{\frac{4 + 140}{10}}$ and proceeded to divide only the discriminant by 10.
– To sketch the curve $y = 5x^2 + 2x - 7$, some candidates generated a table of values instead of simply using the roots, the y-intercept and the minimum point.

Answers:

(a) $5\,(x + \frac{1}{5})^2 - 7\frac{1}{5}$

(b) (i) $-7\frac{1}{5}$ (ii) $-\frac{1}{5}$

(c) $x = -\frac{7}{5},\ 1$

Question 10

The question tested candidates' ability to

- read and interpret speed-time graph and to calculate
  - acceleration over a given period of time
  - distance covered over a given period of time
- read and interpret distance-time graph and to calculate average speed and describe specific situation of an athlete being at rest over a specific period of time.
- write equation of a straight line parallel to the *y*-axis
- write a set of inequalities representing a given region identified by a set of given equations of lines (on graph).

About 51% of the candidates attempted this question. The mean score was 2.90 out of 15.

In Part (a), candidates knew that they had to find the area under the graph in order to obtain the distance travelled/covered. However, many candidates used "distance = speed x time", to find the distance covered instead of calculating the actual area under the curve using only the trapezium rule or area of triangle + area of rectangle.

In Part (b), the candidates knew that somehow acceleration was related to velocity and time but the majority used the wrong formula. Candidates seemed to have been unsure of the correct units to be used.

The horizontal line in the distance-time graph was associated with "something constant or stationary", but were unable to describe the athlete's situation as "at rest".

Some responses given by candidates were

- athlete was moving at constant speed
- running on the spot
- run parallel to the finish line
- kept on a straight track
- distance did not climb.

In Part (c), candidates had difficulty recognising that equations of lines could simply be 'read off' from the graph. The majority of them tried to calculate the 'given' equations of lines using different methods.

Recommendation:

Real life situations should be used to enhance the teaching and learning of Relations, Functions and Graphs.

Answers:

(a) (i) $2\frac{2}{3}$ m/s (ii) 900 m

(b) (i) 6 km/h (ii) athlete stopped (iii) 8 km/h

(c) (i) $x = 6$
(ii) $x \leq 6$

$$y \geq -\frac{5}{8}x + 5$$

$$y \leq \frac{1}{6}x + 5$$

Question 11

This question tested candidates' ability to

- solve geometrical problems involving similar triangles and to find areas of similar triangles
- use geometry and trigonometry to find angles and distances in a composite diagram involving two triangles conjoined to form a trapezium.

The question was attempted by approximately 25% of the candidates. The mean score was 1.96 out of 15.

The area of good performance in this question was the calculation of angle $S\hat{J}M$.

At least one candidate approached the first part of the question by determining the height of D PQX with QX as base and then applying the basic sine ratio to find angle $P\hat{X}Q$.

A few candidates correctly solved Part (b) of the question by drawing perpendiculars from S to MJ and from J to MK and then proceeded to use basic trigonometric ratio.

In Part (e), the majority of candidates generally did not know the relationship between the areas of similar triangles. The fewer candidates who knew were unable to apply the relationship between the areas of similar triangles in calculating the area of Δ YXZ.

In responding to this question, many candidates rounded off the values unnecessarily which resulted in significant deviations from the expected answers.

Recommendation:

- The teacher should use physical or mathematical models to develop an understanding of the relationship between the areas of similar figures.

Answers:

(a) (i) $P\hat{X}Q = 53°$ (ii) Area = 54 cm^2

(b) (i) a) $S\hat{J}M = 22°$ b) $J\hat{K}M = 34°$

(ii) MJ = 92.7 m JK = 62.1 m

Question 12

The question tested candidates' ability to

- draw a sketch of the earth showing the location of two places when given their latitude and longitude
- label on diagram, the equator and the meridian of Greenwich
- calculate the shortest distance between two places measured along their common circle of latitude
- calculate the shortest distance between two places measured along their common circle of longitude.

About 11% of the candidates attempted this question. The mean score was 5.03 out of 15.

Most candidates who attempted this question were able to

- draw and recognise the relative position of the equator and the meridian
- represent the earth as a sphere/circle
- determine the location of places when given their position (degrees latitude and longitude)
- use correct formula in order to find/calculate the arc length (distance).

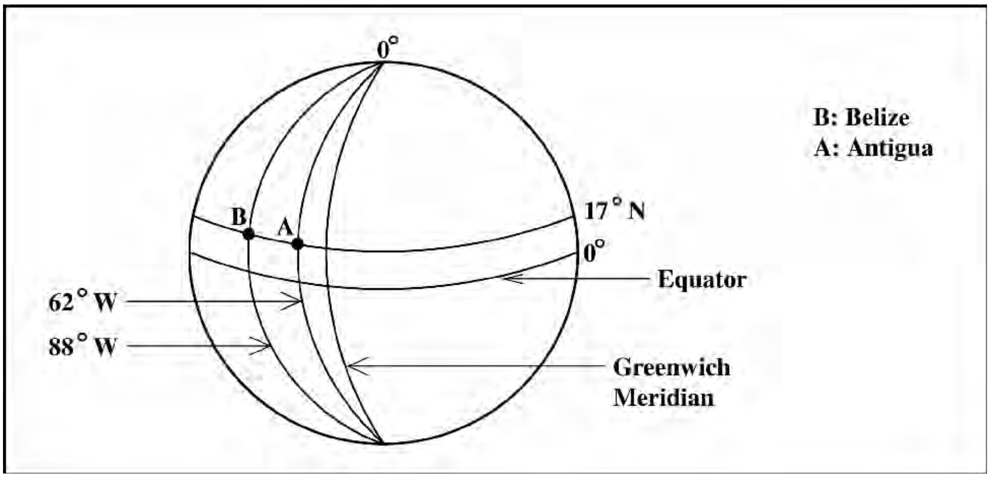
However, quite a number of candidates had difficulty in identifying and using the radius of the circle of latitude 17°N, whereas others knew which formula to use but were unable to manipulate the given information in the formula.

Recommendation:

- The use of a globe should be encouraged in classroom when teaching earth geometry. This will enhance candidates' understanding of equator, meridian, latitude and longitude concepts. Using the globe will also enhance candidates' understanding of East and West of Greenwich.

Answers:

(a)



(b) 2776 km
(c) 6140 km

Question 13

This question tested candidates' ability to use vectors to represent and solve problems in geometry.

The question was attempted by approximately 13% of the candidates. The mean score was 2.44 out of 15.

Candidates generally performed poorly on this question. The model mark was 0 and most candidates failed to secure more than 3 marks.

Generally candidates were able to correctly determine the expression for $\overrightarrow{AB}$ and identify correct routes to define different vectors. There were some unique approaches to solving Parts (c) and (d).

For Part (c), one candidate sought to find the dot product of $\overrightarrow{AP}$ and $\overrightarrow{AE}$ and went on to show that the angle between $\overrightarrow{AD}$ and $\overrightarrow{AE}$ was 0°, thus implying that AP and E are collinear.

For Part (d), most candidates attempted to prove that triangle ADE was isosceles by establishing two equal sides. However, one candidate used a vector approach to calculate the angles in the triangle and thus prove that ADE is isosceles.

In Part (e), the 'reversal' of vectors and changing the signs appeared to be a common area of concern.

In proving collinearity of AP and E, many candidates while establishing the scalar relationship between two vectors taken from AP, PE or AE did not state the fundamental fact that there was a common point.

Recommendation:

- Teachers need to use a more practical approach to teaching candidates about vectors so that the changing of direction as well as collinearity may be better understood.

Answers:

(a) (i) $\overrightarrow{AB} = 3x$ (ii) $\overrightarrow{BD} = -3\underline{x} - 3\underline{y}$ (iii) $\overrightarrow{DP} = \underline{x} + \underline{y}$

(d) $|\overrightarrow{AE}| = |\overrightarrow{DE}| = 3$ units

Question 14

This question was designed to test candidates' ability to

- evaluate the determinant of a 2 x 2 matrix
- identify a 2 x 2 singular matrix
- obtain the inverse of a non-singular 2 x 2 matrix
- use matrices to solve simple problems in Algebra
- determine the matrices associated with reflection, rotation and translation
- perform addition, subtraction and multiplication of matrices
- determine the 2 x 2 matrix representation of the single transformation which is equivalent to the composition of two linear transformations in a plane.

While approximately 27% of the candidates attempted this question, only one in ten responses were satisfactory. The mean score was 3.44 out of 15.

Generally, candidates were able to find the determinant, however, they only saw its relevance in obtaining the inverse matrix. Very few were aware of the association between the determinant and a singular matrix.

Both transposing and stating the inverse were generally well done. Unfortunately, only about 10% of the candidates attempting this part of the question were able to state the identity matrix. Hence, $M \times M^{-1} = I$ was not widely known.

Candidates seemed to have understood that they were required to use matrices to solve problems in algebra. However, many seemed confused by the term "pre-multiply".

About 5% of the candidates opted to use simultaneous equations and the process of elimination to solve (a) (iv).

Almost all the respondents attempted Part (b). Most were able to state satisfactorily the matrices for Parts (i), (ii) and (iii). However, Part (iv) proved to be the most difficult section of the question. It was poorly attempted and those who did encountered problems transposing matrices in the correct order. In some cases, candidates seemed to recognise what the outcome should be and simply stated rather than calculated.

In performing the combined translation NT(P), many sought to multiply NT first rather than perform the translation on P followed by the rotation.

Answers:

(a) (i) $| M | \neq 0, \therefore m$ non-singular

(ii) $M^{-1} = -\frac{1}{5}\begin{pmatrix} 15 & -5 \\ -7 & 2 \end{pmatrix}$

(iii) $MM^{-1} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

(iv) $x = 26$, $y = -11$

(b) (i) $R = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$

(ii) $N = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$

(iii) $T = \begin{pmatrix} -3 \\ 5 \end{pmatrix}$

(iv) P' (6, –11)
P'' (–3, –16)

DETAILED COMMENTS

Basic Proficiency

The Basic Proficiency examination is designed to provide the average citizen with a working knowledge of the subject area. The range of topics tested at the basic level is narrower than that tested at the General Proficiency level.

Candidates continue to demonstrate a lack of knowledge of the fundamental concepts being tested at this level.

Approximately 16 per cent of the candidates achieved Grades I - III. This reflected a 4 per cent decline in performance compared with 2004 where 20 per cent of the candidates achieved Grades I - III.

Paper 01 – Multiple Choice

Paper 01 consisted of 60 multiple-choice items. No candidate earned the maximum mark on this paper. However, one candidate earned 59 out of 60 possible marks. Approximately 45 per cent of the candidates scored at least half the maximum mark for this paper.

Paper 02 – Essay

Paper 02 consisted of 10 compulsory questions. Each question was worth 10 marks. The highest score earned was 96 out of 100. This was earned by two candidates. Five candidates earned at least half the total marks on this paper.

<u>Question 1</u>

The question tested candidates' ability to

- divide chemicals
- write a rational number in standard form
- compare quantities in a given ratio
- solve problems involving percentages.

The question was attempted by 95% of the candidates. Less than 1% of the candidates scored the maximum mark. The mean score was 2.07 out of 10. Thirteen percent of the candidates scored 5 marks or more on this question.

The candidates were generally unable to express a number in standard form or to establish the relationship between the given ratio and the points. Most candidates answered (c) (ii) correctly, where they were required to give the least number of points to be scored by Team B, however a number of candidates had difficulty interpreting the word 'least'.

<u>Answers:</u>

(a) (i) 0.455 (ii) 4.55×10^{-2}
(b) 700
(c) (i) 54 (ii) 111 points

Question 2

The question tested candidates' ability to

- solve problems involving compound interest
- solve problems involving salaries and wages.

The question was attempted by 96% of the candidates, 2% of whom scored the maximum mark. The mean score was 3.02 out of 10. Twenty-seven percent of the candidates scored 5 marks or more on this question.

Candidates had difficulty calculating the compound interest. The majority correctly calculated the interest for the first year but did not proceed any further. A few candidates attempted to use the formula to determine the interest but gave the answer as the total value of the investment after two years instead of subtracting to find the interest.

The majority of the candidates were able to calculate the total weekly wage and quite a few correctly calculated the number of overtime hours worked. However, the candidates were not able to calculate the total wage earned for the week or the number of hours worked overtime when given the wage for the week.

Answers:

(a) $168

(b) (i) $640 (ii) $780 (iii) 51 hours

Question 3

The question tested candidates' ability to

- solve problems involving taxes
- solve problems involving hire purchase.

The question was attempted by 94% of the candidates, 4% of whom scored the maximum mark. The mean score was 3.23 out of 10. Thirty-three percent of the candidates scored at least 5 marks on this question.

Candidates had difficulty calculating the tax-free allowances, taxable income and interpreting the data in the table.

In Part (b), the majority of candidates were able to calculate the cash price and to find the hire purchase price of the car.

Answers:

(a) (i) $5 100 (ii) $26 900 (iii) $3 270
(b) (i) $7 392 (ii) $8 460 (iii) $1 068

Question 4

The question tested candidates' ability to

- solve linear equations in one unknown
- solve simultaneous linear equations in two unknows algebraically.

The question was attempted by 82% of the candidates, less than 1% of whom scored the maximum mark. The mean score was 1.29 out of 10. Ten percent of the candidates scored at least 5 marks on this question.

Most of the candidates were unable to eliminate a variable accurately, but chose an appropriate method to substitute the first value found to determine the second unknown.

Generally, candidates were unable to recognize and use the properties of an isosceles triangle or the sum of the angles in a triangle to determine the value of the angle at C. Further, candidates had difficulty solving a linear equation in one variable.

Answers:

(a) $x = 5$; $y = 2$
(b) (i) $(p + 3)°$ (ii) <A = 58°; <B = 61°; <C = 61°

Question 5

The question tested candidates' ability to

- substitute numbers for algebraic symbols in algebraic expressions
- simplify algebraic expressions
- solve a simple linear inequality in one unknown.

The question was attempted by 86% of the candidates, less than 1% of whom scored the maximum mark. The mean score was 1.61 out of 10. Eleven percent of the candidates scored 5 marks or more on this question.

Most of the candidates correctly completed the substitution although there were errors in evaluating the expression. In Part (b), the candidates were able to determine the Lowest Common Multiple of the algebraic fractions but could not perform the subtraction. The majority of the candidates had difficulty solving the inequality and showing the solution set on a number line.

Answers:

(a) 18 (b) $\frac{x-2}{8}$ (c) $x > -6$

Question 6

The question tested candidates' ability to

- draw and interpret graphs
- determine the gradient of a line
- determine the equation of a line.

The question was attempted by 73% of the candidates, less than 1% of whom scored the maximum mark. The mean mark was 1.68 out of 10. Thirteen percent of the candidates scored at least 5 marks on this question.

Most of the candidates experienced difficulty finding the gradient of the line and writing the equation of the line. While some of the candidates were able to complete the table and insert the missing values; they had difficulty drawing the quadratic graphs. Only a few candidates were able to identify the points of intersection.

Answers:

(a) (i) 1 (ii) $y = x - 1$

(b) (i)

x	1	4
y	0	3

(iii) (1, 0) and (4, 3)

Question 7

The question tested candidates' ability to

- solve simple problems involving distance, time and speed
- calculate the region enclosed by a rectangle
- calculate the volume of a cuboid.

The question was attempted by 87% of the candidates, less than 1 % of whom scored full marks. The mean mark was 1.47 out of 10. Five percent of the candidates scored at least 5 marks on this question.

Most of the candidates were able to calculate the time taken to travel from one point to the next, but very few candidates recognised that the time in minutes should have been converted to hours to calculate the average speed for the journey.

Candidates generally knew how to find the area of the rectangular base, but could not proceed to find the volume of water in the container. In (b) (iii), the majority of candidates could not convert between litres and cubic centimetres and were unable to determine the height of water in the container.

Answers:

(a) (i) 40 minutes (ii) 48 km/h

(b) (i) 3 000 cm^2 (ii) 45 000 cm^3 (iii) 28 cm

Question 8

The question tested candidates' ability to

- use instruments to draw and measure angles and line segments
- use instruments to construct a rectangle
- use Pythagoras' theorem to solve simple problems
- solve geometric problems.

The question was attempted by 70% of the candidates, 1% of whom scored the maximum mark. The mean mark was 2.06 out of 10. Fifteen percent of the candidates scored 5 marks or more on this question.

The majority of candidates were able to construct the line segment accurately and a small percentage accurately constructed the angle of 90 degrees. Most of the candidates who completed a rectangle were able to determine the length of the diagonal.

Very few candidates correctly calculated the length of the side BD, since they did not recognise that Pythagoras' theorem could be applied. Similarly, candidates did not know how to determine the scale factor for the enlargement.

Answers:

(a) (ii) 9.6 $\pm$ 0.1 cm
(b) (i) BD = 20 cm (ii) scale factor is 3

Question 9

The question tested candidates' ability to

- identify and describe a transformation given an object and its image
- use simple trigonomic ratios to solve problems based on measures in the physical world.

The question was attempted by 52% of the candidates, 2% of whom scored the maximum mark. The mean mark was 1.45 out of 10. Eleven percent of the candidates scored 5 marks or more on this question.

Some candidates recognised that the coordinates of the two points could be subtracted to obtain the column vector and a few candidates correctly obtained a value for the *y* value. However, the subtraction with the directed numbers proved challenging.

Even though candidates identified the correct ratios to be used in calculating the length of the flagpole and the angle of elevation, candidates either did not substitute the correct ratios or did not transpose correctly.

Answers:

(a) 10 16
(b) (i) FG = 8 cm (ii) angle of elevation is 53 degrees

Question 10

The question tested candidates' ability to

- interpret data given in a bar graph
- determine probability for simple events
- determine when it is appropriate to use mean, median or mode for a set of data.

The question was attempted by 94% of the candidates, 4% of whom scored the maximum mark. The mean mark was 4.14 out of 10. Forty percent of the candidates scored 5 marks or more on this question.

The majority of candidates were able to read the bar graph and interpret the data. However, some candidates had difficulty in calculating the median and could not determine which average was most appropriate. There was also some difficulty in expressing the required probability.

Answers:

(a)	12 children	(b)	29 children
(c)	The modal size is 5	(d)	The median shoe size is 6
(e)	(i) 14/50	(ii)	21/50
(f)	The mode – the size worn by the most children		

C A R I B B E A N E X A M I N A T I O N S C O U N C I L

REPORT ON CANDIDATES' WORK IN THE SECONDARY EDUCATION CERTIFICATE EXAMINATION MAY/JUNE 2006

MATHEMATICS

MATHEMATICS

MAY/JUNE 2006

GENERAL COMMENTS

The General Proficiency Mathematics examination is offered in January and May/June each year, while the Basic Proficiency examination is offered in May/June only.

In May/June 2006 approximately 86 479 candidates registered for the General Proficiency examination, a decrease of 2 080 over 2005. Candidate entry for the Basic Proficiency examination decreased from 5 803 in 2005 to in 2006.

At the General Proficiency level, approximately 35 per cent of the candidates achieved Grades I – III. This represents a 4 per cent decrease over 2005, however it is consistent with the performance in 2004. Forty per cent of the candidates at the Basic Proficiency level achieved Grades I – III compared with 17 per cent in 2005.

DETAILED COMMENTS

General Proficiency

In general, candidates continue to show lack of knowledge of basic mathematical concepts. The optional section of Paper 02 seemed to have posed the greatest challenge to candidates, particularly the areas of Relation. Function and Graphs; and Geometry and Trigonometry.

Six candidates scored the maximum mark on the overall examination compared with 11 candidates in 2005. Twenty-six per cent of the candidates scored at least half the available marks compared with 31 per cent in 2005.

Paper 01 - Multiple Choice

Paper 01 consisted of 60 multiple-choice items. This year, 84 candidates earned the maximum available mark compared with 167 in 2005. Approximately 47 per cent of the candidates scored at least half the total marks for this paper.

Paper 02 - Essay

Paper 02 consisted of two sections. Section I comprised eight compulsory questions totalling 90 marks. Section II comprised six optional questions; Two each in Relations, Functions and Graphs; Trigonometry and Geometry; Vectors and Matrices. Candidates were required to choose any two questions. Each questions in this section was worth 15 marks.

This year, 17 candidates earned the maximum available mark on Paper 02 compared with 14 in 2005. Approximately 20 per cent of the candidates earned at least half the maximum mark on this paper in 2006 as well as 2005.

Compulsory Section

Question 1

This question tested candidates' ability to

– perform basic operations with decimals
– approximate a value to a given number of significant figures
– calculate the percentage of a quantity
– express one quantity as a percentage of another
– solve problems involving depreciation
– convert from one currency to another given a conversion rate.

The question was attempted by 93 per cent of the candidates, 5 per cent of whom scored the maximum available mark. The mean mark was 6.42 out of 12.

Many candidates demonstrated good use of the calculator in performing the operations. Those candidates who neglected to use electronic calculators frequently made conceptual errors in computation, for example $0.246 \div 3$ was computed as either 0.0082 or 0.82, and $(12.3)^2$ was computed as 24.6.

Some candidates did not apply the principle of order of operations and chose to ignore the brackets completely thereby performing the operations from left to right.

Many candidates showed inaccurate understanding of significant figures. A large number confused significant figures with decimal places and standard form and clearly did not understand the role of zero in retaining the value of the number.

For example: 151. 208 was incorrectly rounded to 15, 150.000, and 151.21.

Even those candidates who obtained 11 marks, failed to earn the mark for approximation to significant figures.

In part (b), the majority of candidates understood how to find the value of the car after one year and computed 12 per cent of \$40 000 correctly.

Calculating the rate of depreciation posed problems for several candidates who gave 85 per cent as the rate instead of 15 per cent.

Many candidates demonstrated a poor understanding of depreciation. A common error was the use of 24 per cent of \$40 000 instead of using the principles of compound interest.

The currency conversion in part (c) was well done by the majority of candidates. However a significant number of candidates chose the same operation for both parts of the question.

Solutions:

(a)	**(i)**	**151.208**	**(ii)**	**150**
(b)	**(i)**	***p*= \$4 800.**	**(ii)**	***q*= 15%**
(c)	**(i)**	**US\$600.00**	**(ii)**	**EC\$2 500.**

Recommendations

- Teachers need to emphasize the difference between decimal places, significant figures and standard form. With regards to significant figures, students need to appreciate that when a number has been rounded its value does not change significantly.

- In the calculating of depreciation, the value of the item reduces with each succeeding year. Hence, interest must be calculated on the reduced value rather than the initial value of the item.

Question 2

The question tested the candidates' ability to

– simplify algebraic fractions
– factorise quadratic expressions using the distributive law and the difference of two squares
– use and apply factorization in simplifying an algebraic fraction
– use simultaneous equations to solve a worded problem.

The question was attempted by 92 per cent of the candidates, 3 per cent of whom scored the maximum available mark. The mean mark was 4.54 out of 12.

Responses were generally unsatisfactory. Candidates generally demonstrated proficiency in computing L.C.M, factorizing using the difference of two squares and translating the worded problems into linear equations. In attempting to simplify the algebraic fraction, a significant number of candidates had difficulty with expanding -3(x-2), they incorrectly obtained -3x-6 while other candidates did not retain the L.C.M in the expression. Some candidates could not factorise $x^2 - 5x$ and incorrectly treated it as a difference of two squares.

Part (b) (ii) was not attempted by many candidates. Only the very able candidates recognized that factorisation of both the numerator and denominator was necessary before the expression can be simplified.

In solving the simultaneous equations, those candidates who used the method of elimination did not always know whether to add or subtract to eliminate a variable while those who used substitution made errors in transposing.

Solutions:

(a) $\dfrac{2x-9}{15}$

(b) **(i)** **a)** $x(x-5)$
b) $(x-9)(x+9)$

(ii) $\dfrac{a}{a-1}$

(c) **$20.00**

Recommendations

- Teachers need to emphasize the meaning of terms such as solve, simplify and factorise. Students must also be able to distinguish between expressions and equations.

- Teachers must encourage students to determine the most efficient method to use in solving simultaneous equations.

Question 3

The question tested the candidates' ability to

– solve geometric problems using the properties of triangles, lines and angles.
– solve problems involving the use of Venn diagrams with not more than two sets.

The question was attempted by 92 per cent of the candidates, 3 per cent of whom scored the maximum available marks. The mean mark was 3.57 out of 11.

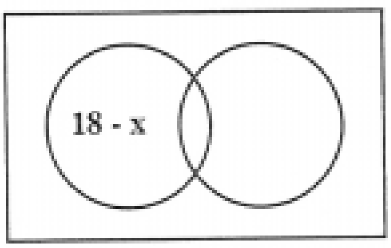
The overall performance on this item was unsatisfactory. In part (a), many candidates failed to use the properties of isosceles triangles and some were unable to identify the specific angles by their names. Reasons were either omitted or inaccurate and candidates had difficulty expressing their ideas using concise mathematical terminology.

In part (b) candidates generally knew how to label the complement of the two sets, however, they omitted to subtract x, the number of elements in the intersection from the number of elements in each set. Hence 18 and 15 were used instead of (18-x) and (15-x) on the Venn Diagram. Candidates were able to calculate x but omitted part (ii) which specifically asked for an expression in x.

Solutions:

(a) **(i)** **40°** **(ii)** **100°** **(iii)** **110°**

(b) **(i)**



(ii) **$2x + 33$** **(iii)** **3**

Recommendations

- Teachers need to focus on the properties of polygons especially quadrilaterals and triangles, describing the features of these polygons according to the lengths of the sides and the measure of the angles.

- Students need to use Venn diagrams to solve worded problems. Emphasis should be placed on the use of algebraic expressions and equations to determine unknown values.

Question 4

The question tested the candidates' ability to

– use instruments to construct a triangle and an angle of 60°
– draw line segments of a given length and measure the length of a line
– draw a line perpendicular to a given line
– use trigonometric ratios to determine the length of one side of a triangle
– calculate the perimeter of a triangle
– calculate the area of a triangle.

The question was attempted by 85 per cent of the candidates, 9 per cent of whom scored the maximum available mark. The mean mark was 5.9 out of 12.

Candidates generally succeeded in drawing the triangle with the prescribed dimensions and in measuring BC. They were also able to calculate the perimeter and area of triangle ABC.

Generally candidates experienced difficulty with the construction of a 60° angle, locating the point D and drawing the line through D, perpendicular to AB.

Some common errors were locating D outside of AB; drawing a perpendicular bisector of AB instead of a line through C, perpendicular to AB; using the ruler to measure lengths in that the initial point of measurement was taken as 1 instead of zero; using a protractor to measure 60° instead of constructing the angle; and using incorrect values in the formula for finding the area of a triangle.

Solutions:

(b) **7.0 to 0.1 cm**

(c) **20 cm**

(e) **4.33 cm**

(f) **17.3 cm^2**

Recommendations

- Teachers need to emphasize the difference between draw and construct and ensure that students can use instruments properly.

- Since a large number of candidates had problems interpreting directions, teachers must present opportunities for students to interpret instructions and pose their own instructions so that they gain competence in the proper use of the language of mathematics.

Question 5

The question tested the candidates' ability to use graphs of a quadratic function to

– determine the elements of the domain that have a given range
– determine the roots of the function
– determine the minimum point of the function
– determine the intervals of the domain for which the elements on the range may be greater than or less than a given value
– estimate the value of the gradient of a curve at a given point given the tangent at the point.

This question was attempted by approximately 62 per cent of the candidates, less than 1 per cent of whom obtained the maximum available mark. The mean mark was 2.49 out of 11.

In general, the performance was very unsatisfactory. It was evident from the responses that candidates were not familiar with interpreting graphs and the majority of them attempted to use calculations to answer the questions posed.

In attempting to identify the values of a and b which define the domain $a \leq$ x $d \leq$, candidates gave coordinates or vectors as answers rather than values.

The following incorrect answers were seen:

$\begin{pmatrix} -2 \\ 4 \end{pmatrix}$, (-2, 4), (-2,5) and (4, 5)

Part (b) required the candidate to state values of x for which $f(x) = 0$.

Many candidates used various algebraic methods such as solving by factorisation, completing the square, and using the quadratic formula.

Many candidates could identify the coordinates of the minimum point but some reversed the coordinates while others wrote them as column matrices.

Some candidates even used $\frac{4ac - b^2}{4a}$ to find the minimum value of y and $\frac{-b}{2a}$ to find the value of x when it occurs.

Although this question did not require candidates to draw a graph of $y = f(x)$, many of them redrew the graph and pointed out the minimum point.

In part (d) candidates were unable to list whole number values for which $f(x) < 1$. This part was omitted by a large number of the candidates. Others misinterpreted this question and listed all the whole numbers in the domain for the function.

Determining the gradient of the function at the point $x = 2$ posed great difficulty for candidates.

Many candidates demonstrated knowledge of the formula $m = \frac{y_2 - y_1}{x_2 - x_1}$ and attempted to use it but the points chosen were often not on the given tangent.

Careless errors in manipulating negative coordinates were also common. A significant number of candidates used the correct formula but interpreted the scales incorrectly.

Solutions:

(a) $\boldsymbol{a = -2 \quad b = 4}$

(b) $\boldsymbol{x = -1}$ **and** $\boldsymbol{x = 3}$

(c) **(1, – 4)**

(d) **0, 1 ,2, 3**

(e) **2**

Recommendations

- Teachers must caution students to pay attention to specific instructions given in a question.

- Since the majority of candidates opted to use calculations rather than read the graphs, it appears that skills in graphical interpretation are not well developed and more time must be devoted to such skills rather than merely drawing graphs.

Question 6

The question tested candidates' ability to

– use Pythagoras' Theorem to solve simple problems
– use trigonometric ratios in the solution of right-angled triangles
– solve problems involving bearings
– solve quadratic equations

The question was attempted by 77 per cent of the candidates, 4 per cent of whom scored the maximum available mark. The mean mark was 2.95 out of 11.

Most of the candidates were able to label the right-angled triangle correctly.

However, a significant number of them applied Pythagoras' Theorem incorrectly by using x as the hypotenuse. Some common errors were

$$x = (x+7)^2 + 13^2 \quad \text{and} \quad x = (x+7)^2 - 13^2$$

Squaring $(x+7)^2$ was also a major source of difficulty for many candidates who obtained the following incorrect results:

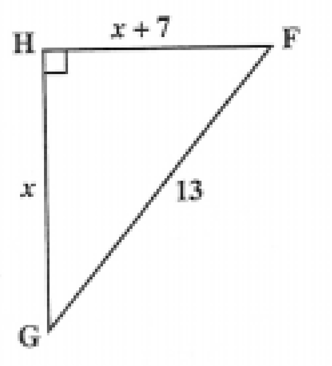
$$x^2 + 49\,; \quad 14x + 49 \quad \text{and} \quad x^2 + 7^2\,.$$

In solving the quadratic equation, those candidates who chose to use the quadratic formula or complete the square made several errors and were not as successful as those who used factorization.

Few candidates were successful at finding the bearing of F from G. However, the use of inefficient methods such as the sine or cosine rule instead of the trigonometric ratios could have resulted in loss of valuable time since these methods require lengthy calculations.

Solutions:

(a)



(b) $x^2 + (x + 7)^2$

(c) **5 km**

(d) **067.4°**

Recommendations

- Teachers need to encourage students to explore all possible methods in solving triangles and in choosing the most efficient method in a given situation.

Question 7

The question tested the candidates' ability to

– use the mid-point of the class interval to estimate the mean of data presented in grouped frequency tables

– complete a frequency polygon

– determine the probability of an event using data from a frequency table.

The question was attempted by 83 per cent of the candidates, 2 per cent of whom scored the maximum available mark. The mean mark was 3.93 out of 11.

The majority of candidates were competent in calculating the mid-interval values but only a small number of them displayed the ability to compute the mean.

Although they were able to correctly state the formula, they failed to use it successfully. Some common errors were failure to multiply mid-interval values by the corresponding frequencies; obtaining incorrect products and subsequent wrong summations; and dividing by the sum of the mid-interval values or by the number of class intervals instead of dividing by the sum of the frequencies

In attempting to complete the frequency polygon, many candidates had difficulties interpreting the given scale. As such, some opted to draw their own graph, and use a scale that they seemed comfortable to work with. In almost all the cases, candidates failed to complete the polygon by right-hand closure showing that they did not have the full concept of a polygon. In many instances, candidates failed to use a ruler. Many candidates chose to draw a histogram first and then construct the frequency polygon.

In the final part of the question, most candidates did not understand how to compute the probability and expressed their answers as whole numbers greater than one.

Solutions:

(a) **22, 27, 32**

(b) **(i) 17.85 kg**

(c) **32/100**

Recommendations

Teachers need to

- emphasize that a frequency polygon must be closed
- allow students to extract raw data from frequency tables and emphasize that for data in grouped frequency tables, some assumptions must be made
- vary the scales used in drawing graphics as much as possible the scales in graphs so that students will develop competence in interpreting scales
- use simple experiments to develop the concept of probability so that students appreciate the range of the probability scale.

Question 8

The question tested the candidates' ability to

– recognize patterns in shapes
– recognize number patterns and sequences
– predict subsequent steps in a pattern
– use algebraic reasoning to generalize a rule for a number pattern.

The question was attempted by 85 per cent of the candidates, 2 per cent of whom scored the maximum available marks. The mean mark was 6.13 out of 10.

The performance in this question was generally good. The candidates who performed well were generally able to draw the next square in the sequence and complete the table of values for $n = 4$ and $n = 7$.

Candidates were less successful in using algebraic thinking to generalise a rule for the series.

Solutions:

(ii) **a)** **40**

b) **112**

(b) **(i)** $\boldsymbol{n(n + 1) \times 2}$

(ii) **10; 10 x 11 x 2**

Recommendations

- Teachers should give students opportunities to create their own shape patterns and number patterns. These patterns should be analyzed so that a general rule emerges; such activities will enable them to develop their reasoning and analytical skills.

Question 9

The question tested the candidates' ability to

– solve simultaneous equations involving one linear and one quadratic equation
– translate verbal statements into algebraic symbols
– change the subject of an algebraic equation
– solve a problem involving a quadratic equation.

The question was attempted by 26 per cent of the candidates, 6.1 per cent of whom scored the maximum available marks. The mean mark was 2.83 out of 15.

In general, this question was poorly done. Many candidates were able to determine the strategy to be used in solving the simultaneous equations. While some experienced problems in eliminating one variable, the majority of candidates used substitution to find the second variable.

The rest of the question proved to be extremely challenging to the candidates. Candidates were unable to use the concepts of perimeter and area to write an algebraic expression for the length of the wire. They also had difficulty in solving the quadratic equation using the formula. Some rounded pre-maturely and obtained inaccurate answers.

Solutions:

(a) **2, – 1**

(b) **(i)** **$2(3 + 6)$**

(ii) **$13 - 2x$**

(iii) **$x^2 - 6x + 39$**

(iv) **3.5, 2.5**

Recommendations

Question 10

The question tested the candidates' ability to

- write inequalities from worded statements
- draw graphs of linear inequalities in one or two variables
- determine the solution of a set of inequalities
- use linear programming techniques to determine the maximum value of an expression.

The question was attempted by 30 per cent of the candidates, 3 per cent of whom scored the maximum available mark. Performance was generally fair with a mean mark of 5.37 out of 15.

Candidates were proficient in using the scale and drawing the straight line graphs although some of these graphs were derived from incorrect equations. In many cases the direction of the inequalities was reversed.

A significant number of candidates were able to identify the region satisfying their inequalities, however, some merely shaded a triangular region whether or not it was consistent with their inequalities.

Candidates were also able to write down the coordinates of their vertices correctly but the majority tested only one point and as such had no basis for comparison even if they had recognized the maximum point.

Solutions:

(i) $x + y \leq 60$

(ii) $y \geqq 10$

(iii) $y \leq 2x$

(v) $6x + 5y$

(vi) **(5, 10) (20, 40) (50, 10)**

(vii) **The maximum fees was $350.**

Recommendations

Students need to be able to differentiate the graph of an inequation in one variable from an inequation in two variables. Students seem to be having a lot of difficulty in drawing lines through the origin, horizontal and vertical lines.

The interpretation of phrases such as no more than, and at most, and at least need to receive particular attention and must be treated as necessary prerequisites for this topic.

Students must show working to justify conclusions, for example, testing a sufficient number of points to determine which point gives the maximum.

Question 11

The question tested the candidates' ability to

- solve practical problems involving heights and distances in three-dimensional situations
- use trigonometric ratios to solve problems involving angles of elevation
- use theorems in circle geometry to calculate the measure of angles.

The question was attempted by 37 per cent of the candidates, 3 per cent of whom scored the maximum available mark. The mean mark was 4.49 out of 15.

Part (a) was generally well done. Candidates showed a reasonable level of proficiency in trigonometry. Most of the candidates labelled the diagram correctly although a significant number had the angles of elevation aligned with the vertical instead of the horizontal.

In calculating the length of TW, some candidates chose the incorrect trigonometric ratios. Those who used the cosine rule chose a longer route and exposed themselves to more computational errors.

Part (b) which tested circle geometry was not as popular as part (a). However, some candidates demonstrated sound knowledge of circle theorems while others used properties of triangles and quadrilaterals to arrive at their answers. A few candidates experienced difficulty in giving reasons to support their answers.

Solutions:

(a) (ii) 15 m

(b) (i) 90°

(ii) 70°

(iii) 140°

(iv) 40°

Recommendations

- Teachers should ensure that candidates use diagrams to represent two and three dimensional situations, highlighting important lines and angles, as well as lines of latitude and longitude.

- Candidates also need to be exposed to different orientations of the right angled triangle so that trigonometric ratios can be easily determined for any triangle.

Question 12

The question tested the candidates' ability to

– use the cosine rule to solve problem involving triangles
– determine the area of a triangle given two sides and an included angle
– show, on a diagram of the earth, the equator, meridian of greenwich and two points on the same latitude but with different longitudes
– calculate the circumference of a circle of latitude and the distance between two points on the surface of the earth along their common circle of latitude.

The question was attempted by 13 per cent of the candidates with only one per cent scoring the maximum available mark. The mean mark was 2.88 out of 15.

In general, this question was poorly done. In part (a), many candidates did not recognize the triangle to be non-right angled and hence incorrectly used Pythagoras' Theorem instead of the cosine rule to calculate HF. Some of the candidates who correctly substituted into the cosine rule made computational errors in simplifying their terms. Weaker candidates used 4.2 cm, the length of one side of the parallelogram as the perpendicular height of the parallelogram, hence displaying poor understanding of the concept of height.

Although the majority of candidates could label the diagram, far too many candidates could not perform the calculations which followed. Even though many candidates knew the correct formula to use for circumference of a circle, they could not substitute the correct value for the radius, neither could they substitute the correct value for the angular distance.

Solutions:

(a)	**(i)**	**6.03 cm**	**(ii)**	**23.68 cm^2**
(b)	**(ii)**	**a) 30 191 km**	**b)**	**6 541 km**

Recommendations

The use of practical models is recommended in the teaching of circle geometry so that students can visualize the angles and distances before applying the formulae.

Question 13

The question tested the candidates' ability to

- write vectors using column matrix notation
- add vectors using column matrix notation
- use a vector method to prove three points are collinear
- use vectors to represent and solve problems in geometry.

The question was attempted by 23 per cent of the candidates, less than 1 per cent of whom scored the maximum available mark. The mean mark was 3.58 out of 15.

Performance was generally poor. Candidates were able to copy the given diagram, and in some cases complete the parallelogram. However, very few candidates labelled the vector, ***u***, correctly on their diagram; they also neglected to insert the arrow to show the direction of the vector.

Candidates successfully wrote down the column vectors for OA and OC but many had difficulty in determining AC.

Although G was correctly located, some candidates wrote the coordinates in column matrix form.

Proving the three point to be collinear was indeed a challenge for the majority of candidates. Of those who attempted to find AC, GC and AG, few were unable to follow through the proof to completion. An extremely small number of candidates used the fact that there is a common point.

Solutions:

(b) **(i)** $\begin{pmatrix} 6 \\ 2 \end{pmatrix}$

(ii) $\begin{pmatrix} 0 \\ 4 \end{pmatrix}$

(iii) $\begin{pmatrix} -6 \\ 2 \end{pmatrix}$

(c) **(i)** **(3, 3)**

Recommendations

Emphasis should be placed on

- recognition of the properties of quadrilaterals prior to teaching this topic
- differentiating between a column vector and the coordinates of a point
- the technique of using a vector method in formulating a proof.

Question 14

The question tested the candidates' ability to

– evaluate the determinant of a 2 x 2 matrix
– obtain the inverse of a non-singular 2 x 2 matrix
– use matrices to solve simple problems in geometry
– perform multiplication of matrices
– determine the elements of a 2 x 2 matrix which transform two points into two given images.

The question was attempted by 26 per cent of the candidates, 2 per cent of whom scored the maximum available mark. The mean mark was 5.07 out of 15.

Responses to this question were fair. Most of the candidates knew how to obtain the determinant of a matrix but poor algebraic skills hindered their attempts to calculate the value of x.

Showing that $MM^{-1} = I$ also proved to be challenging for many candidates due to errors in handling negative numbers and failure to use their determinant in computing the inverse.

Obtaining the equation to represent the transformation posed the most difficulty for the majority of candidates. A large number of them omitted this part of the question and went on to write down the values of p, q, r, and s, thus demonstrating proficiency in recognizing the matrix for the reflection in the X-axis.

Solutions:

(a) (i) $x = 3$ **(ii)** $\frac{1}{9}\begin{pmatrix} 3 & -3 \\ 1 & 2 \end{pmatrix}$ **(iii)** $MM^{-1} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

(b) (i) a) $\begin{pmatrix} 2 & 5 \\ 4 & 3 \end{pmatrix}$ **b)** $\begin{pmatrix} 2 & 5 \\ -4 & -3 \end{pmatrix}$

(ii) $\begin{pmatrix} p & q \\ r & s \end{pmatrix}\begin{pmatrix} 2 & 5 \\ 4 & 3 \end{pmatrix} = \begin{pmatrix} 2 & 5 \\ -4 & -3 \end{pmatrix}$

(iii) $p = 1,\ q = 0,\ r = 0,\ s = -1$

Recommendations

- Teachers need to emphasize that matrix multiplication is order specific and once matrices are set up for multiplication students should be encouraged to verify that multiplication is possible before proceeding to multiply.

DETAILED COMMENTS

Basic Proficiency

The Basic Proficiency examination is designed to provide the average citizen with a working knowledge of the subject area. The range of topics tested at the basic level is narrower than that tested at the General Proficiency level.

This year candidates demonstrated a fairly good understanding of the concepts tested.

Approximately 42 per cent of the candidates achieved Grades I - III. This represents a significant increase of 26 per cent over 2005.

Paper 01 – Multiple Choice

Paper 01 consisted of 60 multiple-choice items. The maximum mark was not attained by any of the candidates. However, two candidates earned 58 out of 60 possible marks. Approxi-mately 35 per cent of the candidates scored at least half the maximum mark on this paper. This was achieved by 45 per cent of the candidates in 2005.

Paper 02 – Essay

Paper 02 consisted of 10 compulsory questions. Each question was worth 10 marks. The highest score earned was 98 out of 100. This was earned by one candidate. Twenty-five per cent of the candidates earned at least half the total marks on this paper.

Question 1

The question tested the candidates' ability to

- perform the basic operations with rational numbers
- convert from one currency to another
- express one quantity as a fraction of another.

The question was attempted by 97 per cent of the candidates, 1 per cent of whom scored the maximum available mark The mean mark was 4.6 out of 10.

The candidates were generally able to express one decimal as a fraction of the other, although some candidates did not know which fraction should be in the numerator/denominator. They were also able to add the fractional quantities but in dividing, inverted the incorrect fraction. Most candidates were able to convert from US dollars to Barbados dollars but were unable to reconvert the remaining Barbados dollars to US dollars.

Solutions:

(a) $\frac{2}{25}$

(b) $\frac{5}{2}$

(c) **(i)** **BDS $170**

(ii) **US $38**

Recommendations

- Students need to be exposed to mathematical terms and the language used in mathematics.
- There should also be a greater focus on the basic computational skills that will be needed for further work in mathematics.

Question 2

The question tested the candidates' ability to

- use symbols to represent binary operations
- use the laws of indices to manipulate expressions with integral indices
- solve simultaneous linear equations in two unknowns algebraically.

The question was attempted by 92 per cent of the candidates, 2 per cent of whom scored the maximum available mark. The mean score was 2.81 out of 10.

Candidates were generally able to manipulate the indices to obtain the correct solution. They were also able to determine the value of the binary expression but could not show that the operation was commutative. Most of the candidates attempted to eliminate a variable, they were able to correctly substitute the first value to determine the second unknown. However, errors were made in manipulating the equations.

Solutions:

(a) x^6

(b) **(i) 9**

(c) $x = 5;\ y = 1$

Recommendations

- Emphasis should be placed on the laws of indices and directed numbers with practical application of both.

Question 3

The question tested the candidates' ability to

- solve problems involving payments by installments in the case of hire purchase
- calculate depreciation
- solve problems involving simple interest, compound interest and depreciation.

The question was attempted by 96 per cent of the candidates, 9 per cent of whom scored the maximum available mark. The mean score was 4.89 out of 10.

The candidates were generally able to calculate the percentage of a given amount but did not understand the concept of depreciation. The majority of the candidates calculated the time taken to save the given sum of money and correctly computed the total installments. However, the deposit was omitted in calculating the total hire purchase price. The reasons for using hire purchase were not well written.

Solutions:

(a) $32 400

(b) (i) 13 weeks

(ii) $86.50

(iii) Hire purchase requires less money to be paid each week; Cash payment costs less overall.

Recommendations

- More time should be spent on the distinction between simple and compound interest as well as between compound interest and depreciation.

- Students should also be encouraged to reflect on mathematical processes and to express their ideas both orally and in written form.

Question 4

The question tested the candidates' ability to

- calculate the area of a region enclosed by a square
- calculate the volume of a simple right prism
- convert units of capacity within the SI system
- solve problems involving measurements.

The question was attempted by 87 percent of the candidates, 1 percent of whom scored the maximum available mark. The mean score was 2.53 out of 10.

Most candidates were able to calculate the length of the side of the square. Candidates experienced difficulty calculating the volume of the tank and converting between units. Candidates were however able to express the volume of water as a percentage of the volume of the tank.

Solutions:

(a) **(i)** **5.2 cm**

(ii) **27.04 cm^2**

(b) **(i)** **90 000 cm^3**

(ii) **60%**

Recommendations

- Students need exposure to practical work in measurement.

- The properties of two and three dimensional shapes and solids should also be emphasized.

Question 5

The question tested the candidates' ability to

- perform operations involving directed numbers
- translate verbal phrases into algebraic symbols and vice versa
- simplify algebraic fractions
- use a linear equation to solve a word problem.

The question was attempted by 91 per cent of the candidates, 3 per cent of whom scored the maximum available mark. The mean score was 3.22 out of 10.

The candidates were generally able to substitute numbers into the given expression but experienced difficulty computing the directed numbers. Generally, candidates were not able to simplify the algebraic expression without errors including attempts to cross multiply and incorrectly applying the distributive law. Candidates experienced difficulty in determining the algebraic expressions for Pam's age and the sum of the ages.

Solutions:

(a) **4**

(b) $\dfrac{x+3}{6}$

(c) **(i)** $x + 12$

(ii) $2x + 12$

Recommendations

- Teachers need to find innovative ways of introducing algebra, making reference to real life applications, where possible.

Question 6

The question tested candidates' ability to solve problems involving rates.

The question was attempted by 89 per cent of the candidates, 14 per cent of whom scored the maximum available mark. The mean score was 4.03 out of 10.

The candidates were generally able to calculate the cost of typing 10 and 15 pages but had difficulty calculating the cost of typing 23 pages, where the calculation required the use of two different rates. Some candidates were able to determine the number of pages in the document in part (b) although a trial and error method was used by some candidates. A common error in this section was using one rate instead of two to determine the solution.

Solutions:

(a) **(i)** **\$4.00**

(ii) **\$6.00**

(iii) **\$8.40**

(b) **55 pages**

Recommendations

- Teachers need to use everyday examples such as actual bills, when teaching consumer arithmetic.
- Students should also be encouraged to simulate buying and selling activities in the classroom.

Question 7

The question tested the candidates' ability to

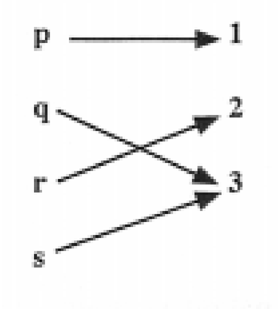
- use arrow diagrams to show relations
- define a function as a many-to-one or one-to-one relation
- interpret data presented in a graphical form.

The question was attempted by 94 per cent of the candidates, 3 per cent of whom scored the maximum available mark. The mean score was 5.02 out of 10.

The majority of the candidates completed the arrow graph correctly although some repeated the element "3" in the range. As a result, these candidates stated that the relation was one-to-one. The candidates were generally able to read the points from the line graph and identify the steepest slope. There was difficulty however, in determining the rate of decrease and stating the period of time.

Solutions:

(a) **(i)**



(ii) **many to one since both q and s map to 3.**

(b) **(i)** **25°C**

(ii) **15°C**

(iii) **7.5°C/hr**

(iv) **4:00 to 5:00**

Recommendations

- Greater emphasis need to be placed on the types of mapping diagrams and the description of a function.
- Teachers need to engage students in discussions, including interpretation and analysis, on graphs and diagrams.

Question 8

The question tested the candidates' ability to

- use instruments to draw and measure angles and line segments
- use instruments to construct a triangle
- state the relationship between an object and its image in a plane when it undergoes a translation in that plane
- identify and describe a transformation given the object and its image.

The question was attempted by 72 per cent of the candidates, less than 1 per cent scored the maximum available mark. The mean score was 3.32 out of 10.

The majority of the candidates were able to draw the line segments PQ and PR accurately, but a number of them did not complete the triangle or state the length of the side QR. In many cases there was no evidence that a compass was used to construct the angles. The candidates generally had difficulty completing part (b). While some could identify the angle centre of the rotation, they were unable to state the direction using terms such as right, left, north and east. Further, many of the candidates did not recognise that the dimensions of an object did not change after a translation.

Solutions:

(a) QR **= 7.2 cm**

(b) **(i)** ***ABC*** **is mapped onto** ***A′B′C′*** **by a rotation of 90 degrees about the Origin in a clockwise direction.**

(ii) **a)** $A''B'' = 6$ **cm**

b) $\angle C''A''B''$ **is 90 degrees**

Recommendations

- Students need more practice in drawing polygons using geometrical tools with special emphasis on constructing angles with the compasses.

- Transformation geometry should be taught both on graph paper and on plain paper so that students become familiar with different orientations without depending on graph lines.

Question 9

The question tested the candidates' ability to

- draw and use bar charts
- determine the mean for a set of data
- determine experimental and theoretical probabilities of simple events.

The question was attempted by 95 per cent of the candidates, 4 per cent of whom scored the maximum available mark. The mean score was 6.08 out of 10.13 per cent of the candidates scored at least half the marks on this question.

The candidates were generally able to calculate how many more chocolates were sold, the total number sold for the week and the mean number sold for the week, although a few candidates calculated the median instead of the mean. There was some difficulty computing the probability.

Solutions:

(a) **15**

(b) **220**

(c) **44**

(e) $\frac{3}{5}$

Recommendations

- Teachers should expose students to more experimental probability conducting simple experiments in the classroom.

- The measures of central tendency (mean, median, mode) also need to be clearly defined and sufficient examples and exercises provided so that students can differentiate among them.

Question 10

The question tested the candidates' ability to

- use the properties of perpendicular and parallel lines to solve problems
- use Pythagoras' Theorem to solve problems
- use the sine, cosine and tangent ratios in the solution of right-angled triangles
- solve problems involving bearings.

The question was attempted by 67 per cent of the candidates, less than 1 per cent of them scored the maximum available mark. The mean mark was 2.09 out of 10.

.

The candidates were generally able to determine the values of the unknown angles. However, they were unable to use Pythagoras' theorem correctly, use trigonometric ratios to determine the unknown angle or determine the bearing.

Solutions:

(a) $x = 110^{o}$; $y = 110^{o}$

(b) **(i)** **15 m**

(ii) 58^{o}

(iii) 32^{o}

Recommendations

- Students need more practice in the real life application of Pythagoras' Theorem, trigonometric ratios and bearings to complement the work done in the classroom.

C A R I B B E A N   E X A M I N A T I O N S   C O U N C I L

REPORT ON CANDIDATES' WORK IN THE SECONDARY EDUCATION CERTIFICATE EXAMINATION JANUARY 2007

MATHEMATICS

MATHEMATICS

GENERAL PROFICIENCY EXAMINATION

JANUARY 2007

GENERAL COMMENTS

The General Proficiency Mathematics examination is offered in January and May/June each year. The Basic Proficiency is offered in May/June only.

There was a candidate entry of approximately 12 650 in January 2007. This year, forty-four per cent of the candidates achieved Grades I – III. The mean percentage for the examination was 44.

DETAILED COMMENTS

The examination consists of two papers. Paper 01 consists of 60 multiple choice items. The highest mark was 59 and approximately 61 per cent of the candidates scored at least half the available marks for this paper.

Paper 02 consists of two sections. Section I comprised eight compulsory questions totalling 90 marks. Section II comprised six optional questions: two each in Relations, Functions and Graphs; Trigonometry and Geometry; Vectors and Matrices. Candidates were required to choose any two questions. Each question was worth 15 marks.

This year, two candidates earned the maximum available marks on Paper 02 and approximately 26 per cent of the candidates earned at least half the maximum mark on this paper.

<u>Question 1</u>

This question tested candidates' ability to

- perform basic operations with decimals
- approximate a decimal number to the nearest whole number
- solve problems involving ratio, rates and proportion.

The question was attempted by 99 per cent of the candidates, 29 per cent of whom scored the maximum available mark. The mean score was 9 out of 12.

Generally, performance on this item was good, particularly in part (a) where many candidates demonstrated competence in performing the operations on decimals. Incorrect answers were mainly due to careless errors and in some cases $(1.5)^2$ was computed as 2 x 1.5.

In part (b), the majority of candidates demonstrated a sound understanding of ratio although many did not recognize that $60.00 represented two shares

Using the given rate to calculate the cost of 3 litres of gasoline in part (c) was well done by the majority of candidates. However, a significant number of candidates could not calculate the number of litres which could be bought with $50.00.

Many candidates had difficulty completing the approximation, since they were unable to round their answers to the nearest whole number.

Solutions

(a) (i) 12.2092 (ii) 2.1

(b) $210.00

(c) (i) $17.33 (ii) 14 litres

Recommendations

Teachers need to encourage candidates to show all working by writing their solutions to sub-parts of a question. In addition, the candidates need to be exposed to problems which could be solved using ratio or proportion. There should also be emphasis on approximation, especially rounding to whole numbers.

Question 2

This question tested candidates' ability to

- perform operations involving directed numbers
- substitute numbers for algebraic symbols in simple algebraic expressions
- solve linear equations in one unknown
- simplify algebraic fractions
- solve a simple linear inequality in one unknown.

The question was attempted by 99 per cent of the candidates, 6 per cent of whom scored the maximum available mark. The mean score was 6.15 out of 12.

Candidates generally demonstrated proficiency in the substitution in part (a) although some of the weaker candidates could not multiply negative numbers accurately.

In attempting to simplify the algebraic fraction in part (b), a significant number of candidates used an LCM of 5 instead of 6 while others incorrectly cross-multiplied and obtained
$3x + 2x = 5$. Solving the in-equation proved challenging for candidates. Some treated it as an equation, while others solved for $-x$ instead of x.

In part (c), candidates were able to use symbols to represent the cost of 5 muffins although some incorrectly wrote $5m + m$ or $5m \times m$. Stating the cost of 6 cupcakes given the cost of three, was more challenging and some candidates wrote 6c, ignoring the information given. The majority of candidates were successful in writing an equation to represent the total cost of the muffins and cupcakes.

Solutions:

(a)	**(i)**	**6**	**(ii)**	**–12**
(b)	**(i)**	**x = 6**	**(ii)**	**x e” –9**

(c) **(i)** **a)** **\$5m** **b)** **\$4m**

(ii) **5m + 4m = \$31.50**

Recommendations

Teachers need to teach candidates to simplify and perform the basic operations on algebraic expressions. Candidates must recognise that the solution to an in-equation is different from the solution of an equation.

Question 3

The question tested candidates’ ability to

- use set notation to describe a given set
- draw Venn diagrams to represent information
- list the members of the union, intersection and complement of a set
- determine the number of elements in the union, intersection and complement of a set.

The question was attempted by 99 per cent of the candidates, less that 1 per cent of whom scored the maximum available mark. The mean mark was 4.15 out of 10.

Candidates generally were able to describe the shaded region in part (a)(i), but parrts (ii) and (iii) were poorly done with some candidates describing the un-shaded regions instead of the shaded regions.

In part (b), candidates were able to identify odd and even numbers. However, the candidates had difficulty identifying the prime numbers, with many candidates listing 1 as a prime number.

In part (c), a number of candidates interpreted the numbers in each region on the Venn diagram as elements instead of the number of elements**.** For example, the number of elements in AUB was given as {10, 4, 3} or 3 instead of 17.

Solutions:

(i) $\mathbf{A \cup B}$ **(ii)** $\mathbf{(A \cup B)'}$ **or** $\mathbf{A' \cap B'}$ **(iii) A**

(c) **(i)** **17** **(ii)** **4** **(ii)** **21** **(iv)** **25**

Recommendations

Teachers need to emphasise the importance of set notation in the teaching of sets. In addition, candidates must be given opportunities to (i) represent practical situations using Venn diagrams and (ii) shade given regions. Describing these regions in words is also necessary if a full understanding of the concept is to be attained.

Question 4

(a) The question tested candidates' ability to

- draw line segments
- construct a triangle given the lengths of the sides
- measure angles in a polygon
- calculate the gradient of a straight line given two points
- calculate the coordinates of the mid-point given two points.

The question was attempted by 95 per cent of the candidates, 8 per cent of whom scored the maximum available mark. The mean mark was 5.1 out of 11.

Candidates were generally able to draw the triangle with the prescribed dimensions. However, many candidates did not show the appropriate construction lines. Many candidates experienced difficulty in locating the point D and hence could not proceed to measure AD. In some instances candidates measured angle BAC instead of angle ABC.

Part (b) was generally well done with candidates showing strengths in the choice of formula for finding the gradient and mid-point of a line. However, some of the candidates experienced problems in substituting values in the formula because of failure to define or set up their coordinates in terms of (x_1, x_2) and (y_1, y_2). Also writing the mid-point as a coordinate was often omitted.

Solutions:

(iii) a) 7.4 cm b) 68^0

(i) 1.5 (ii) (4, 7)

Recommendations

Teachers need to teach candidates to develop proper techniques to measure line-segments and angles and to use the appropriate instruments to construct plane figures. Candidates also need to become familiar with the language and terms used in geometry.

Question 5

The question tested candidates' ability to

- find the output of a function given an input value
- find the input of a function given the output value
- locate the position of the mirror line given an object and its image
- locate the image of an object under a rotation
- describe a transformation given an object and its image.

The question was attempted by 94 per cent of the candidates, 3 per cent of whom scored the maximum available mark. The mean mark was 3.83 out of 11.

Candidates were able to evaluate g(3) and f(-2) correctly. They, however, experienced difficulty with working with inverse functions. In many cases, candidates were unable to complete the procedure for finding the inverse. For example, after interchanging x and y, they did not proceed to make y the subject. Those candidates who chose to equate f(x) to 11 and then solve for x, experienced more success.

Locating the mirror line was well done although a number of candidates drew the mirror line but did not write its equation.

Part (b) was poorly done with the majority of candidates being unable to perform a rotation about a given point. Many candidates mistakenly used (5,4) as an image point instead of the centre of rotation. Describing the transformation which mapped the original figure onto its final image also posed serious problems for candidates. Some candidates used the rotation matrix to determine the coordinates of the image, not recognising that this matrix may only be used when the centre of rotation is (0,0).

Solutions:

(a) (i) $\frac{1}{6}$ **(ii) -10 (iii) 1**

(b) (i) x = 5

(ii) A″(1,2) B″ (3,2) C″ (3,-1)

(iii) Reflection in the line y = 4

Recommendations

It is recommended that candidates understand how to perform geometrical transformations before they are exposed to matrix methods. Further, they should perform transformations using actual shapes on plane paper or using geo-boards, before performing transformations on the Cartesian plane.

Question 6

The question tested candidates' ability to

- construct a cumulative table for a given set of data
- draw a cumulative frequency curve
- use a cumulative frequency curve to estimate the median
- calculate the probability of an event using data in a frequency distribution.

The question was attempted by 92 per cent of the candidates, 1 per cent of whom scored the maximum available mark. The mean mark was 4.57 out of 12.

Candidates were able to calculate the cumulative frequencies, use the given scale, plot points correctly and draw the curve. Most of the candidates did not use the boundaries to plot the cumulative frequency curve, but used mid-points or upper limits instead. Many candidates omitted to find the median and those who attempted to do so often made errors in interpreting the scale.

Although it was generally understood that the probability must be expressed as a fraction, some candidates had difficulty calculating the number of candidates whose score was greater than 40 while others used 40 as the total number of candidates in the sample.

Solutions:

(a) (i) Cumulative frequencies [5, 23, 46, 68, 89, 100]

(iii) Median score: 36

(iv) $\frac{32}{100}$

Recommendations

Teachers should give candidates opportunities to draw graphs using a variety of scales so that they are familiar with interpreting graphs. The use of class boundaries in drawing cumulative frequency curves also needs to be emphasized.

Question 7

The question tested candidates' ability to calculate the

- volume of a cuboid
- total surface area of a cuboid
- length of an arc
- perimeter of a sector
- area of a sector.

The question was attempted by 88 per cent of the candidates, 8 per cent of whom scored the maximum available mark. The mean mark was 4.26 out of 12.

Candidates generally knew how to select and apply the formula to calculate the volume of the cuboid. The perimeter of the sector was also well done. Many candidates were unable to select the appropriate formula to calculate the surface area of the cuboid. In some cases, candidates calculated the volume instead of the surface area.

In computing the area of the sector and the arc length, there was a tendency to omit the fraction in the formula. A few candidates opted to use $r\theta$ but failed to convert to radians.

Solutions:

(a) **(i)** **4320 cm^2**
(ii) **1728 cm^2**

(b) **(i)** **11.78 cm**
(iii) **41.78 cm**
(iii) **88.31 cm^2**

<u>Recommendations</u>

The poor choice of formulae in finding surface area, arc length and area of the sector suggests that candidates are using measurement formulae without understanding how they are derived. Teachers should spend more time in developing formulae using practical approaches and emphasising the units of the attribute being measured.

<u>Question 8</u>

The question tested candidates' ability to

- create and use number patterns
- determine the nth term in a number sequence.

The question was attempted by 81 per cent of the candidates, 2 per cent of whom scored the maximum available mark. The mean mark was 2.95 out of 10.

This question was poorly done by most of the candidates. Although some candidates were able to recognize a pattern, they did not test their pattern to see if it applied to all the cases. For example, candidates gave 256 as the result for n = 3, assuming the pattern to be n^2. Those who recognized the correct pattern were unable to calculate the value for n = 6, or the power to which 4 must be raised to give a result of 65 536. A limited number of candidates were able to compute the result of the mth term

Solutions

(i) **64**
(ii) **4 096**
(iii) **8**
(iv) **4^m**

Recommendations

Teachers should prepare candidates for questions of this type by allowing candidates to generate their own number sequences and have other candidates predict the general rule. Candidates must also be given opportunities to hypothesize and test conjectures so that they can observe instances when the pattern breaks down.

Teachers should also expose candidates to problem solving activities regularly, in an effort to develop the analytical skills of the candidates.

Question 9

The question tested candidates' ability to

- factorise a quadratic expression
- use common factors to factorise an algebraic expression
- expand an expression of the form $(x + a)^2 (x + b)$
- express a quadratic equation in the form $a(x + b)^2 + c$ and state the axis of symmetry and the minimum point
- sketch the graph of a quadratic function.

The question was attempted by 51 per cent of the candidates, less than 1 per cent of whom scored the maximum available mark. The mean mark was 2.32 out of 15.

Although this question was the most popular among the optional questions, candidates performed poorly with 41 per cent scoring no marks. The candidates were able to expand $(x+3)^2$ correctly and many knew the procedure for completing the square. Some candidates were also able to sketch the curve although they did not show the maximum point nor the axis of symmetry on the diagram.

Candidates had difficulty factorising, especially the difference of two squares, and expanding $(x^2 +6x+9)(x-4)$. Identifying the minimum point from the expression $2(x+1)^2 -7$ and stating the axis of symmetry were also poorly done.

Solutions:

(a) **(i)** **$(2p – 1) (p – 3)$**

(ii) **$(p + q) (5 + p – q)$**

(b) $\mathbf{x^3 + 2x^2 - 15x - 36}$

(c) **(i)** $\mathbf{2(x+1)^2 - 7}$

(ii) **Axis of symmetry x** $= -1$

(iii) **Minimum point (– 1, – 7)**

Recommendations

Teachers need to link the use of algebraic and graphical methods in solving quadratic equations. Candidates should be able to match expressions of the form $a(x+b)^2 + c$ with the graph of the function and interpret the meanings of the constants.

Question 10

The question tested candidates' ability to

- write inequalities from worded statements
- draw graphs of linear inequalities in one or two variables
- determine the solution of a set of inequalities
- use linear programming techniques to determine the maximum value of an expression.

The question was attempted by 40 per cent of the candidates, 1 per cent of whom scored the maximum available mark. The mean mark was 4.44 out of 15.

Most candidates were able to write the inequalities $x \geq 3$ and $x + y \leq 10$. They were also able to obtain the profit expression and to draw the line $x = 3$. Candidates experienced problems in describing the inequality $5x + 2y \leq 35$ in their own words, the most common statement being: "the total of 5 pens and 2 pencils is less than 35". Candidates omitted to mention the words *cost* and *dollars* in their explanations.

Candidates also had difficulty drawing the line $x+y=10$, identifying the region common to all the inequalities and extracting information from the graph to obtain the maximum number of pencils that could be bought.

Solutions:

(a) **(i)** $\mathbf{x \geq 3}$

(ii) $\mathbf{x + y \leq 10}$

(iii) **The total cost of x pens and y pencils is no more than $35.00**

(b) (ii) (3, 0) (7, 0) (5, 5) (3, 7)

(c) (i) Profit : 1.5x + y

(ii) The maximum profit: $12.50

(iii) Maximum number of pencils: 6

Recommendations

Teachers need to encourage candidates to test points in order to identify the correct regions and to use a consistent system of shading to define the wanted/unwanted region. In finding the maximum or minimum, the use of the fundamental theorem of testing points at the vertices of the feasible region must also be reinforced.

Question 11

The question tested candidates' ability to

- use theorems in circle geometry to calculate the measure of angles
- use the properties of circles to perform simple calculations
- recognize and use Pythagoras' theorem to calculate the length of a side of a right-angled triangle.

The question was attempted by 14 per cent of the candidates, less than 1 per cent of whom scored the maximum available mark. The mean mark was 4.54 out of 15.

Generally, candidates were successful in stating the lengths of PQ and PN. Many did not recognize that RS= NQ and hence were unable to proceed to use Pythagoras' theorem to calculate NQ. Providing reasons for answers to part (a) posed major problems for candidates who merely described the information shown in the diagram without supporting their answers by referring to circle theorems.

Candidates were able to calculate the angles in part (b) although they continued to experience difficulty in giving reasons to support their answers.

Solutions:

(a)

(i) a) PQT is a straight line because T is the common point of contact of two circles and a common tangent drawn through T will be perpendicular to both PT and TQ. Hence, angle PTQ = $90^o + 90^o = 180^0$

b) PQ = 7cm

c) PS is parallel to QR because PSR = QRS = 90^o (Tangent XY is perpendicular to both radii). The corresponding angles are equal; hence the lines must be parallel.

(b) (i) angle MNL = 55^o
angle LMO = 35^o

Recommendations

Candidates should be given more opportunities in the classroom to justify their solutions using properties of polygons and the relevant theorems. In addition, teachers should expose candidates to the vocabulary associated with the concepts and principles in geometry through the use of strategies which incorporate oral exercises.

Question 12

The question tested candidates' ability to

- draw a diagram to represent information involving bearings and distances
- calculate distances using the sine and cosine rule
- solve problems involving bearings.

The question was attempted by 33 per cent of the candidates, 2 per cent of whom scored the maximum available mark. The mean mark was 5.43 out of 15.

Candidates were generally able to sketch and label the diagram to illustrate the journey of the ship. There was some difficulty illustrating the bearings of 135^0 and 60^0 with 135^0 drawn as an acute angle in some cases. Quite a significant number of candidates used scale drawings and these were mainly well done yielding accurate results.

Part (b) of the question was poorly done. Candidates generally chose to use trigonometric ratios associated with right-angled triangles. In several instances where they chose the correct rule for solving the problem they proceeded to substitute values and angles incorrectly. For example, in using the cosine rule, many substituted 60^0 instead of 105^0.

Solutions:

(b) (i) AC = 18.7 km
(ii) angle BCA = 50.8^0
(iii) Bearing of A from C is 290.8^0

Recommendations

Teachers should encourage candidates to construct drawings which include distances and bearings, where the focus would be on realistic representation of line segments and angles. Such diagrams should lead to more reasonable solutions to the problems.

Question 13

The question tested candidates’ ability to

- locate points on a diagram given information
- add vectors
- use vectors to solve a problems in geometry.

The question was attempted by 13 per cent of the candidates, 1 per cent of whom scored the maximum available mark. The mean mark was 3.36 out of 15.

Candidates were generally able to approximate the position of X on the diagram and state a correct route for the vectors and . However, they were unable to locate the point Q given the instructions. The handling of fractions also posed major challenges for candidates in their attempts to express vectors in terms of *r* and *s*. Few candidates attempted part (c) where a proof was required.

Solutions:

(b) **(i)**

$\overrightarrow{DNE}$ **(ii)** $\frac{1}{3}\underline{s} - \frac{2}{3}\underline{r}$

(iii) $\frac{5}{6}\underline{r} - \frac{7}{12}\underline{s}$

Recommendations

The responses to this question suggest that candidates are not familiar with applying basic concepts of ratio and fractions to locate points on line segments when drawing diagrams. These types of exercises must be addressed prior to the teaching of vectors using strategies which involve actual measurements and the use of squared paper.

Question 14

The question tested candidates’ ability to:

- identify a singular matrix
- calculate the determinant of a 2 $\times$ 2 matrix
- find the inverse of a 2 $\times$ 2 matrix
- use matrix methods to solve a system of 2 linear equations

The question was attempted by 34 per cent of the candidates, 2 per cent of whom scored the maximum available mark. The mean mark was 7 out of 15.

Most candidates performed well on this question, especially solving the linear equation by a matrix method. In part (a), candidates generally knew that a singular matrix had a determinant of zero but only the stronger candidates were able to solve the quadratic equation 4 “ $9p^2 = 0$ to obtain two roots. Quite a few candidates found only the positive root.

In part (b), candidates knew how to convert the pair of simultaneous equations into matrix form and how to find the determinant and inverse of the matrix. Some errors were made in obtaining the adjunct with candidates changing the signs along the leading diagonal. Candidates were generally able to solve for x and y, although performing operations on directed numbers was a major weakness demonstrated by some candidates.

Solutions:

(a) **p =**

(b) **(i)** $\begin{pmatrix} 2 & 5 \\ 3 & 4 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 6 \\ 8 \end{pmatrix}$

(ii) **a)** **Determinant : “ 7**

c) **x =** $\frac{16}{7}$**, y =** $\frac{2}{7}$

<u>Recommendations</u>

Teachers should emphasise the processes for finding the determinant and the inverse of a matrix. Candidates should also be exposed to more problems where linear equations are solved by the matrix method.

C A R I B B E A N   E X A M I N A T I O N S   C O U N C I L

REPORT ON CANDIDATES' WORK IN THE SECONDARY EDUCATION CERTIFICATE EXAMINATIONS MAY/JUNE 2007

MATHEMATICS

MATHEMATICS

GENERAL AND BASIC PROFICIENCY EXAMINATIONS

MAY/JUNE 2007

GENERAL COMMENTS

The General Proficiency Mathematics examination is offered in January and May/June each year, while the Basic Proficiency examination is offered in May/June only.

In May/June 2007 approximately 86 835 candidates registered for the General Proficiency examination. Candidate entry for the Basic Proficiency examination was approximately 5 513.

At the General Proficiency level, approximately 34 per cent of the candidates achieved Grades I - III. This represents a 2 per cent decrease over 2006. Thirty-two per cent of the candidates at the Basic Proficiency level achieved Grades I - III compared with 44 per cent in 2006.

DETAILED COMMENTS

General Proficiency

In general, candidates' work revealed a lack of knowledge of basic mathematical concepts, especially the areas of Measurement, Algebra and Transformation Geometry. The optional section of Paper 02 seemed to have posed the greatest challenge to candidates, particularly the areas of Relations, Functions and Graphs; and Geometry and Trigonometry.

One candidate scored the maximum mark on the overall examination compared with six candidates in 2006. Twenty-three per cent of the candidates scored at least half the available marks compared with 26 per cent in 2006.

Paper 01 - Multiple Choice

Paper 01 consisted of 60 multiple-choice items. This year, 120 candidates earned the maximum available mark compared with 84 in 2006. Approximately 60 per cent of the candidates scored at least half the total marks for this paper.

Paper 02 – Essay

Paper 02 consisted of two sections. Section I comprised eight compulsory questions totalling 90 marks. Section II comprised six optional questions: two each in Relations, Functions and Graphs; Trigonometry and Geometry; Vectors and Matrices. Candidates were required to choose any two questions. Each question in this section was worth 15 marks.

This year, one candidate earned the maximum available mark on Paper 02 compared with 17 in 2006. Approximately 15 per cent of the candidates earned at least half the maximum mark on this paper.

Compulsory Section

Question 1

This question tested the candidates' ability to:

– perform basic operations with decimals
– express a common fraction in its lowest terms
– calculate the fraction and the percentage of a given quantity
– express one quantity as a fraction of another
– solve problems involving ratios.

The question was attempted by 98 per cent of the candidates, 13 per cent of whom scored the maximum available mark. The mean mark was 6.9 out of 11.

Responses were generally good with the majority of candidates displaying competence in using the calculator to perform computations and in calculating the fraction or percentage of a quantity.

Those candidates who performed the computation without the use of calculators made errors such as evaluating 3.7^2 as 7.4 or $(6.24 \div 1.3)$ as 0.48. A few who used calculators ignored the brackets and performed the operations from left to right.

In part (b) where candidates had to find the missing part of a ratio, many did not understand that 30 parts had to be equated to 1200 and interpreted it as sharing 1200 into 31 parts, hence they divided 1200 by 31 instead of 30.

A large number of candidates correctly calculated the number of students who own personal computers but omitted to subtract from 1200 to calculate the number of students who do not own computers.

A major problem faced by candidates in calculating the fraction of students in the school who own play-stations, was recognizing that the whole consists of 1200 students. Many candidates were able to reduce the fraction to its lowest terms but quite a large number omitted this part of the question.

Solutions:

(a) **8.89**

(b) **(i)** **40** **(ii)** **720** **(iii)** $\mathbf{\frac{3}{25}}$

Recommendations

Teachers need to emphasize the difference between:

- Calculating the missing part of a ratio and dividing a quantity in a given ratio
- A part-part comparison (ratio) and a part-whole comparison (fraction).

Question 2

This question tested the candidates' ability to:

– perform operations involving directed numbers
– use symbols to represent binary operations
– divide algebraic fractions

– substitute numbers for algebraic symbols in simple algebraic expressions

– translate verbal phrases into algebraic symbols

– use simultaneous equations in two variables to solve a real world problem.

The question was attempted by 97 per cent of the candidates, 7 per cent of whom scored the maximum available mark. The mean score was 5.19 out of 11.

Although many candidates were able to evaluate 4*8 correctly, far too many of them interpreted 4*8 as 4×8 and 2*(4*8) as $2 \times$ (4*8). Few candidates made the connection between parts (i) and (ii), failing to substitute the result for 4*8 in part (ii). Computing $4 \times 8 - \frac{8}{4}$ posed a problem for many candidates; a common error was to subtract 8 from 32 and divide the result by 4, hence obtaining 6 instead of 30.

In carrying out the operation of division on fractions, many candidates inverted correctly but only the very able candidates expressed the fraction in its simplest form. In attempting the division, some candidates cross-multiplied displaying poor understanding of operations on algebraic symbols.

Responses to part (c) on simultaneous equations were very good. The majority of the candidates correctly derived the two equations, although some of the candidates wrote inequalities instead of equations.

In solving the equations, both elimination and substitution were used although the majority chose the method of elimination. Common errors made included selecting the operation to eliminate a variable and simplifying the algebraic expressions.

When using the substitution method where a fractional expression was involved, some candidates experienced difficulty. A small percentage of candidates used the matrix method, although only a few were able to obtain the correct solution using this method.

Solutions**:**

(a) (i) 30 (ii) 45

(b) $\mathbf{\frac{5}{12\boldsymbol{p}}}$

(c) (i) 5a + 3b = 105, 4a + b = 63

(ii) a = 12, b = 15

Recommendations

Teachers need to

- emphasize the difference between operations in algebra and operations in arithmetic.

- connect the teaching of operations on fractions in arithmetic with the operations on fractions in algebra.

- encourage students to determine the most efficient strategy to use in solving equations. Both elimination and substitution methods should be taught but students must be able to decide which is better in a given situation.

- emphasize the difference between ***solve*** and ***simplify***, ***equation*** and ***expression***, ***equality*** and ***inequality***.

Question 3

This question tested the candidates' ability to:

- use a phrase to describe a set
- use Venn Diagrams to solve practical problems with not more than three sets
- use instruments to draw and measure angles and line segments
- use instruments to construct a triangle given three sides
- use instruments to construct a perpendicular.

The question was attempted by 93 per cent of the candidates, 1 per cent of whom scored the maximum available mark. The mean score was 5.91 out of 12.

The majority of candidates were able to interpret the Venn Diagram in part (a) by stating correctly the games played by each member. However, describing the region $H' \cap S$ posed problems for many candidates.

Candidates generally succeeded in drawing the triangle with the prescribed dimensions although many omitted to use instruments as evidenced by absence of construction arcs. Weaker candidates experienced problems in measuring the angle and the line segment. Several of them attempted to calculate rather than measure. Generally candidates experienced difficulty constructing the perpendicular from P to meet QR at T.

Some common errors in attempting to locate the point T were constructing the perpendicular bisector of QR and labelling the mid-point as T and bisecting angle PQR and labelling the point at which the bisector cuts QR as T.

Solutions:

(a) **(i)** **a) Tennis and Hockey** **b) Squash, Tennis and Hockey** **c) Hockey**

(ii) Those members who play Squash (and Tennis) but not Hockey

(b) **(ii)** **a)** **$59^{\circ} \pm 1^{\circ}$** **b)** **5.1 cm + 0.1 cm**

Recommendations

Teachers need to present meaningful contexts in the teaching of set theory giving student opportunities to create their own Venn Diagrams from everyday situations. Oral descriptions of regions should precede written ones so that students grasp the language of sets.

With respect to the teaching of constructions, there is need to emphasize the difference between draw and construct and ensure that students can use instruments properly.

Question 4

This question tested the candidates' ability to:

– make suitable measurements on a map and use them to determine distance and area

– convert centimetres to metres and square centimetres to square metres

– estimate the area of an irregularly shaped figure

– calculate the length of an edge of a square-based prism given its volume and height

– calculate the total surface area of a square-based prism.

The question was attempted by 86 per cent of the candidates, 1 per cent of whom scored the maximum available mark. The mean score was 2.47 out of 11.

Generally this question was poorly done mainly because candidates experienced difficulties with conversion of metric units, and using the linear scale factor to calculate the area represented by 1 cm^2 on the map. Few candidates counted the number of squares to derive the area.

Candidates were able to find the distance between two points on the map and use the scale to convert to the actual distance.

In part (b), the majority of candidates were able to find the area of the cross-section of the prism but they often failed to use the square root to calculate the length of the side AB. Candidates who calculated AB were able to determine the total surface area of the prism.

Solutions:

(a) (i) 120 m (ii) 220 – 232 m (iii) 1 600 m^2 (iv) 416 000 – 46 400 m^2

(b) (i) 8 cm (ii) 608 cm^2

Recommendations

Teachers should delay the introduction of measurement formulae as much as possible and allow students to fully explore basic concepts in measurement. Measuring the area of irregular shapes allows students to grasp the concept of area and how it is measured. The concept of a square unit and the difference between square centimetres and other metric square units also needs to be emphasized.

Question 5

This question tested the candidates' ability to:

– represent inverse variation symbolically

– perform calculations involving inverse variation

– determine the gradient of a line parallel to a given line

– determine the equation of a line given one point and the equation of a line parallel to it.

This question was attempted by approximately 65 per cent of the candidates, 3 per cent of whom scored the maximum available mark.. The mean mark was 2.05 out of 12.

In general the performance was very unsatisfactory. It was evident from the responses that candidates were not familiar with the inverse variation. Some incorrect responses were:

$y = kx, y = \frac{k}{x}$ $y = kx^2$, $y = k\sqrt{x}$

In calculating the table values the majority of the candidates were able to make the correct substitution but some experienced difficulties in transposing.

In part (c) many candidates demonstrated knowledge of the formula y = mx + c and attempted to use it but errors in substitution resulted in an incorrect equation.

Solutions:

(a) $y = \frac{k}{x^2}$

(b) (i) K = 18 (ii) r = 5.6 (iii) f = 1.5

(c) y = 2x – 1

Recommendations

The teaching of variation can be used to show the application of mathematics in other disciplines and in real life. The principles of variation are exemplified in many laws in the pure sciences as well as the social sciences and teachers can take the opportunity to design projects using this topic as a foundation.

Question 6

This question tested candidates' ability to:

– determine the centre and scale factor of an enlargement
– locate the image of an object under a refection in the line y = -x
– solve problems involving bearings
– use the cosine rule to calculate an unknown side of a triangle.

The question was attempted by 87 per cent of the candidates, 1 per cent of whom scored the maximum available mark. The mean mark was 1.95 out of 11.

Many candidates attempted either Part (a) or Part (b), but only a few candidates answered both parts.

Most of the candidates could not correctly identify the scale factor or the centre of the enlargement. However, the major difficulty experienced by candidates was the inability to reflect the triangle LMN in the given line y = -x. Many candidates performed a translation instead, and merely shifted the triangle diagonally to a new position in the third quadrant.

Some candidates attempted to use a matrix method to calculate the coordinates of the image but were unsuccessful since an incorrect matrix was used.

In part (b), many candidates correctly chose the Cosine Rule to calculate the length PR. However they could not interpret the given bearings to calculate angle PQR and used cos 70° instead of cos 20° in the formula. In evaluating the result, a common computational error was equating

125 – 100 cos 20° to *25 cos 20°*.

In calculating the bearing, many candidates made the false assumption that the triangle was isosceles and based their calculations on this. Only a few candidates were successful in stating the bearing of R from P.

Solutions:

(a) (i) a) 2 b) (0,0)

(ii) L´´(-4,-1) M´´(-2,-2) N´´(-3, -4)

(b) (i) 5.6 km (ii) 108°

Recommendations

Teachers should give students practical experience in

- Performing transformations in a variety of situations. For example, reflecting in mirror lines of different orientations should be done so that students can generalize the properties of reflection. Also paper-folding activities involving the use of plain paper should precede the use of graph paper on the coordinate plane.

- Performing out-door activities in which they determine the bearing from one position to another using actual measurements. Stating the bearing using appropriate language and terminology should be explored.

Question 7

This question tested the candidates' ability to:

– complete a frequency table from a set of data

– determine the range from a set of data

– construct a frequency polygon using given scales

– determine the probability of an event using data from a frequency table.

The question was attempted by 92 per cent of the candidates, 1 per cent of whom scored the maximum available mark. The mean mark was 4.17 out of 12.

The majority of candidates were competent in completing the frequency table and in using the correct scales. However, a significant number of candidates did not know that the horizontal axis was the x-axis and the vertical axis was the y-axis. They also failed to use the mid-points on the horizontal scale and used the limits or boundaries instead.

Candidates were generally aware that the frequency polygon was constructed using straight lines. However, some candidates closed the polygon by joining the first and last points, while others did not connect the points in sequence. A few candidates chose to draw a histogram first and then construct the frequency polygon.

A major weakness was the inability to determine the range for the data. Some candidates merely listed the upper-class boundaries, while others interpreted the range as an interval.

The majority of the candidates were able to successfully use the total frequency in calculating the probability, but many could not determine the number of candidates who completed the race in less than 60 seconds.

Solutions:

(a) **3, 7, 4, 5**

(b) **32**

(d) $\mathbf{\frac{7}{32}}$

Recommendations:

- Teachers need to ensure that students can differentiate between a bar chart, a histogram and a frequency polygon.
- Teachers should encourage students to construct a frequency polygon independent of a histogram emphasizing the use of mid-points of the class intervals on the horizontal axis.
- The concept of range as a value needs to be emphasized and students should be taught methods of determining the range using raw data as well as data from frequency tables.

Question 8

This question tested the candidates' ability to:

– express one quantity as a fraction of a given quantity

– calculate perimeter of plane shapes

– calculate the area of a trapezium

– draw a trapezium given its component parts.

The question was attempted by 66 per cent of the candidates, 3 per cent of whom scored the maximum available mark. The mean mark was 2.34 out of 10.

Performance on this question was generally poor. Candidates could not express the parts of the rectangle as a fraction of the whole rectangle. Answers consisted mainly of unit fractions with relatively large denominators.

In ordering the shapes by perimeter, most of the candidates recognized which shapes should be placed first and last and determined the area of the three shapes which had to be rearranged to form the trapezium. Attempts to re-arrange the shapes to form a trapezium proved to be futile and many either omitted this part or failed to form a new arrangement.

Solutions:

(a) **(A, B, D, E, F) =** $\left(\frac{1}{4}, \frac{1}{6}, \frac{5}{24}, \frac{1}{9}, \frac{1}{6}\right)$

(b) **A, D, F, B, E, C, G or reverse order.**

(c) **12 square units**

Recommendations

The use of investigative methods in developing reasoning and problem-solving processes needs to be addressed more aggressively in the classroom. Students must be given opportunities to reason with space as well as with quantity using physical props, where necessary.

Optional Section

Question 9

This question tested the candidates' ability to:

– interpret and make use of the functional notation f(x), $f^{-1}(x)$ and fg(x)
– find the inverse of a simple function
– find an expression for a composite function
– obtain the solution of a quadratic equation by factorization or by formula
– solve a word problem involving a quadratic equation.

The question was attempted by 59 per cent of the candidates, 4 per cent of whom scored the maximum available marks. The mean mark was 4.73 out of 15.

In general this question was not well done. The more able candidates obtained full marks for part (a) of the question. The weaker candidates were able to substitute correctly to evaluate f(-2) but made careless errors in simplifying their result. Candidates also successfully applied the first two steps of the algorithm to find the inverse of a function but in many instances failed to complete it because they lacked skills to manipulate algebraic expressions. Similar problems were encountered in simplifying their expression for gf(x).

In part (b), the majority of candidates recognized that the area of the rectangle was (2x-1)(x+3), but only a small percentage of these were able to expand this expression. Equating their quadratic expression to 294 was not a major problem for many candidates but some attempted to solve without transposing the 294.

Most of the candidates chose to use the formula to solve the quadratic equation and some candidates used the negative **x** value in calculating the dimensions even though it gave a meaningless result.

Solutions:

(a) **(i)** $\frac{-3}{5}$ **(ii)** $\frac{2x+9}{5}$ **(iii)** $\frac{5x-1}{2}$

(b) **(i)** **(2x – 1)(x + 3)** **(ii)** **x = 11** **(iii)** **21 cm × 14 cm**

Recommendations

While this optional question was by far the most popular, teachers must caution students that basic algebraic skills are required in order to attempt these questions successfully. The use of factorization as the first method to try in solving quadratic equations must be emphasized. Students must also be reminded that unlike linear equations, quadratic equations can only be solved when all terms are equated to zero.

Question 10

This question tested the candidates' ability to:

- translate algebraic inequalities into worded statements
- translate verbal statements from and into inequalities
- draw graphs of linear inequalities in one or two variables
- determine whether a set of points satisfy all conditions described by a set of inequalities
- use linear programming techniques to solve problems in two variables.

The question was attempted by 42 per cent of the candidates, 0.2 per cent of whom scored the maximum available mark. Performance was generally fair with a mean mark of 4.82 out of 15.

Candidates were generally able to translate verbal statements into algebraic inequalities. However, most candidates were unable to demonstrate the reverse process. Interpreting the given scale posed no problems for the majority of candidates.

Although quite a significant number of candidates correctly drew all the graphs and obtained the correct region, some made common errors such as:

- writing the inequality $x > 15$ instead of $y > 15$
- writing the inequality $x + y < 60$ instead of $x + y > 60$
- interchanging the variables, gold stars with silver stars
- drawing the line $y = 20$ instead of $x = 20$ and $x = 15$ for $y = 15$
- drawing the line $y = \frac{1}{2}x$ as $x = \frac{1}{2}y$
- using a 'solid' line instead of a 'broken' line for the graphs of inequalities.

The majority of the candidates were able to identify at least one of the two points satisfying the four conditions.

Solutions:

(a) $\mathbf{y \geq 15,\ x + y \leq 60}$

(b) The number of gold stars must be less than twice the number of silver stars.

(d) A and C lie in the region which satisfies all conditions.

Recommendations

Teachers need to allow students to construct their own inequalities using a variety of authentic situations so that they can develop the vocabulary relating to inequalities. Terms such as *at least, at most, not greater than, less than or equal to,* need to be emphasized.

Identifying the region defined by an inequality also needs to be more carefully addressed. Students should understand when to use broken lines as boundaries and how to test points to determine the feasible region.

Question 11

This question tested the candidates' ability to:

– recognize and use the trigonometrical ratios of special angles
– calculate the circumference of a circle of latitude
– determine the longitude of a point on the surface of the earth given its distance from another point on the same parallel of latitude.

The question was attempted by 9 per cent of the candidates, 2 per cent of whom scored the maximum available mark. The mean mark was 1.49 out of 15.

It was evident that candidates were not familiar with the surd and fractional form of the trigonometric ratios. Some attempted to use the calculator to find $\sin^{-1} \frac{\sqrt{3}}{2}$ but encountered problems because of incorrect use of the order of operations when inputting the inverse function. A few candidates were able to successfully use the trigonometric identity $\sin^2 x + \cos^2 x = 1$. A number of candidates did not use Pythagoras' Theorem to find the unknown side of the triangle.

In part (b), a number of candidates substituted R instead of $R\cos\theta$ in calculating the radius of the circle latitude 37^0 N.

Using the given arc length to find the difference in latitude posed a problem for many candidates; many were unable to rearrange their expression to evaluate the angle. Some who were successful failed to subtract 50^0 to obtain the value of x.

Solutions:

(a) (i) a) $\frac{1}{2}$ b) $\sqrt{3}$

(ii) $\frac{1}{2}$

(b) (i) 31 948 km

(ii) 10.7^o

Recommendations

Teachers should allow students to derive the trigonometric ratios for special angles using drawings and measurements prior to using the calculators.

The use of three-dimensional models in illustrating various concepts in relation to earth geometry must also be explored so that students can have mental props in visualizing the angles and distances in solving these type of problems.

Question 12

This question tested the candidates' ability to:

– solve problems using the properties of regular polygons
– calculate the area of a triangle given two sides and an included angle
– calculate the area of regular polygon (octagon) from the area of one triangle
– use theorems in circle geometry to calculate the measure of angles.

The question was attempted by 14.5 per cent of the candidates 1 per cent of whom scored the maximum available mark. The mean mark was 3.2 out of 15.

Responses to this question were poor. Candidates were generally able to find the area of the octagon by multiplying the area of one triangle by 8.

Many candidates did not understand the properties of a regular polygon and failed to recognize that the angle at the centre was $\frac{1}{8}$ of 360^0. It was assumed that triangle XYZ was equilateral and hence many gave 60^0 as their response.

Some candidates calculated the interior angles of the octagon and used a lengthy route to obtain the required angle.

In calculating the area of the triangle, those who did not recognize the use of $\frac{1}{2}$ ab sinθ attempted to calculate YZ but these attempts generally did not produce correct results, because of computational errors.

Solutions:

(a) **(i)** **45^0** **(ii)** **12.7 cm^2** **(iii)** **101.6 cm^2**

(b) **a)** **90^0, angle in a semi-circle**

b) **90^0, angle between a tangent and a radius**

c) **23^0, angle between tangent and chord is equal to the angle subtended by the same chord in the alternate segment**

d) **113^0, opposite angles of a cyclic quadrilateral are supplementary.**

Recommendations

The topic of circle theorems continues to pose challenges for many students and proficiency in this area is necessary if students are to develop logical thinking and problem-solving skills. Teachers need to give students opportunities to make oral presentations in order to clarify their reasons, and to be concise in their explanations.

Question 13

This question tested the candidates’ ability to:

– show the relative position of points on a diagram
– add vectors using the triangle law
– use a vector method to prove two lines are parallel
– use vectors to represent and solve problems in geometry.

The question was attempted by 15 per cent of the candidates 1 per cent of whom scored the maximum available mark. The mean mark was 4.91 out of 15.

Most candidates were able to locate the position of R, the midpoint of a line but the fraction ⅓ gave some difficulty and the point ‘S’ was not always placed nearer to O along OM. Routes for resultant vectors were arrived at in a variety of ways and although these were mostly correct, incorrect substitution of vectors resulted in the loss of marks. In many cases candidates did not know the direction of the vector.

Candidates knew the condition for vectors to be parallel but could not simplify their expressions to express one vector as a scalar product of the other.

Solution:

(b) **(i)** $\underline{\mathbf{k}} - \underline{\mathbf{m}}$ **(ii)** $\underline{\mathbf{m}} - \frac{1}{2}\underline{\mathbf{k}}$ **(iii)** $\frac{1}{3}\underline{\mathbf{m}} - \underline{\mathbf{k}}$ **(iv)** $\frac{1}{3}\underline{\mathbf{m}} - \frac{1}{2}\underline{\mathbf{k}}$

Recommendations

The abstract nature of this topic demands that teachers place greater emphasis on practical situations. Particular attention to the use of proper notation in writing vectors is also critical to developing proficiency in this topic.

Question 14

This question tested the candidates' ability to:

– evaluate the determinant of a 2×2 matrix
– obtain the inverse of a non-singular 2×2 matrix
– perform addition, subtraction and multiplication of matrices
– perform multiplication of matrices by a scalar
– describe a transformation geometrically given an object and an image
– determine the matrices associated with translation and rotation
– use matrices to solve simple problems in geometry.

The question was attempted by 27 per cent of the candidates, 1 per cent of whom scored the maximum available mark. The mean mark was 4.01 out of 15.

Responses to this question were barely satisfactory although some parts were well done. In part (a), many candidates were able to perform the scalar multiplication and to solve their resulting matrix equation. Although candidates were familiar with the method of finding the inverse, careless errors were common in stating the adjunct or in calculating the determinant.

In solving the matrix equation, candidates were able to equate corresponding terms in their 2×2 matrix, but poor algebraic skills prevented them from arriving at the correct solution. For example 3a + 2 was often expressed as 5a.

Describing the geometric transformations which represented the transformations posed difficulties for the majority of candidates. A large number of candidates omitted this part of the question while others gave an incomplete description by simply naming the transformation without stating the specific characteristics.

A number of candidates successfully obtained the matrix for the rotation. However, there was a tendency to use a 2×2 matrix instead of a column matrix to describe the translation.

Solutions:

(a) **(i)** $\begin{pmatrix} 3a & 3b \\ 3c & 3d \end{pmatrix}$ **(ii)** $\begin{pmatrix} 2 & -3 \\ -3 & 5 \end{pmatrix}$ **(iii)** $\begin{pmatrix} 3a+2 & 3b-3 \\ 3c-3 & 3d+5 \end{pmatrix}$

(iv) **a = 4, b = 1, c = -2, d = 0**

(b) **(i)** **a)** **A translation of -4 units parallel to the y-axis**

b) **A rotation of 180^0, about the origin OR an enlargement, scale factor -1 about the origin.**

(ii) **a)** $\begin{pmatrix} 0 \\ -4 \end{pmatrix}$ **b)** $\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$

(iii) **P´ (6, -2)**

(iv) **Q´ (-5, 4)**

Recommendations

Teachers must seek to strengthen basic algebraic skills prior to teaching topics in matrices. The principles involved in solving simple algebraic equations still apply when solving matrix equations and students must be allowed to make this connection.

Similarly, the understanding of basic geometric transformations is a necessary prerequisite for the understanding of matrix transformations. Hence teachers should take the necessary steps to ensure that basic conceptual knowledge is in place before introducing students to these abstract topics.

DETAILED COMMENTS

Basic Proficiency

The Basic Proficiency examination is designed to provide the average citizen with a working knowledge of the subject area. The range of topics tested at the basic level is narrower than that tested at the General Proficiency level.

This year approximately 32 per cent of the candidates achieved Grades I - III.

Paper 01 - Multiple Choice

Paper 01 consisted of 60 multiple-choice items. The maximum mark was not attained by any of the candidates. Approximately 41 per cent of the candidates scored at least half the maximum mark on this paper.

Paper 02 – Essay

Paper 02 consisted of 10 compulsory questions. Each question was worth 10 marks. The highest score earned was 95 out of 100. This was earned by two candidates. Thirteen per cent of the candidates earned at least half the total marks on this paper.

Question 1

This question tested candidates' ability to:

– perform any of the basic operations with rational numbers

– approximate a value to given number of significant figures

– solve problems involving fractions and decimals.

The question was attempted by 98 per cent of the candidates, 0.5 per cent of whom scored the maximum available mark. The mean mark was 3.12 out of 10.

The candidates were able to use calculators to evaluate 1.05^2 but they generally lacked knowledge of significant figures. Examples of incorrect responses were 690.000 or 69. Some candidates wrote their answer in standard form.

In part (b), candidates were able to recognize that the total number of shares was 7 and that 3 shares were equivalent to $45. However, some candidates assumed that one share was the $45 while others equated $45 to 7 shares.

In part (c), most candidates knew that they had to add the fractions but the weaker candidates associated the 'and' with multiplication and proceed to multiply $\frac{1}{4}$ by $\frac{3}{8}$ instead of $\frac{1}{4} + \frac{3}{8}$. Candidates were generally aware that they had to find the fraction that played basketball by subtracting before they found the required percentage.

Solutions:

(a) **(i)** **687.96** **(ii)** **690**

(b) **$105**

(c) **(i)** $\mathbf{\frac{5}{8}}$ **(ii)** **37.5%**

Recommendations

- The concept of significant figures should be reinforced and more practice exercises given.
- More non-routine problems based on ratio must be given to students, and more word problems involving fractions.

Question 2

This question tested candidates' ability to:

– perform operations involving directed numbers

– use symbols to represent binary operations and perform simple computations

– use the laws of indices to manipulate expressions with integral indices

– apply the distributive law to insert or remove brackets in algebraic expressions

– solve a simple linear inequality in one unknown.

The question was attempted by 95 per cent of the candidates, 1 per cent of whom scored the maximum available mark. The mean mark was 2.3 out of 10.

Generally, candidates performed unsatisfactorily. They experienced difficulty in all parts of the question.

Candidates were generally unable to apply the laws of indices. For example, they incorrectly equated $2p^2 \times 3p^3$ with $6p^6$ or 5p6.

In other cases, $2p^2 \times 3p^3$ was interpreted as $2^2 \times 3^3 \times p = 4 \times 27 \times p = 108p$.

In part (iii), candidates multiplied the entire expression by 3 instead of the terms in the bracket. Thus $3(2x + 1) - 4x$ was equated to $6x + 3 - 12x$, instead of $6x + 3 - 4x$. They also experienced difficulty in collecting the like terms and simplifying.

In part (b) $4 \times (-5^2)$ was interpreted to mean $(4\times - 5)^2$.

Solutions:

(a) (i) $6p^5$ (ii) p^2 (iii) $2x + 3$

(b) 100

(c) (i) $x < 2$ (ii) $x = 1$

Recommendations

- Emphasis should be placed on the laws of indices and operations with directed numbers.

Question 3

This question tested candidates' ability to:

– solve simple problems involving payments in installments as in the case of hire purchase

– solve problems involving insurances

– solve problems involving compound interest.

The question was attempted by 95 per cent of the candidates, 5 per cent of whom scored the maximum available mark. The mean mark was 3.31 out of 10.

Candidates showed a fairly good understanding of the topic hire purchase. However, many did not complete the question by finding the difference between the cash price and the hire purchase price.

Candidates showed very limited knowledge of insurance calculations. Most added the fixed land charge to the price of the house and then tried to find 0.5% of the total. Some found 5% instead of 0.5%.

Compound interest was very often calculated as simple interest.

Solutions :

(a) \$35

(b) \$1 350

(c) \$6 615

Recommendations

- Students should be taught to distinguish between simple and compound interest.
- More emphasis should be placed on all areas of consumer arithmetic.

Question 4

This question tested candidates' ability to:

– calculate the area enclosed by a trapezium
– use measurements on a map and a scale to calculate actual distance and vice versa
– solve problems involving measurements .

The question was attempted by 93 percent of the candidates, 7 per cent of whom scored the maximum available mark. The mean mark was 3.83 out of 10.
.
In part (a) most candidates were able to calculate the actual distance in km. However many could not do the calculation to find the distance on the map.

Solutions:

(a) (i) 11.2 km (ii) 7.5 cm

(b) (i) 6 cm (ii) 88 cm²

Recommendations

- Students need to do more examples in measurement involving irregular shapes.
- More emphasis on the use of scale drawings and converting from maps to actual measures and vice versa.

Question 5

This question tested candidates' ability to:

– solve problems involving cost price, percentages and discount
– solve problems involving salaries and wages.

The question was attempted by 95 per cent of the candidates, 13 per cent of whom scored the maximum available mark. The mean mark was 5.06 out of 10.

The majority of the candidates were able to calculate the total cost of books and magazines. They also understood the concept of doubling to obtain the overtime hourly rate. Although many of the candidates were able to calculate the discount, they were unable to follow through to determine the actual discount price. Many of the candidates were unable to calculate the overtime wage of $80 for the week.

Solutions:

(a) (i) $8 (ii) $16 (iii) $440

(b) (i) a) $150
b) $45

(iii) $179.40

Recommendations

- Students should be given more practice in solving word problems in consumer arithmetic.

Question 6

This question tested candidates' ability to:

– translate verbal phrases into algebraic symbols and vice versa

– solve simultaneous linear equations in two unknowns algebraically

– use a linear equation to solve word problems.

The question was attempted by 89 per cent of the candidates, 3 per cent of whom scored the maximum available mark. The mean mark was 1.97 out of 10.

Candidates generally had knowledge of methods used in solving simultaneous equations. However, some candidates failed to subtract correctly after equating the coefficients. A number of candidates obtained the result below.

$$\begin{aligned} x + 2y &= 7 \\ 6x + 2y &= 12 \\ \hline -5x &= 5 \\ &\text{or} \\ 5x &= -5 \end{aligned}$$

After the candidates found the value for one variable, they incorrectly substituted it for the other variable, for example, after obtaining $x = 1$, some mistakenly used this value to substitute for y instead of x in the equation.

In part (b), candidates interpreted the statement "more than" as the inequality symbol and wrote $p > 36$ instead of $p + 36$. In simplifying algebraic expressions, some common errors were: Equating $p + 36$ with $36p$ or with $36p^2$. Forming an equation from the given information also posed problems for candidates.

Solutions:

(a) x = 1, y = 3

(b) (i) a) p + 36 b) 2p + 36

(ii) p = 9

Recommendations

- The simplification of algebraic expressions needs to be practised by students, as well as the solution of simultaneous equations.

Question 7

This question tested candidates' ability to:

– use instruments to draw and measure angles and line segments

– use instruments to construct triangles

– use Pythagoras' Theorem to solve simple problems

– use trigonometric ratios in the solution of right-angled triangles.

The question was attempted by 79 per cent of the candidates, 1 per cent of whom scored the maximum available mark. The mean mark was 2.19 out of 10.

The responses to this question were fair. However, a number of candidates had difficulty drawing line segments and measuring angles accurately. Some candidates summed all three sides of the triangle to determine the angle BAC.

The majority of candidates had some knowledge of Pythagoras' Theorem and a number of candidates wrote XY = 13cm without showing any working. This may be as a result of previous knowledge of Pythagorean Triples.

The trigonometric ratios were not well understood by many candidates. A significant number did not attempt this part of the question.

Solutions:

(a) (ii) 57^{o}

(b) (i) 13 cm (ii) 9.96 cm

Recommendations

- Students must develop the ability to construct polygons using geometrical tools and to measure angles.
- Students need more practice in the applications of Pythagoras' Theorem and the trigonometric ratios.

Question 8

This question tested candidates' ability to:

– interpret graphical data

– recognize the gradient of a line given its equation

– determine the equation of a line given the gradient and one point on the line.

The question was attempted by 93 per cent of the candidates, 0.25 per cent of whom scored the maximum available mark. The mean mark was 2.19 out of 10.

In (a) (i) quite a few of the candidates were able to obtain the 35 seconds.

In b (ii) many candidates were familiar with the equation of a straight line $y = mx + c$. However, they were unable to make the appropriate substitution.

Most candidates were unable to interpret and calculate the distance in a(ii) and a few candidates stated the time rather than the distance

Most candidates were unable to obtain 12.5 seconds for (a) (iii) but instead stated 10, 10.5 and 15 seconds. Some candidates incorrectly calculated speed as $\frac{time}{distance}$ while others stated their speed as 200 metres in 8 seconds.

In part (b), many of the candidates were unable to identify the gradient as the coefficient of x in the equation $y = 2x – 1$. They also failed to substitute the given point in the equation to determine the value of c, thus stating the equation of the line incorrectly.

Solutions:

(a)	**(i)**	**35 secs**	**(iv)**	**5 secs (Chris fell)**
	(ii)	**75 m**	**(v)**	**8 m s $^{-1}$**
	(iii)	**12.5 secs**		
(b)	**(i)**	**2**	**(ii)**	**y = 2x + 3**

Recommendations

- Students need to be exposed to more problems involving graphs.
- Teachers need to emphasize that the coefficient of x is always the gradient of a straight line in the equation $y = mx + c$.
- Students need more practice in determining the equation of parallel and perpendicular lines.

Question 9

This question tested candidates' ability to:

– describe a translation using column vectors

– perform an enlargement given the centre and scale factor

– state the relationship between the area of an object and an image under an enlargement.

The question was attempted by 76 per cent of the candidates, 1 per cent of whom scored the maximum available mark. The mean mark was 1.54 out of 10.

The performance in this question was generally poor. Most of the candidates were able to identify the coordinate of the point K (2, 2) but were unable to determine the column vector $\begin{pmatrix} 2 \\ -6 \end{pmatrix}$ for the translation.

The majority of the candidates were unable to draw the correct image of the triangle KLM after the enlargement.

Although many candidates correctly stated the formula for the area of a triangle, they were unable to calculate the area of the triangle drawn.

Solutions:

(a) **K (2, 2), K´(4, -4)**

(b) $\begin{pmatrix} \mathbf{2} \\ \mathbf{-6} \end{pmatrix}$

(c) **K´´ (4,4), L´´(4,10), M´´(8,4)**

(d) **12 square units**

<u>Recommendations</u>

- Teachers must ensure that students know the difference between column vectors and coordinates.
- Enlargements and the other transformations should be taught using both graph paper and paper with no grid lines.

<u>Question 10</u>

This question tested candidates' ability to:

– construct a simple frequency table for a given set of data

– interpret data from a pie-chart

– determine the median of a set of data presented in a frequency table

– determine the probability of an event from data presented in frequency table.

The question was attempted by 88 per cent of the candidates, 1 per cent of whom scored the maximum available mark. The mean mark was 3.39 out of 10.

The majority of candidates were able to recognize that 90 out of 360 was $\frac{1}{4}$ to determine the number of candidates whose favourite music was j …………………………………..azz. Completion of the frequency table was also well done. However, many candidates could not determine the median value from the table or calculate the probability.
The majority of the candidates also had difficulty converting degrees from the pie chart into percentages.

<u>Solution:</u>

(a) **(i)** **15 students** **(ii)** **30%**

(b) **(i)**

Height	**Freq**
6	**4**
7	**3**
8	**1**

(ii) **a)** **6 cm** **b)** $\mathbf{\frac{1}{4}}$

<u>Recommendations</u>

- Measures of central tendency (mean, mode, median) need to be clearly defined and sufficient examples provided so that students can differentiate between them.
- Students should be exposed to more experimental probability by conducting simple experiments in the classroom.

CARIBBEAN EXAMINATIONS COUNCIL

REPORT ON CANDIDATES' WORK IN THE CARIBBEAN SECONDARY EDUCATION CERTIFICATE JANUARY 2008

MATHEMATICS

MATHEMATICS

GENERAL PROFICIENCY EXAMINATIONS

JANUARY 2008

GENERAL COMMENTS

The General Proficiency Mathematics examination is offered in January and May/June each year. The Basic Proficiency examination is offered in May/June only.

There was a candidate entry of approximately 14 800 in January 2008. This year, fifty-seven per cent of the candidates achieved Grades I – III. The mean percentage for the examination was 87.6.

DETAILED COMMENTS

Paper 01 – Multiple Choice

Paper 01 consists of 60 multiple-choice items. This year, thirty-one candidates earned the maximum available mark and seventy-seven per cent of the candidates scored at least half the total marks for this paper.

Paper 02 – Essay

Paper 02 consisted of two sections. Section I comprised eight compulsory questions totalling 90 marks. Section II comprised six optional questions: two each in Relations, Functions and Graphs; Trigonometry and Geometry; Vectors and Matrices. Candidates were required to choose any two questions. Each question in this section was worth 15 marks.

This year, no candidate earned the maximum available mark on Paper 02. However, three candidates scored 119 marks out of a possible 120. Approximately thirty-two per cent of the candidates earned at least half the maximum mark on this paper.

Compulsory Section

<u>Question 1</u>

This question tested candidates’ ability to:

- perform basic operations with fractions
- perform basic operations with decimals
- calculate the hire purchase price
- express one quantity as a percentage of another

The question was attempted by 99 per cent of the candidates, 28 per cent of whom scored the maximum available mark. The mean mark was 8.21 out of 11.

Generally, performance on this item was good. The majority of candidates demonstrated proficiency in performing operations on fractions and decimals. In part (a)(i), some candidates converted the fractions to decimals and then simplified. However, this procedure could not yield an exact answer in this situation.

In part (a)(ii), some of the weaker candidates simplified $2-\frac{0.24}{0.15}$ by first subtracting 0.24 from 2 and then proceeded to divide the result by 0.15.

In part (b), many candidates scored full marks. Errors were seen in expressing the difference between the hire purchase price and the cash price as a percentage of the cash price. For example, some candidates did not use the cash price in the denominator while others set up the percentage with 100 in the denominator.

Solutions:

(a) **(i)** $\frac{11}{14}$ **(ii) 0.4**

(b) **(i) \$354.00** **(ii) \$34.05** **(iii) 10.64%**

Recommendations

Teachers need to point out to students that computations involving fractions always yield exact results and converting fractions to decimals will sometimes yield an approximate result. They should also emphasize that in expressing one quantity as a percentage of another, the quantities to be compared must first be written as a fraction, with the whole as the numerator, before multiplying by 100%.

Question 2

This question tested the candidates' ability to:

- solve a linear inequality in one unknown
- identify sets of numbers, in particular whole numbers
- factorise expressions of the form ax+bx, a^2- b^2, ax+bx+ay+by
- use algebraic expressions to represent information
- use linear equations to solve worded problems

The question was attempted by 99 per cent of the candidates, less than 1 per cent of whom scored the maximum available mark. The mean score was 4.84 out of 12.

Solving the inequality in part (a), proved to be the most challenging part of this question. Candidates omitted to change the direction of the inequality when dividing by a negative quantity. They also had difficulty in identifying the smallest whole number that satisfied the inequality.

Factorisation by common factors or by the difference of two squares appeared to be easier than factorisation by grouping. The major difficulty experienced by candidates in using the grouping technique was extracting a negative factor.

There were several good attempts in part (c), where candidates were able to write the expression and equation for the total amount of money collected for the sale of sponge cakes. A few candidates did not multiply the cost by the quantity, while others collected their terms incorrectly while solving the equation.

Solutions:

(a)	**(i)**	**$x > -2$**	**(ii)**	**$x = 0$**		
(b)	**(i)**	**$x(x - y)$**	**(ii)**	**$(a + 1)(a - 1)$**	**(iii)**	**$(p - q)(2 - p)$**
(c)	**(i)**	**$2(k + 5)$**		**(ii)**	**$2(k + 5) + 10k + 4(2k)$**	
	(iii)	**$k = \$6.50$**				

Recommendations

Teachers need to emphasize the difference in techniques used to solve equations and inequations. The practice of verifying solutions when solving word problems must also be emphasized.

Question 3

This question tested the candidates' ability to:

- construct and use Venn diagrams
- solve problems involving the use of Venn diagrams
- determine the elements in the union, intersection and complement of two sets
- use the properties of parallel lines to calculate unknown angles
- use the properties of isosceles triangles to calculate unknown angles

The question was attempted by 99 per cent of the candidates, 5.34 per cent of whom scored the maximum available mark. The mean score was 5.66 out of 12.

Responses to part (a) were generally good. Candidates displayed strengths in constructing the Venn diagram and listing elements of the subsets. A few candidates omitted some elements of the union in their list while others mistakenly listed the members of the intersection.

Part (b) was poorly done. Candidates made assumptions in calculating angles and were unable to provide any reasons for their calculations. The properties of angles on parallel lines were hardly mentioned and there was difficulty in naming the angles.

Solutions:

(a)	**(ii)**	**a)**	**SUT ={l,m,k,p,q}**			
		b)	**S2 ={q,n,r}**			
(b)	**(i)**	**90^0**	**(ii)**	**48^0**	**(iii)**	**84^0**

Recommendations

Teachers need to teach Set Theory using practical examples to determine the intersection, union and complement of sets.

The properties of angles, parallel lines and polygons should be reinforced through regular practice and varied examples.

Question 4

This question tested the candidates' ability to:

- interpret the 24 hour clock notation and use it to calculate elapsed time
- calculate average speed
- calculate the area of a square, circle and a sector

The question was attempted by 98 per cent of the candidates, 4 per cent of whom scored the maximum available mark. The mean score was 3.66 out of 10.

In part (a), the majority of candidates were able set up the subtraction to calculate the length of the journey but many failed to obtain the correct result. Substituting their time into the formula for average speed did not pose problems for candidates. However, they failed to express 6 hours 50 minutes as hours and hence, could not obtain the correct speed.

In part (b), candidates were successful in calculating the area of the circle and square but did not recognize that the area of the sector was required to compute the area of the shaded region. They incorrectly subtracted the area of the square from the area of the circle to obtain the area of the shaded region.

Solutions:

(a) (i) **6 hours 50 min** **(ii)** **60 km/h**

(b) (i) **38.5 cm^2** **(ii)** **12.25 cm^2** **(iii)** **2.625 cm^2**

Recommendations

Teachers need to emphasize that in time, the base unit is 60 and not 100. The importance of consistency in units in calculating speed should also be addressed.

Question 5

This question tested the candidates' ability to:

- construct a frequency table from a bar graph
- interpret information from a bar graph
- determine the mean from a bar graph
- calculate the probability of a simple event

This question was attempted by approximately 94 per cent of the candidates, 4.65 per cent of whom scored the maximum available mark. The mean mark was 4.89 out of 12.

Candidates displayed strengths in stating the mode, computing the mean, and calculating the probability. In setting up the frequency table, some candidates had incorrect labeling, for example, the use of the term "frequency" was often incorrect. Interpreting the data in parts (b) and (e) posed most problems for candidates. Although many candidates correctly used the formula, $\frac{\sum fx}{\sum f}$ in computing the mean, they could not state how many boys were in the club, nor could they state how many books were read. In the latter case they merely added the scores from 0 to 4 and obtained an answer of 10.

Solutions:

(a)

Number of Books	**0**	**1**	**2**	**3**	**4**
Number of Boys	**2**	**6**	**17**	**8**	**3**

(b) **36 boys**
(c) **Mode : 2**
(d) **76 books**
(e) **Mean: 2.1**
(f) $\frac{11}{36}$

Recommendations

Teachers must pay closer attention to developing skills in interpreting data. Students must be given opportunities to experience statistics by generating their own data on relevant and interesting situations. Statistical concepts such as the mean and the mode should be introduced and understood before formulae are introduced.

Question 6

This question tested candidates' ability to:

- identify and describe transformations given an object and its image
- use instruments to draw and measure angles and line segments
- use instruments to construct a parallelogram given two adjacent sides and one angle

The question was attempted by 86 per cent of the candidates, less than 1% per cent of whom scored the maximum available mark. The mean mark was 3.61 out of 12. Responses to this question were mainly poor. Candidates seemed unable to recognize transformations in this particular context. In cases where the transformation was named correctly, candidates could not state the characteristics which define the transformation.

In constructing the parallelogram, although candidates were able to make accurate measurements, many were unfamiliar with the techniques for constructing parallel lines. Some candidates constructed the 60^0 angle correctly and proceeded to draw the parallelogram without the use of instruments.

Solutions:

(a) (i) A rotation of 180^0 about the point L
An enlargement, scale factor -1, about L
(ii) A translation of 4 units horizontally to the right and 1 unit vertically downwards

(b) (ii) WY = 10.9 cm

Recommendations

In the teaching of transformation geometry, teachers need to re-introduce informal approaches without the use of graph paper, before they use formal approaches on the Cartesian plane. The critical attributes of each transformation and the particular vocabulary associated with describing these transformations must be emphasized. Techniques in the proper use of geometrical instruments to construct parallel lines must be explored using different approaches.

Question 7

This question tested the candidates' ability to:

- use substitution to complete a table of values for a given function
- draw the graph of a quadratic function given a specific domain
- draw the graph of a linear function of the type y = k
- state the solution of a pair of equations – one linear and one quadratic
- derive a quadratic equation given its roots

The question was attempted by 91 per cent of the candidates, 1.55 per cent of whom scored the maximum available mark. The mean mark was 6.26 out of 11.

In general, the performance on this question was satisfactory. Candidates displayed strengths in calculating the table values, using the given scale, plotting points and drawing the graphs. In some cases, incorrect graphs were obtained because of errors in the values calculated or plotting the points. In some cases, candidates drew a vertical line for the line y = 2.

In stating the x-coordinates of the points of intersection of the line and the curve, some candidates gave the x-intercepts instead. Hence, a limited number of candidates was able to attempt the last part of the question.

Solutions:

(a) when x = 0, y = 0 ; when x = 3, y = -3 ; when x = 5, y = 5

(b) (ii) x = -0.4 and 4.4 (iii) $x^2 - 4x - 2 = 0$

Recommendations

Teachers should ensure that the students can determine the basic shape of the graph on inspecting the equation. This would assist them in identifying errors made in completing their table of values. Teachers must emphasize that rulers cannot be used to draw curves.

Question 8

This question tested the candidates' ability to:

- recognise number patterns
- calculate unknown terms in a number sequence
- state the formula for the n^{th} term in a sequence
- use a pattern to generate the sum of a series

The question was attempted by 95 per cent of the candidates, 1.41 per cent of whom scored the maximum available mark. The mean mark was 6.08 out of 10. Responses to this question were generally good. The majority of candidates recognised the number pattern and were able to calculate specific terms or the sum of the series. Deriving formulae in terms of **n** posed the most difficulty for candidates and some substituted a specific number instead of using a generalized result.

Solutions:

(a) **(i)** **21, ½ (6)(6 + 1)** **(ii)** **½ (n)(n + 1)**

(b) **(i)** **36^2** **(ii)** **$[½ (n)(n + 1)]^2$**

(c) **$[½ (12)(12 + 1)]^2 = 6024$**

Recommendations

Teachers should continue to create opportunities for students to recognise patterns and solve for unknown terms. However, more attention must be placed on generalization of the $\mathbf{n}^{th}$ term of a sequence.

Question 9

This question tested the candidates' ability to:

- represent inverse variations symbolically
- perform calculations involving the inverse variation
- apply Pythagoras' Theorem to obtain an algebraic relationship between the sides of a right-angled triangle
- expand a binomial expression and simplify algebraic terms
- solve a quadratic equation and interpret the solution in a given context
- solve problems involving quadratic equations

The question was attempted by 28 per cent of the candidates, 5.4 per cent of whom scored the maximum available marks. The mean mark was 4.78 out of 15.

Responses to this question were mainly unsatisfactory. In part (a), candidates were unable to represent the inverse variation symbolically and few had successful attempts in finding the constant of variation.

In part (b), a limited number of candidates was able to apply Pythagoras' Theorem to state the correct relationship among the sides of the triangle. Expanding the binomials and simplifying their result proved to be challenging for candidates. Many candidates gave $(a-7)^2$ as $(a^2 + 49)$. Those who went on to solve the quadratic equation preferred to use the formula rather than factorise. Interpreting the result was difficult for many candidates and negative lengths of sides were often given.

Solutions:

(a) **(i)** $V = \frac{K}{P}$ **(ii)** **K =6 400** **(iii)** **V = 13.3**

(b) **(i)** $(a-7)^2 + a^2 = (a+1)^2$ **(ii)** **a = 12 or 4**

(iii) **Reject a =4, since (4-7) is negative, hence a = 12**
Sides are 5, 12 and 13

Recommendations

Teachers should use integrated approaches to teach the topic of variation, emphasizing graphical approaches to determining the constant of variation. In treating the expansion of binomials, teachers should use visual approaches using the sides of rectangles to represent linear expressions of the form (x+k) so that students can appreciate that the expansion has three distinct terms.

Question 10

This question tested the candidates' ability to:

- write inequalities from worded statements
- draw graphs of linear inequalities in one or two variables
- determine the solution of a set of inequalities
- use linear programming techniques to determine the maximum value of an expression

The question was attempted by 42 per cent of the candidates, 4.7 per cent of whom scored the maximum available mark. The mean mark was 5.58 out of 15.

The majority of candidates were able to write the inequalities, although the weaker candidates omitted the variables in the second inequality and incorrectly wrote
$6 + $24 ≤ 360. Those candidates who gave a full response to the question displayed strengths in using the scale, drawing the line x+y = 30, stating the coordinates of their region and calculating the maximum profit.

A significant number of candidates lost marks when they drew the line 6x + 24y = 360 incorrectly or shaded the incorrect region.

Solutions:

(a) (i) $x + y \leq 30$, $6x + 24y \leq 360$

(b) (iii) (0, 0) (0, 15) (20, 10) (30, 0)

(c) (i) 0 ; \$45.00 ; \$50.00 ; \$30.00

(ii) Maximum Profit: \$50.00

Recommendations

Teachers should emphasize that the solution of an inequation is a region and that the correct region can be obtained by testing points on both sides of the line. Further, once a region has been identified, students should test points in the region to verify that all inequalities are satisfied. The use of a consistent system for identifying the feasible region must also be addressed.

Question 11

This question tested the candidates' ability to:

- use theorems in circle geometry to calculate the measure of angles
- draw a diagram to represent information involving bearings and distances
- solve problems involving bearings
- use the sine and cosine rules in the solution of problems involving triangles

The question was attempted by 23 per cent of the candidates 3.47 per cent of whom scored the maximum available mark. The mean mark was 5.45 out of 15.

In part (a), most candidates were able to calculate the unknown angles although many of the candidates did not give reasons for their answers. The drawing of the diagram in part (b) posed particularly difficult for candidates, especially showing the bearing. In calculating the distances, most candidates correctly chose the cosine rule and substituted the lengths of the sides of the triangle correctly. However, many were unable to calculate the angle ABC, others made errors in simplifying their result. The last part of the question which required candidates to calculate the bearing was omitted by most candidates.

Solutions:

(a) (i) 100^0 [The angle at the centre is twice that at the circumference]

(ii) 40^0 [Triangle WOY is isosceles, $\frac{180^0 - 100^0}{2}$]

(b) (ii) 134.1 m (iii) 123^0

Recommendations

Teachers should allow students to represent situations involving bearings using models, before drawing them on paper. Approaches to teaching bearings need to be practical and done in an outdoor setting. Basic geometrical skills in calculating angles should be reviewed prior to teaching this topic.

Question 12

This question tested the candidates' ability to:

- recognize and use the trigonometrical ratios to solve right-angled triangles
- solve problems involving angles of elevation and depression
- represent lines of latitude and longitude, and points given their positions on a sketch of the Earth
- calculate the distance between two points on the Earth measured along a circle of longitude
- calculate the circumference of a circle of latitude

The question was attempted by 8 per cent of the candidates, 1.64 per cent of whom scored the maximum available mark. The mean mark was 4.03 out of 15.

In part (a), candidates had difficulty in identifying the angles of elevation and depression. However, once these angles were inserted they were able to correctly select the trigonometric ratio needed to solve the problem. In part (b), most candidates were able to draw the lines of latitude and longitude, although the line of longitude did not always pass through the North and South Poles. Locating the points P and Q was well done. In calculating the distance PQ along the circle of latitude, some candidates did not use the radius of a great circle and many could not calculate the angular difference.

Solutions:

(a) (i) 28.6 m (ii) 12.9 m

(b) (iii) a) 8 890 km b) 25 714 km

Recommendations

In teaching the application of trigonometry to solving problems in the real world, teachers should construct apparatus to measure angles of elevation and use models to represent the Earth. Students should be encouraged to use these materials to construct diagrams and solve problems. In addition, scale drawings should be done to reinforce these concepts.

Question 13

This question tested the candidates' ability to:

- locate points on a diagram given the relevant information
- add vectors
- use vectors to represent and solve problems in geometry

The question was attempted by 24 per cent of the candidates 1.38 per cent of whom scored the maximum available mark, The mean mark was 4.83 out of 15.

In part (a), candidates were able to locate the points on their diagram and write vectors for AC and PQ. Many candidates demonstrated weaknesses in simplifying their vectors and could not derive the proof.

In determining the position vectors for RT and SR in part (b), many candidates did not reverse their vectors in writing the route, for example RT was written as OR + OT instead of RO + OT.

Candidates were unable to choose an appropriate strategy to determine the position vector of F. Hence, this part was either omitted or poorly done.

Solutions:

(a) **(ii) a)** $\mathbf{2x + 3y}$ **b)** $\mathbf{x + \frac{3}{2}\,y}$

(b) **(i)** **a)** $\begin{pmatrix} 2 \\ -6 \end{pmatrix}$ **b)** $\begin{pmatrix} -4 \\ 2 \end{pmatrix}$

(ii) **b)** **F(4, 1)**

Recommendations

The resolution of vectors should be taught using concrete examples such as airplane routes. Teachers should challenge students to find alternate routes to the same journey. Students should also be given sufficient opportunities to communicate their routes using vector notation, emphasizing that vectors can only be added end to end.

Question 14

This question tested the candidates' ability to:

- multiply matrices
- identify a singular matrix
- calculate the determinant of a 2×2 matrix
- find the inverse of a non-singular 2×2 matrix
- use matrix methods to solve a system of linear equations

The question was attempted by 50 per cent of the candidates, 4.35 per cent of whom scored the maximum available mark. The mean mark was 5.42 out of 15.

In part (a), the multiplication of a 1×2 matrix by a 2×2 matrix proved to be very challenging for candidates. The majority of candidates obtained a column matrix instead of a row matrix.

In showing that R is non-singular, many candidates merely found the determinant and did not make a statement to the effect that since the determinant was not zero, the matrix was non-singular. In multiplying R by R^{-1}, many candidates worked with the fraction inside the matrix thus setting themselves up for computational errors in multiplying the elements. Some candidates were unfamiliar with **I** as the identity matrix.

There were better attempts at part (b) where candidates had to use matrices to solve a pair of simultaneous equations. However, many candidates did not use their calculations of the inverse from the earlier question and treated the last part of the question as an independent question, losing valuable time redoing the calculations.

Solutions:

(a) **$x = 4$, $y = 3$**

(b) **(i)** **Determinant of R is 7.**
Since the determinant of R is not equal to zero, R is non-singular.

(iv) **$x = 1$, $y = 2$**

Recommendations

Teachers should emphasize that there is a condition for the multiplication of matrices. Students should be given opportunities to create their own matrices and determine if it is possible to multiply any two of these matrices. In solving simultaneous equations, students should appreciate that it is more efficient to perform the multiplication by $\frac{1}{\text{det}}$ in the final stage of the solution.

C A R I B B E A N   E X A M I N A T I O N S   C O U N C I L

REPORT ON CANDIDATES' WORK IN THE SECONDARY EDUCATION CERTIFICATE EXAMINATIONS

JUNE 2008

MATHEMATICS (T & T)

MATHEMATICS

GENERAL PROFICIENCY – TRINIDAD AND TOBAGO

MAY/JUNE 2008

The General Proficiency Mathematics examination is offered in January and May/June each year, while the Basic Proficiency examination is offered in May/June only.

In May/June 2008 approximately 20 000 candidates registered for the General Proficiency examination. At the General Proficiency level, approximately 47 per cent of the candidates achieved Grades I – III.

DETAILED COMMENTS

General Proficiency

No candidate scored the maximum mark on the overall examination. Forty per cent of the candidates scored at least half the available marks.

Paper 01 – Multiple Choice

Paper 01 consisted of 60 multiple-choice items. Approximately 57 per cent of the candidates scored at least half the total marks for this paper.

Paper 02 – Essay

Paper 02 consisted of two sections. Section I comprised eight compulsory questions totaling 90 marks. Section II comprised six optional questions: two each in Relations, Functions and Graphs; Trigonometry and Geometry; Vectors and Matrices. Candidates were required to choose any two questions. Each question in this section was worth 15 marks.

This year, fourteen candidates earned the maximum available mark on Paper 02. Approximately 33 per cent of the candidates earned at least half the maximum mark on this paper.

Question 1

This question tested candidates' ability to:

- Perform basic operations on mixed numbers
- Solve problems associated with income tax
- Calculate a percentage of a derived quantity
- Write as a percentage the ratio of two quantities

The question was attempted by 100 per cent of the candidates, 15 per cent of whom scored the maximum available mark. The mean score was 7.42 out of 12.

Responses were satisfactory. Candidates demonstrated competence in:

- Choosing an appropriate strategy for subtracting common fractions
- Converting a mixed number into an improper fraction
- Inverting and multiplying when dividing improper fractions
- Calculating annual salary given monthly salary
- Calculating 22 per cent of a quantity
- Expressing one quantity as a percentage of another

Some candidates employed incorrect algorithms for performing operations on mixed numbers.

e.g. $1\frac{5}{8}$ was equated to $\frac{40}{8}$

$2\frac{1}{5}-1\frac{1}{3}$ was simplified to $\frac{13-5}{15}=\frac{8}{15}$

Many of the candidates disregarded the fact that the family had 3 children and so computed the total taxable allowance on the assumption that there was only one child. Some candidates did not recognize the difference between taxable allowance and income tax payable. They therefore, calculated 22 % of the taxable allowances.

Recommendations

It would be useful to expose students to more real-life situations based on income tax.

Solutions:

(a) 8/15

(b) (i) $90000.00 (ii) $22500.00 (iii) $14850.00
(iv) 16.5 %

Question 2

This question tested candidates' ability to:

- Square a binomial
- Change the subject of the formula
- Factorize expressions using common factors and the difference of two squares
- Solve a pair of linear simultaneous equations

The question was attempted by 97 per cent of the candidates, 8 per cent of whom scored the maximum available mark. The mean score was 4.2 out of 12.

A high proportion of candidates scored zero. The areas of good performance were:

- Factorizing the expression $3mn - 6n^2$
- Choosing strategy for solving simultaneous equations

Students in general experienced difficulty in:

- Squaring the binomial
- Changing the subject of the formula
- Factorizing the difference of two squares
- Applying strategy for solving simultaneous equations

Recommendations

Topics which are covered in the junior school need to be reviewed systematically.

Solutions:

(a) $\mathbf{9a^2 - 6a + 1}$

(b) $p = \dfrac{5-3q}{q}$

(c) (i) $\mathbf{3n\,(m - 2n)}$ **(ii)** $\mathbf{(5p - q)(5p + q)}$

(d) $\boldsymbol{x = 5}$; $\boldsymbol{y = -2}$

Question 3

This question tested candidates' ability to:

- Describe a set
- Identify the cardinal number of a set
- Determine the complement of a given set
- Determine the elements in the intersection of a set
- Use a ruler and protractor to draw angles and measure line segments

The question was attempted by 98 per cent of the candidates, 2 per cent of whom scored the maximum available mark. The mean score was 6.01 out of 12.

Students were able to:

- List the members of the intersection
- Measure lines and angles with some degree of accuracy

Students in general:

- Failed to demonstrate an understanding of the complement of a set
- Were unable to find the cardinal number for a set
- Were unable to recognize relations among numbers, hence, the terms ‘multiples’ and ‘factors’ were used interchangeably
- Experienced difficulty in describing the range of a set
- Appeared to have measured from ‘1’ rather than ‘0’ on their rulers
- Attempted to construct the quadrilateral rather than draw it

Recommendations

Teachers should pay attention to instructional terms such as draw, sketch, construct, describe, define.

Solutions:

(a) (i) a) { 12, 24 } **b) { 4, 8, 16, 20 }**

(ii) 9

(iii) a) even numbers less than or equal to 30

b) multiples of 6 less than or equal to 30

Question 4

This question tested candidates’ ability to:

- Calculate the area of a right-angled triangle
- Calculate the length of a prism given its volume and cross-section
- Use Pythagoras theorem to find the length of a line segment
- Identify the number of faces, edges and vertices in a given prism
- Use, correctly, the S.I. units of measure for area and length

The question was attempted by 92 per cent of the candidates, 14 per cent of whom scored the maximum available mark. The mean score was 4.64 out of 10.

Candidates were generally able to calculate the area of the triangle. However, many candidates either omitted the units of measure for area (cm^2) or wrote the incorrect S.I. units of measure (cm). Many of them chose an appropriate strategy for finding the length of AC (use of Pythagoras or trigonometric ratio). However, they encountered difficulty following through with the computation.

Candidates found difficulty in calculating the length of the prism given its volume and area. Many misinterpreted the length of the prism to be the height of its cross-section. Several candidates interchanged the number of edges and vertices.

Recommendations

Teachers may wish to make greater use of concrete objects in teaching the properties of solids.

Solutions:

(a) 8 cm^2

(b) 9 cm

(c) 5.7 cm

(d) 5 faces, 9 edges, 6 vertices

Question 5

This question tested candidates' ability to:

- State the relation between an object and its image as a combination of transformations
- Locate the image of a set of points when the transformations (reflection and enlargement) are performed
- Determine the ratio of the area of the object and its image under an enlargement

The question was attempted by 89 per cent of the candidates, 3 per cent of whom scored the maximum available mark. The mean score was 5.38 out of 12.

The responses to this question were generally satisfactory. Candidates knew how to reflect the original triangle in a line. Most of them were able to plot the points of the triangle representing the enlargement.

The majority of the candidates were able to identify the transformation as an enlargement. However, they were unable to describe the transformation fully, particularly the negative aspect of the scale factor and the centre of enlargement (the origin). Students were unfamiliar with the use of the square of the scale factor to find the ratio of the areas of the triangle and its enlarged image.

Solutions:

(c) (ii) Enlargement; centre (0, 0); scale factor -2

(d) 4

Question 6

This question tested candidates' ability to:

- Represent given statistical data graphically, using specified graph and scale
- Use a graph to make estimates and to draw inferences from it

The question was attempted by 88 per cent of the candidates, 11 per cent of whom scored the maximum available mark. The mean score was 5.9 out of 11.

Most candidates were able to identify the five-year periods where there were increases in the population. Generally they were able to show on the graph how to estimate the population of the country in 1989. Most candidates were able also to draw correctly, the scales on both axes. However, a large number of them

- Interchanged the axes
- Used the values from the table as intervals on the population axis
- Drew bar charts or histograms
- Joined the points without using a straight edge

Most candidates were unable to explain in a concise manner (using terms such as gradient, slope) two facts about the population.

Recommendations

Teachers may wish to expose their students to the full range of graphs as specified by the syllabus. Interpretation of information represented graphically needs to be reinforced.

Solutions:

(b) 1.75 to 1.85 million
(c) (i) 1990 – 1995 (ii) 1995 - 2000

Question 7

This question tested candidates' ability to:

- Calculate the coordinates of the end point of a line segment given the midpoint and the other end point
- Determine the image on to which a particular function maps a member of the domain

- Find the inverse of a linear function

- Find the domain value for which the composite function is equal to a real number

The question was attempted by 77 per cent of the candidates, 8.62 percent of whom scored the maximum available mark. The mean score was 4.24 out of 11.

Candidates were generally able to employ a suitable strategy to find the values of j and k, some used $\frac{1+j}{2}=4$ and $\frac{8+k}{2}=5$ while others plotted the coordinates and extended the line segment.

Candidates were able to substitute correctly for $f(0)$ and $g(2)$ but multiplication by zero proved problematic, for example, 5×0 was commonly written as 5 and $\frac{4}{2+1}$ was equated to 2.

Responses to finding the inverse function were generally satisfactory. However, after candidates interchanged the domain and range, transposition of terms proved difficult for them.

Candidates experienced difficulty with the composite function. They simply multiplied the functions. In some cases they substituted 1 for *x* rather than equating the composite function to 1. Many candidates could not proceed beyond $5(\frac{4}{x+1})-2$ to solve for x.

<u>Solutions:</u>

(a) **j = 7 ; k = 2**

(b) **(i)** **-2** **(ii)** **4/3** **(iii)** $f^{-1}(x)=\frac{x+2}{5}$ **(iv)** $\frac{17}{5}$

<u>Question 8</u>

This question tested candidates' ability to:

- Continue a sequence of diagrams given three terms in the sequence

- Follow number patterns in order to make generalizations

The question was attempted by 87 per cent of the candidates, 10 per cent of whom scored the maximum available mark. The mean score was 4.67 out of 10.

Most candidates were able to

- Sketch the fourth pattern in the sequence although in some cases they did not distinguish between grey and white triangles

- Deduce the pattern for the total number of triangular shapes and the number of grey triangular shapes. However, they had difficulty in generalizing the number of white triangular shapes

Recommendations

Teachers might choose to expose students to a wide range of problem solving experiences involving number patters and generalizations. Investigative methods should prove to be helpful.

Solutions:

(b) (i) 36, 21, 15 **(ii) 400, 190**

(c) $\boldsymbol{n^2}$, $\frac{n(n+1)}{2}$

Question 9

This question tested candidates' ability to:

- Solve a pair of simultaneous equations – one linear and one quadratic
- Establish an identity
- Determine the minimum value of a function
- Sketch the graph of a quadratic function to show roots of the function and the values of x at which the function is a minimum

The question was attempted by 25 per cent of the candidates, 12 per cent of whom scored the maximum available mark. The mean score was 5.32 out of 15.

Most candidates were able to:

- Eliminate one of the variables and to solve for x and y
- Solve the quadratic equation and to sketch the curve showing the minimum value and the intercept on the x-axis

Candidates generally had difficulty:

- Completing the square
- Identifying the x coordinate of the minimum point

Recommendations

Teachers might consider teaching the completion of the square as a means to an end which may include curve sketching, solving quadratic equations, maximizing and minimizing equations.

Solutions:

(a) x = 1, y = -14; x = 4, y = -8

(b) (i) $(x-\frac{3}{2})^2-1\,4\frac{1}{4}$ **(ii)** $\boldsymbol{-14^{\frac{1}{4}}}$ **(iii) 5.3; -2**.3

Question 10

This question tested candidates' ability to:

- Draw and use the graph of the function y = a x^{-2}
- Draw a tangent to a curve at a point
- Estimate the gradient of a curve at a point

The question was attempted by 46 per cent of the candidates, 10 per cent of whom scored the maximum available mark. The mean score was 9.26 out of 15.

The majority of the candidates who attempted this question were able to do part (a) quite well. They were also able to plot points on the Cartesian plane and connect the points to form a smooth curve.

An area of difficulty was in part (a) when calculating the y-values for the corresponding x-values. Candidates were unable to convert the fractional values into decimals (they simply divided by the smaller number).

Drawing axes and using the given scales also posed some problems, candidates simply used the values of y at intervals on the y-axis.

Part (d) posed the greatest difficulty. In many cases candidates were unable to draw a tangent at the given point and calculate the gradient of that tangent. Some candidates used points on the curve itself, to calculate the gradient. Also, when calculating the gradient of the tangent, candidates neglected the negative sign and when they quoted the formula $\frac{y_2 - y_1}{x_2 - x_1}$, they interchanged x and y values. In some cases they inverted the formula to obtain $\Delta x/\Delta y$.

Solutions:

(a) **5; 0.8; 0.4**

(c) **(i)** **y = 1** **(ii)** **x = 2.4**

(d) **- 1.5**

Question 11

This question tested candidates' ability to:

- Sketch a diagram given bearings and distances
- Calculate unknown angles
- Calculate lengths of two sides of a figure using trigonometry

- Read bearings

The question was attempted by 29 per cent of the candidates, 5 per cent of whom scored the maximum available mark. The mean score was 4.37 out of 15.

Stating the size of angle QRS and reading the bearings at Q and S were well done. Even if the angle QSR was inaccurately calculated, candidates were able to obtain the bearing of Q.

Many candidates were unable to read angles RQS or QST. Obtaining QT was difficult for some candidates. The cosine rule was necessary to obtain it.

Recommendations

- More attention must be given to:

- Reading and understanding written instructions. (some candidates sketched four diagrams instead of one)

- Locating the position of a point , given bearings

- Reading bearings from a given sketch

- Assisting candidates in recognizing when to use the sine rule and the cosine rule. (They should know that the trigonometric ratios sine, cosine, and tangent can only be used with right angled triangles.)

- Reading and referring to an angle using three vertices (some candidates found angle QSR instead of angle QST.

Solutions:

(c) (i) 57^0 (ii) 95^0 (iii) 152^0

(d) (i) 7 km (ii) 9 km

(e) 298^0

Question 12

This question tested candidates' ability to:

- To solve problems related to the properties of a circle

- Know and use the properties of an isosceles triangle

- Calculate the distance between two points on the earth along lines of longitude or latitude

The question was attempted by 13 per cent of the candidates, 2 per cent of whom scored the maximum available mark. The mean score was 4.06 out of 15.

The majority of the candidates who attempted this question were able to successfully copy the diagram and correctly indicate the 60 degree north latitude and the 25 degree west longitude. Another area of good performance was in part (a) where the students were able to recognize that triangle KLN was isosceles and to use its properties.

A common error made by candidates was incorrectly assuming that the line NL was a diameter of the circle and hence angles NKL and NML were both 90 degrees. Students demonstrated lack of understanding of the concept of the alternate segment in a circle. Although candidates recognized that NKLM was a cyclic quadrilateral, they wrongly stated that the opposite angles were equal as opposed to being supplementary.

In part (b) candidates failed to correctly state the formula for finding arc length. They showed great deficiencies in calculating angle θ as being 65 degrees and in calculating the radius of the circle of latitude. Most times the angle for the centre (65 degrees) was used instead of 60 degrees.

Teachers should ensure that students review the basics of angles, sectors and circles before attempting questions related to earth geometry.

Solutions:

(a) (i) 24^0 (ii) 132^0 (iii) 48^0

(b) (iii) 3611 km

Question 13

This question tested candidates' ability to:

- Write position vectors of given points
- Add vectors
- Find the magnitude of a vector
- Prove three points are collinear

The question was attempted by 21 per cent of the candidates, 5 per cent of whom scored the maximum available mark. The mean score was 3.47 out of 15.

Candidates were able to state the position vectors with little difficulty.

However, many were unable to find the value of p. Some did not know the magnitude formula for a vector while others who knew the formula had difficulty solving the resulting quadratic equation.

Recommendations

Teachers must encourage students to:

- Use appropriate mathematical vocabulary relevant to the topic when teaching

- Use and interpret vector notation, for example, the difference between a point and its position vector

- Use a vector method to establish a proof

<u>Solutions:</u>

(i) $\begin{pmatrix} -1 \\ 3 \end{pmatrix}$ **(ii)** $\begin{pmatrix} -5 \\ 4 \end{pmatrix}$ **(iii)** $\begin{pmatrix} 7 \\ p \end{pmatrix}$

(b) **(i)** $\begin{pmatrix} 4 \\ =1 \end{pmatrix}$ **(ii)** $\begin{pmatrix} 8 \\ -3+p \end{pmatrix}$

(c) ***p* = 9, -3**

<u>Question 14</u>

This question tested candidates' ability to:

- Identify an 2 × 2 singular matrix

- Perform multiplication of matrices

- Use matrices to solve problems in algebra

- Use matrices to solve problems in transformation geometry

The question was attempted by 23 per cent of the candidates, 7 per cent of whom scored the maximum available mark. The mean score was 4.04 out of 15.

In part (a), most candidates associated the singularity of a matrix with its determinant. Part (b) proved to be a great challenge to candidates as they were unable to perform the correct matrix multiplication. Many candidates left out the addition sign between 10 and b and 5a and 4b. There were many instances where arbitrary guesses were made with no working shown, for instance candidates wrote a = 1, b = 2.

In part (c) many candidates simply stated the matrix that represented the transformation mapping E onto E' as $\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$. In other cases, they showed a good working knowledge of the choice of strategy to be used in (c) (i) but failed to correctly apply this strategy, for instance, they wrote 5y = 5. There were a few cases where candidates used a graphical approach to get values for x and y.

Part(c)(ii) was well done, but an area of difficulty was sometimes seen in transposing points from coordinate form to column matrix form in order to perform the necessary calculations.

Candidates showed weakness in obtaining the coordinates of H. They were unable to perform this inverse translation. Examples of errors were adding H' to T or adding H to T, forming equations and equating to H'

Candidates showed a lack of understanding of the concept of a combined transformation. Some common errors were performing T first then S or attempting to combine S and T first.

Recommendations

Teachers should help students to follow instructions given in a logical order and practise using matrices to solve problems.

Solutions:

(b) $a = 3$; $b = -2$

(c) **(i)** $x = 1$; $y = -1$ **(ii)** $p = 5$; $q = 2$ **(iii)** P' **(9, 6).**

C A R I B B E A N   E X A M I N A T I O N S   C O U N C I L

REPORT ON CANDIDATES' WORK IN THE
SECONDARY EDUCATION CERTIFICATE EXAMINATIONS
MAY/JUNE 2008

MATHEMATICS - (ROR SCRIPTS)

MATHEMATICS

GENERAL AND BASIC PROFICIENCY EXAMINATIONS

MAY/JUNE 2008

GENERAL COMMENTS

The General Proficiency Mathematics examination is offered in January and May/June each year, while the Basic Proficiency examination is offered in May/June only.

In May/June 2008 approximately 57 000 candidates registered for the General Proficiency examination. Candidate entry for the Basic Proficiency examination was approximately 4 000. At the General Proficiency level, approximately 37 per cent of the candidates achieved Grades I – III. At the Basic Proficiency level, approximately 32 per cent of the candidates achieved Grades I – III.

DETAILED COMMENTS

General Proficiency

One candidate scored the maximum mark on the overall examination. Twenty-eight per cent of the candidates scored at least half the available marks.

Paper 01 – Multiple Choice

Paper 01 consisted of 60 multiple choice items. This year, 53 candidates earned the maximum available mark. Approximately 54 per cent o the candidates scored at least half the total marks for this paper.

Paper 02 – Essay Choice

Paper 02 consisted of two sections. Section I comprised eight compulsory questions totaling 90 marks. Section II comprised six optional questions: two each in Relation, Functions and Graphs; Trigonometry and Geometry; Vectors and Matrices. Candidates were required to choose any two questions. Each question in this section was worth 15 marks.

This year, four candidates earned the maximum available mark on Paper 02. Approximately 18 per cent of the candidates earned at least half the maximum mark on this paper.

Compulsory Section

Question 1

This question was designed to test candidates' ability to:

- perform basic operations with decimals
- perform basic operations with fractions
- convert from one currency to another given the exchange rate

The question was attempted by 100 percent of the candidates, 33 per cent of whom scored the maximum available mark. The mean mark was 7.48 out of 10.

Responses were generally good with the majority of candidates obtaining correct solutions to all parts of the question.

In part (a), errors were made mainly by candidates who performed the computations without the use of calculators. They often failed to place the decimal point in the correct position when multiplying or when computing the square root. Candidates were more proficient in dividing fractions than in subtracting fractions.

In converting currencies using the given rate of exchange, some candidates used $72.00 instead of $72.50 and hence could not obtain the exact answer. They also ignored the cents and gave answers correct to the nearest dollar.

In part (b) (ii), a significant number of candidates correctly obtained the amount left (in $JA) on the credit card by subtracting what was spent from $30 000. However, they did not proceed to convert this amount to Canadian dollars.

Solutions

(a) **(i)** **1.873** **(ii)** $\mathbf{2\frac{4}{15}}$

(b) **(i)** **JA $18 125.00** **(ii)** **CAN $163.79**

Recommendations

In order to avoid errors with respect to the position of the decimal point, students must be reminded to check for the reasonableness of their answers when performing operations on decimals. Unless a question specifies a particular degree of accuracy, students must leave their answers correct to the nearest cent when performing calculations with money.

Question 2

The question tested the candidates' ability to:

- perform operations involving directed numbers
- substitute numbers for algebraic symbols in simple algebraic expressions
- translate verbal phrases into algebraic symbols
- solve simple equations in one variable
- factorise simple algebraic expressions using common factors
- factorise a quadratic expression

The question was attempted by 99.6 per cent of the candidates, 2 per cent of whom scored the maximum available mark. The mean score was 5.3 out of 12.

Most candidates substituted the values correctly in the expressions but a significant number failed to simplify the answer. Weaknesses in operating with directed numbers were evident. For example $2(-1 + 3)$ was equated to $2(-4)$ or $2(-2)$. Candidates also made errors in simplifying $4(-1)^2$. This was often written as: $(-4 \text{ x } -4 \text{ x } -1 \text{ x } -1)$ or (-4^2) or $4(-1)$.

Too many candidates had poor responses to part (b) of this question. In part (i), four times the sum of x and 5 was stated as $4x + 5$, again ignoring the brackets. Far too many candidates wrote an inequality in response to part (ii) instead of an expression. A popular response was $16 > ab$.

In solving the equation in part (c), many candidates had problems in using the distributive law while others made errors in transposing terms. Hence, $2(3x + 1)$ was often seen as $6x + 1$ and $15 - 4x = 6x + 2$ was simplified to $-4x + 6x = 2 + 15$.

In using the method of common factors to factorise the expression, only a few candidates were able to get the correct HCF. Errors were frequently made in dividing algebraic terms. Weaker candidates had no idea what factorise meant and attempted to add the terms to give $18a^6b^4$.

Factorizing the quadratic expression also presented problems for candidates. Errors in signs were common. A significant number of candidates assumed it was an equation and attempted to use the quadratic formula or the method of completing the squares to solve for m.

Solutions

(a)	**(i)**	**4**	**(ii)**	**–2**
(b)	**(i)**	**4 (x + 5)**	**(ii)**	**16 + ab**
(c)	**(i)**	$\mathbf{\frac{13}{10}}$		
(d)	**(i)**	$\mathbf{6a^2b\,(b^2 + 2a^2)}$	**(ii)**	$\mathbf{(2m - 1)\,(m + 5)}$

Recommendations

Teachers need to ensure that students are familiar with the necessary vocabulary associated with algebra. Terms such as expression, equation, factorize, simplify and solve must be explored fully. In the teaching of translation of worded statements to symbols, students need to reverse the process and translate symbols to words as well. When solving equations, students should verify their solutions. Careless mistakes in substitution can be avoided if students use brackets in making substitutions and then simplify each term separately before collecting them.

Question 3

The question tested the candidates' ability to:

- calculate the frequency of an item given the total frequency and a frequency distribution
- calculate the angles in sectors of a Pie Chart
- construct a Pie Chart

The question was attempted by 97 per cent of the candidates, 27 per cent of whom scored the maximum available mark. The mean score was 5.51 out of 12.

Almost all of the candidates who attempted this question were able to calculate the value of the unknown frequency, t.

Calculating the angles for the Pie Chart in (b) (i) posed the most difficult for candidates. Many failed to recognize that each frequency had to be expressed as a fraction of the total before multiplying by 360°. A large number calculated percentages for each category and then multiplied these percentages by 360° in an attempt to convert them to angles.

In constructing the Pie Chart, far too many candidates did not use protractors to measure angles. Those who attempted to measure frequently made errors by reading the complement of the angle on the protractor, for example 80° was read as 100°. Incorrect angle calculations often resulted in Pie Charts in which angles did not add up to 360°. Hence it was quite common to see six instead of five sectors. There were also several instances where sectors were not labelled.

Solutions

(a) 105

(b) Lawyer: 80º; Teacher: 63º; Doctor 35º; Artist: 72º; Salesperson: 110º

Recommendations

Teachers must emphasize that a Pie Chart is a statistical graph in which the sectors of the circle represent fractions of the sample surveyed. Hence the calculation of fractional parts must precede calculation of angles. Students who used faulty algorithms to compute angles were unable to make this connection and could not detect their errors. The use of instruments in drawing accurate diagrams must also be emphasized. In particular, care must be taken in selecting the correct scale (inner or outer) when using a protractor.

Question 4

The question tested the candidates' ability to:

- list members of a set from a given description
- construct Venn Diagrams to show relationships between two sets within a Universal set
- list members of the union of two sets and the members of the complement of a set
- identify prime, odd and even numbers
- use instruments to construct angles and measure line segments
- use instruments to construct a parallelogram

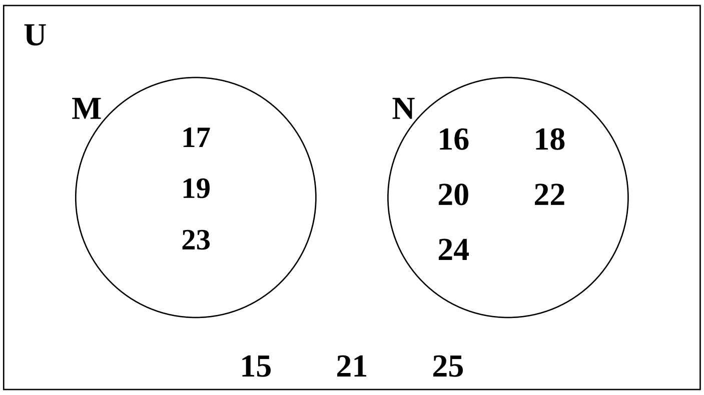
This question was attempted by approximately 98 per cent of the candidates 8 per cent of whom scored the maximum available mark. The mean mark was 6.80 out of 12.

Candidates had fairly good responses to part (a) in which concepts in sets was tested. They displayed strengths in identifying even numbers and prime numbers. Setting up the Venn Diagram also appeared to be an easy task for candidates although many did not recognize that the intersection was empty. Identifying members of the region $(M \cup N)'$ posed the most challenging for candidates who either omitted this part or gave members of the union instead.

In constructing the parallelogram, candidates generally were successful in constructing the 60° angle and measuring the lengths of sides AB and AC. Only a few candidates were able to use instruments to accurately construct the parallelogram. Errors in labelling the parallelogram were common and many located D incorrectly to produce a quadrilateral ACBD instead of ABCD.

Solutions

(a) **(i)**



(ii) $(M \cup N)' = \{15, 21, 25\}$

(b) **AC = 12.1 ± 0.1**

Question 5

The question tested the candidates' ability to use graphs of a quadratic function to:

- calculate the perimeter of a compound shape
- calculate the area of a compound shape
- solve problems in measurement involving tiling

This question was attempted by approximately 88 per cent of the candidates 5 per cent of whom scored the maximum available mark. The mean mark was 4.38 out of 12.

Although candidates were familiar with the formulae for finding perimeter and area of rectangles they often applied them incorrectly, especially in finding the perimeter of the compound shape. There was a tendency to add the perimeter of rectangle A to the perimeter of rectangle B, thus including the length of a line which was not along the boundary of the shape.

Candidates were more successful in calculating the area of the compound shape, but correct answers to part (c) were hardly seen. Many candidates omitted this part completely and some who attempted used inconsistent units in their calculations. For example the area of one flooring board was often written as 1 m x 20 cm.

Solutions

(a) **(i)** **6 metres** **(ii)** **4 metres**

(b) **40 metres**

(c) **74 m^2**

(d) **250 flooring boards**

Recommendations

Teachers need to emphasize the concept of perimeter using examples of irregular figures prior to using formulae to calculate the perimeter of a shape.

Practical work in measurement should be done using authentic situations such as tiling and carpeting.

Question 6

The question tested candidates' ability to:

- identify angles of elevation on a diagram
- use trigonometric ratios to solve a right a right-angled triangle
- describe a translation given the object and image
- draw the line $y = x$
- locate the image of an object under a refection in the line $y = x$

This question was attempted by approximately 91 per cent of the candidates 1 per cent of whom scored the maximum available mark. The mean mark was 2.3 out of 12.

Inserting the angles of elevation in part (a) of this question was done correctly by less than half of the candidates. Common errors were interchanging the angles or inserting them between the vertical and the line of vision. Although many candidates recognized that they had to use the tangent ratio to calculate the unknown sides, they sometimes set up the ratio incorrectly or transposed incorrectly in solving for HK. A significant number did not attempt to find JK. Weaker candidates assumed triangle JKG was right-angled and used trigonometric ratios to find JK.

Part (b) was less popular than part (a) and few candidates had a completely correct response to this part of the question. Drawing the line $y = x$ posed great difficulty for candidates. Others could not successfully perform the reflection and only the very strong candidates described the translation correctly. There was a tendency to use informal language to describe the translation, and column matrices were rarely seen.

Solutions

(a) (ii) a) 19.2 metres b) 4.4 metres

(b) (i) A translation $\begin{pmatrix} \mathbf{2} \\ \mathbf{-7} \end{pmatrix}$

(ii) Coordinates of the image of P: (4, 4) (4, 6) (2, 4) and (2, 7)

Recommendations

Trigonometry should be approached through out-door activities whereby students can actually measure the heights of buildings using three-dimensional models to represent situations. Teachers should emphasize that angles of elevation and depression are measured in relation to a horizontal line.

Transformations can be performed using manipulatives and moving them on a plane. Although informal verbal descriptions are necessary these should be followed with formal descriptions using the appropriate language so that students appreciate the need for precision and accuracy in representing mathematical ideas.

Question 7

The question tested the candidates' ability to:

- interpret the graph of a straight line by
  - stating the y-intercept
  - determining its gradient
  - determining the coordinates of the mid-point
- use the equation to calculate the value of one ordinate given the other
-
- determine the coordinates of the point of intersection of two lines given their equations.

The question was attempted by 91 per cent of the candidates, 2 per cent of whom scored the maximum available mark. The mean mark was 1.7 out of 12.

Very few candidates presented a completely correct solution to this question and many omitted it entirely. Only a small proportion used the graph to read off the values of c and m. Many attempted to calculate the gradient using points on the line but the coordinates were often read incorrectly. Errors were due to poor interpretation of the scale or poor choice of points on the line. A few who chose to use the x and y intercepts reversed the coordinates.

There were better attempts at finding the mid point of AB as many candidates chose the correct formulae. Errors were mainly due to substituting incorrect coordinates.

Those who attempted parts (b) and (c) failed to link these parts with the first part hence appropriate strategies were rarely seen. In part (c) some candidates completed a table of values for the line $y = x - 2$ but few proceeded to check if any of these coordinates were on the line AB. An extremely small number of candidates recognized that they had to solve the two equations simultaneously to find the coordinates of the point of intersection.

Solutions

(a) **(i)** **7** **(ii)** $\mathbf{\frac{-7}{2}}$ **(iii)** **(1, 3.5)**

(b) $\boldsymbol{k} = \mathbf{14}$

(c) **(2, 0)**

Recommendations

The poor response to this question suggests that students are unfamiliar with writing down the equation of a linear function from its graph. Determining the intercepts and gradient from observation should be explored using a variety of graphs so that students can become proficient in analysing a graph and relating it to its equation.

Question 8

The question tested the candidates' ability to:

- perform computations with money
- use different combinations of bills to make up an exact amount
- solve problems involving bills

The question was attempted by 63 per cent of the candidates, 3 per cent of whom scored the maximum available mark. The mean mark was 3.43 out of 10.

Nearly all the candidates who attempted this question were able to find the total cost of all the stamps. In solving the remaining parts candidates did not always work within the constraints and assumed that there were no limits as to the choice of stamps to make up the amount. Hence many candidates obtained a total of \$25.70 using stamps that exceeded what was in the collection.

Few candidates recognized that in order to obtain a total of \$25.70, there must be partial-sums ending in 50 cents and 20 cents.

Only the very able candidates recognized that in order to maximize the number of stamps, they must use as many of the stamps that are smaller in value, as possible.

Solutions

(a) **\$52.20**

(b) **(i)** **5 x \$4.00, 1 x \$2.50, 1 x \$1.20, 2 x \$1.00**

(ii) **4 x \$4.00, 1 x \$2.50, 1 x \$1.20, 6 x \$1.00**

(c) **(i)** **17 stamps**

(ii) **5 x \$2.50, 6 x \$1.20, 6 x \$1.00**

Recommendations

This question called for high levels of quantitative reasoning in a quite familiar situation. Students' lack of familiarity with the reasoning processes required in solving this problem suggests that teachers need to pay closer attention to the development of thinking skills in their daily lessons. Teachers can make use of familiar situations in which decisions have to be made based on certain constraints and allow students to come up with a variety of options that satisfy all the constraints. Testing of solutions is also critical to this type of activity.

Question 9

The question tested the candidates' ability to:

- use the laws of indices to simplify algebraic expressions with integral and fractional indices
- interpret and make use of the functional notation $f(x)$, $f^{-1}(x)$ and $fg(x)$
- interpret a given scale

- draw the graph of a non-linear function given a table of values
- use a graph to
  - (i) estimate values
  - (ii) determine the gradient at a point

The question was attempted by 67 per cent of the candidates, less than 1 per cent of whom scored the maximum available marks. The mean mark was 5.3 out of 15.

This question was the most popular optional question and candidates demonstrated strengths in simplifying algebraic expressions involving integral indices, evaluating *f(2)*, interpreting the given scale, plotting points, drawing the temperature/time graph, and using the graph to estimate a value of one variable.

Candidates were generally unable to substitute the square root with a fractional index and this part of the question was poorly done.

Although the majority of candidates used the correct procedure for finding $f^{-1}(x)$, some made errors in transposing while others did not substitute $x = 0$, to find $f^{-1}(0)$. Candidates who solved for x from the equation $2x - 3 = 0$ had more success than those who attempted to find the inverse.

Candidates failed to recognize that $f^{-1} f(2) = 2$. They also did not link all parts of the question and proceeded to re-evaluate $f(2)$, and then apply the inverse. Some of the weaker candidates interpreted the inverse as the reciprocal of the function while others interpreted the composite $f^{-1} f(2)$ as a product of two functions.

Calculating the rate of cooling at $t = 30$ minutes was omitted by almost all the candidates. The few who attempted it merely read the graph instead of drawing a tangent at the point.

Solutions

(a) **(i)** x **(ii)** a^2b^4

(b) **(i)** **1** **(ii)** $\frac{3}{2}$ **(iii)** **2**

(c) **(ii)** **a)** **49° ± 1°** **b)** **0.66° per minute**

Recommendations

Teachers need to emphasize the meaning of functional notation, and give students opportunities to verbalise expressions and equations using this notation. They should understand that $f^{-1}(x)$ is not the same as the reciprocal of $f^{-1}(x)$.

The importance of acquiring proficiency in basic algebraic skills as a necessary pre-requisite for attempting this option must be emphasized.

Question 10

The question tested the candidates' ability to:

- solve a pair of equations in two variables when one is linear and one is non-linear

- determine whether a set of points satisfy all conditions described by a set of inequalities
- write a set of inequalities to define a given region
- use linear programming techniques to solve problems in two variables

The question was attempted by 24 per cent of the candidates, less than 1 per cent of whom scored the maximum available marks. The mean mark was 2.3 out of 15.

Almost all candidates attempted to eliminate one of the variables in solving the pair of equations but many could not do so successfully and ended up with an incorrect quadratic equation. Errors were more predominant when a fractional expression was used at the substitution stage. The majority used the formula to solve the quadratic and substituted their values to obtain the value of the second variable.

In part (b), when correct conclusions were made these were not supported by reasons. In describing the region defined by the set of inequalities, errors were made in stating the direction of the inequalities and in interpreting the scale so that the inequality $y \geq 2$ was written as $y \geq 4$.

Candidates generally had no problems in writing the profit equation, but in calculating the minimum profit they sometimes used non-integer values.

Solutions

(a) $x = (5, 7)$ **and (2, 19)**

(b) (i) a) (10, 5) is not in the region b) (6, 6) is in the region

(ii) $y \leq \frac{4}{5}x + 12, \quad y \leq 2x, \quad y \geq 2$

(iii) a) $3x + 5y$ **b) $13.00**

Recommendations

Teachers need to encourage students to devise methods for determining the most efficient strategy in solving simultaneous equations in a given situation.

Students need to be encouraged to test points to verify their solutions when identifying regions common to a system of inequations.

Question 11

The question tested the candidates' ability to:

- use the properties of parallel lines and theorems in circle geometry to calculate the measure of angles
- use trigonometric ratios to determine the size of an angle in a sector of a circle
- calculate the area of a triangle given two sides and the included angle
- calculate the area of a sector and a segment of a circle

- calculate the length of an arc of a circle

The question was attempted by 15 per cent of the candidates, 1 per cent of whom scored the maximum available mark. The mean mark was 2.0 out of 15.

Responses to part (a) was extremely poor. Candidates could not explicitly give reasons for their answers. Although some candidates were familiar with circle theorems, they could not use them to justify their answers.

In part (b), attempts to use trigonometrical ratios to solve for the angle in the sector were unsuccessful because candidates failed to divide the triangle into two right-angled triangles and solve for one half of angle AOB. Those who used the cosine rule also experienced difficulties by substituting incorrect values into the formula.

Similar problems were encountered in the other parts of the problem, where candidates chose the correct formula for calculating the area of the triangle, area of the segment and the length of the arc but made incorrect substitutions into these formulae. For example in using ½ *ab* sin *C* many candidates substituted ½ x 8.5 x 14.5 sin *AOB* instead of ½ x 8.5 x 8.5 sin *AOB.* A few calculated the length of the minor arc instead of the major arc.

Solutions

(a) **(i)** **64°** **(ii) 38°**

(b) **(i)** **117°** **(ii)** **32.2 cm^2** **(iii)** **41.5 cm^2** **(iv)** **36 cm**

Recommendations

The use of deductive reasoning in solving problems in geometry continues to be a challenge for students. Teachers need to address this need by exposing students to techniques using discourse and arguments before introducing them to formal proofs.

The application of formulae in solving problems in trigonometry also needs to be addressed through systematic approaches focusing on what information is given so as to avoid candidates merely plugging in values in formulae without analysing the particular situation.

Question 12

The question tested the candidates' ability to:

- represent the relative position of points on a diagram given bearings
- solve non-right angled triangles using the sine rule
- solve right-angled triangles using trigonometric ratios
- determine the bearing of one point relative to another

The question was attempted by 25 per cent of the candidates 2 per cent of whom scored the maximum available mark. The mean mark was 4.02 out of 15.

There were marked improvements this year in drawing of diagrams and many candidates got only this part of the problem correct. A few still had problems in representing the bearings while others labelled the points incorrectly. Some candidates did not realise that the journey started at R, and positioned R at the end of the first line segment drawn. Once a correct diagram was obtained, locating the position of X was usually correct.

Calculating angle RST proved to be difficult for the majority of candidates. Therefore correct answers to the rest of the question were hardly seen since this incorrect value was substituted in their further calculations. The use of the sine rule was recognised but not always applied correctly. The bearing of R from T proved to be very challenging for candidates who either omitted this part or measured the bearing starting clockwise instead of anti-clockwise from the north direction.

Only a few candidates managed to get to the final part of the question but once again these had difficulties with calculating the angle required to solve the problem.

Those students who used scale drawings produced excellent diagrams but when asked to calculate they resorted to the use of measurements instead.

Solutions

(b) **(i)** **101°** **(ii)** **47.1°** **(iii)** **260.1°**

(c) **TX = 73.88 km**

Recommendations

Teachers need to strengthen student skills in representing three-dimensional drawings on a plane through the use of models. Calculating angles can be made easier if students draw all the North/South lines at each vertex of the triangle. This would enable them to see which angles are equal and so facilitate their calculations.

Question 13

The question tested the candidates' ability to:

- show the relative position of points on a diagram
- add vectors using the triangle law
- use a vector method to prove collinearity
- calculate the length of a vector
- use vectors to represent and solve problems in geometry

The question was attempted by 13 per cent of the candidates less than 1 per cent of whom scored the maximum available mark. The mean mark was 2.15 out of 15.

In completing the diagram most candidates were able to locate the position of M, the midpoint of a line and the approximate position of P. However, the point N was hardly seen.

Writing an expression for the vector, AB did not pose problems for the majority of candidates. However, incorrect responses were common for those vectors in which routes had to be expressed in fractional parts of a line segment.

Proving collinearity continues to be a challenge for even the more able candidates. While some selected the correct line segments they had difficulties proving the vectors were parallel mainly because of algebraic and arithmetical errors in simplifying terms. Making a concluding remark as to why the vectors were parallel was rarely seen.

The few candidates who attempted part (d) knew how to find the length of a vector but incorrect routes or values substituted often resulted in incorrect responses.

Solutions

(b) **(ii)** $\overrightarrow{AB} = -\boldsymbol{a} + \boldsymbol{b}$ $\overrightarrow{PA} = \frac{1}{3}\boldsymbol{a}$ $\overrightarrow{PM} = -\frac{1}{6}\boldsymbol{a} + \frac{1}{2}\boldsymbol{b}$

(d) $\sqrt{20}$ **or 4.47**

Recommendations

Prior to the study of vectors, teachers need to present situations to students whereby they can describe the relative position of one point relative to another along a straight line. Comparing the lengths of line segments using fractions are necessary pre-requisites for the teaching of this topic. Concepts of parallelism and collinearity also need to be strengthened.

Question 14

The question tested the candidates' ability to:

- evaluate the determinant of a 2 x 2 matrix
- perform addition, subtraction and multiplication of matrices
- compute the square of a 2 x 2 matrix
- obtain the inverse of a non-singular matrix
- determine the matrices associated with enlargement and rotation
- determine the 2 x 2 matrix for a single transformation which is equivalent to the combination of two successive transformations
- use matrices to solve simple problems in geometry.

The question was attempted by 27 per cent of the candidates, less than 1 per cent of whom scored the maximum available mark. The mean mark was 2.88 out of 15.

Responses to this question were barely satisfactory with candidates demonstrating lack of knowledge of basic concepts and principles in relation to matrices. For example, almost all of the candidates could not evaluate the square of a matrix. They merely squared each element. Those who set up the matrices correctly for squaring made errors in multiplication. Addition of matrices appeared to be the only skill well understood.

In part (b), even the stronger candidates were stumped and did not recognise that an inverse was required.

In determining the value of k, the scale factor of the enlargement, candidates failed to recognise that a ratio was required and some subtracted the coordinates instead.

Although candidates were familiar with the 2 × 2 matrices for rotation and enlargement, they often reversed the order in obtaining the matrix for the combined transformation.

Solutions

(a) $\begin{pmatrix} \mathbf{8} & \mathbf{-1} \\ \mathbf{-2} & \mathbf{8} \end{pmatrix}$

(b) $\begin{pmatrix} \mathbf{3} & \mathbf{-2} \\ \mathbf{-1} & \mathbf{1} \end{pmatrix}$

(c) (i) a) $\boldsymbol{k = 1.5}$ **b)** $\boldsymbol{E' = (3, 10.5)}$ $\boldsymbol{F'}$ **(12, 6)**

(ii) a) $\begin{pmatrix} \mathbf{0} & \mathbf{1} \\ \mathbf{-1} & \mathbf{0} \end{pmatrix}$

b) **D” (18, –7.5)** **E” (10.5, –3)** **F” (6, –12)**

c) $\begin{pmatrix} \mathbf{0} & \mathbf{\frac{3}{2}} \\ \mathbf{-\frac{3}{2}} & \mathbf{0} \end{pmatrix}$

Recommendations

In the treatment of matrices, teachers need to emphasize that such objects cannot always be treated in the same way as other systems. The meaning of concepts in Number Theory such as inverse and identity element should be reviewed prior to the introduction of similar concepts in matrices to enable students to make the necessary connections.

Similar practices should be adopted in the treatment of matrix transformations whereby links between geometric and matrix transformations should be established.

DETAILED COMMENTS

Basic Proficiency

The Basic Proficiency examination is designed to provide the average citizen with a working knowledge of the subject area. The range of topics tested at the Basic level is narrower than at the General Proficiency level.

This year approximately 32 per cent of the candidates achieved Grades I to III.

Paper 01 – Multiple Choice

Paper 01 consisted of 60 multiple choice items. This year, 53 candidates earned the maximum available mark. Approximately 43 per cent of the candidates scored at least half the total marks for this paper.

Paper 02 – Essay Choice

Paper 02 consisted of 10 compulsory questions. Each question was worth 10 marks. The highest score earned was 96 out of 100. This was earned by one candidate. Thirteen per cent of the candidates earned at least half the total marks on this paper.

<u>Question 1</u>

This question tested candidates' ability to:

- perform basic operations with rational numbers, namely whole numbers and fractions
- compare two quantities in a given ratio
- express one quality as a percentage of another

The question was attempted by 97 per cent of the candidates, 9 per cent of whom scored the maximum available mark. The mean score was 3.93 out of 10.

Many candidates exhibited very good knowledge of the order of operations (BODMAS). In part (b) they knew that all quantities had to be added in order to get the total. Percentage was fairly done in part (b) (iii).

Candidates subtracted numerators and denominations in order to solve Part (a). For example:

$$\left(\frac{5}{8} \div 1\frac{1}{4}\right) = \frac{5-1}{8-4} = \frac{4}{4} = 1\,.$$

Many candidates found difficulty in converting mixed numbers to improper fractions and inverting in order to divide by a fraction. For example:

$$1\frac{1}{3} - \left(\frac{5}{8} \div 1\frac{1}{4}\right) \text{ was done as } 1\frac{1}{3} - \frac{5}{8} \times \frac{4}{4} \quad \text{or} \quad \frac{4}{3} - \frac{5}{8} \times \frac{5}{4} \quad \text{with LCM of 32.}$$

They also found difficulty with: $\frac{4}{3} - \frac{1}{2} = \frac{8-3}{6}$. They did $\frac{4}{3} - \frac{1}{2} = 1\,\frac{2-3}{6} = 1\,\frac{5}{6}$ or $1\,\frac{1}{6}$.

They had difficulty calculating ratio. The majority could have calculated the quantity for one proportional part – that is, $6\,\frac{5}{13} = 5$ in part (b) (i) but did not multiply by 2 to get the correct answer.

<u>Solutions</u>

(a) $\mathbf{\frac{5}{6}}$

(b) **(i)** **10** **(ii)** **200** **(ii)** **88%**

<u>Recommendations</u>

Teachers should ensure that students practice the conversion of fractions, worded problems and complex ratios.

<u>Question 2</u>

The question tested the candidates' ability to:

- substitute numbers for algebraic symbols in simple algebraic expressions
- perform operations involving integers
- perform basic operations with algebraic expressions
- solve a simple linear inequality in one unknown

The question was attempted by 97 per cent of the candidates, 4 per cent of whom scored the maximum available mark. The mean score was 4.37 out of 10.

Generally, candidates performed satisfactorily. However, some candidates experienced difficulty in some parts of the question.

In part (a) (i), some candidates were unable to substitute a number for an algebraic symbol in expression $2b + c$. For example, they incorrectly substituted $b = 5$ and $c = -2$ by writing $25 + (-2) = 23$ instead of $2 \times 5 + (-2) = 10 - 2 = 8$.

In part (a) (ii), many candidates were unable to compute $2\,(-2)^2$. Instead they wrote:

$$\frac{-2^2}{4} = \frac{-4}{4} = 1.$$

In part (b), candidates multiplied the entire expression by 3 instead of the terms in the brackets. Thus $3\,(4y + 1) - 5y$ was equated to $12y + 3 - 15y$. They experienced difficulty collecting the like terms and simplifying.

In part (c), candidates were unable to transpose correctly thus not obtaining the correct solution. Many candidates did not change the inequality sign when dividing by a negative.

<u>Solutions</u>

(a) **(i)** **8** **(ii)** **1**

(b) $\mathbf{7}\boldsymbol{y}\mathbf{ + 3}$

(c) $\boldsymbol{x \leq 3}$ **or** $\boldsymbol{3 \geq x}$

<u>Recommendations</u>

More emphasis should be placed on operations with directed numbers and substituting numerals in algebraic expressions.

Teachers need to revisit the simplification of algebraic expressions at the first form and second form level.

<u>Question 3</u>

The question tested the candidates' ability to:

- solve simple problems involving payments by instalments as in the case of hire purchase
- use the simple interest formula to calculate simple interest for a specific time period
- determine the amount after simple interest has been added in a ½ yearly arrangement
- solve problems involving fractions, decimals and percentages

The question was attempted by 97 per cent of the candidates, 4 per cent of whom scored the maximum available mark. The mean score was out of 10.

In part (a) (i), most candidates demonstrated an understanding of the monthly payments of the hire purchase. However, they experienced difficulty adding $100 to get the total hire purchase cost.

In part (a) (ii), most candidates understood the concept of EXTRA amount paid although the difference involved incorrect numbers or showed a reverse, e.g. $510 – $540 instead of $640 – $510 or $540 – $510 instead of $640 – $510. In part (b) (i), most candidates were able to obtain the total amount deposited for six months. In part (b) (ii), most candidates experienced problems with the use of the formula as it related to calculating ½ yearly simple interest. Most wrote $\frac{4 \text{ x } 240 \text{ x } 6}{100}$ rather than $\frac{4 \text{ x } 240 \text{ x } 0.5}{100}$.

However, for the last part, most candidates were able to sum the interest obtained to the principal in order to find the amount.

<u>Solutions</u>

(a) **(i)** **$640** **(ii)** **$30**

(b) **(i)** **$240** **(ii)** **$244.80**

Recommendations

(i) Greater emphasis should be placed on calculating half yearly simple interest.

(ii) Students should be exposed to more consumer arithmetic problems involving percentages, decimals and fractions.

Question 4

The question tested the candidates' ability to:

- solve simultaneous linear equations in two unknown algebraically
- use symbols to represent numbers, operations, variables and relations
- translate several phrases into algebraic symbols

The question was attempted by 93 per cent of the candidates, 2 per cent of whom scored the maximum available mark. The mean score was 1.97 out of 10.

In part (a), candidates were able to arrive at a method for solving the equations simultaneously (most used the elimination method). However they were unable to use the method correctly.

In part (b), most candidates were able to

a) express Sue's age (p + 3); and

b) arrive at the sum of the ages of Dwight, Sue and Joan

In part (a), candidates were unable to do the operation to arrive at the correct answers. In part (b) they were not able to see Joan's age as 2 (p + 3), or twice Sue's age and hence were not successful in getting the sum of the ages correctly.

Solutions

(a) $x = 4$; $y = 3$

(b) **(i)** $p + 3$ **(ii)** $2(p + 3)$

(iii) $p + (p + 3) + 2(p + 3)$ **(iv)** $p = 7$ **years**

Recommendations

(i) More practice using different types of examples.

(ii) Teachers should use real life situations to teach algebra.

Question 5

The question tested candidates' ability to:

- solve problems involving salaries and wages

- solve problems involving measures and money including exchange rates

The question was attempted by 95 per cent of the candidates, 19 per cent of whom scored the maximum available mark. The mean score was 5.54 out of 10.

Many candidates were able to calculate the overtime wage yet unable to determine the overtime rate of pay. They were able to find the quantity of US currency and to convert to EC currency. However, some wrongly converted to BDS currency. The 2 per cent bank charge on the EC currency proved difficult to calculate with some candidates subtracting 0.02 from the total amount of EC dollars. In converting the US $ to EC $ students did the calculation on the four individual quantities rather than on their sum. A similar procedure was repeated in finding the 2 per cent charge.

Solutions

(a) **(i)** **$37.50** **(ii)** **$7.50**

(b) **(i)** **$610** **(ii)** **$1 614.06**

Recommendations

- Newspaper clippings on percentages and other practical examples should be used to arouse candidates' interest in this topic.
- More practice is needed in problems involving salaries and wages.

Question 6

The question tested candidates' ability to:

- use instruments to draw and measure angles and line segments and to construct triangles
- use Pythagoras' theorem to solve problems
- use simple trigonometrical ratios to solve right-angled triangles and problems based on measures in the physical world.

The question was attempted by 83 per cent of the candidates, 1 per cent of whom scored the maximum available mark. The mean score was 2.86 out of 10.

Candidates showed greater proficiency at measuring line segments rather than constructing line segments. Too many candidates did not attempt to construct the 90° angle. While candidates recognised the need to use trigonometry to solve for an angle, they found difficulty determining which ratio to use and how to get the required angle using the inverse trigonometric function. Incorrect application of Pythagoras' theorem was often seen. Some candidates found the area of the triangle, instead of the height. Few students attempted to present their answer to 2 significant figures.

Solutions

(a) **(ii)** **8.6 cm**

(b) **(i)** **2.7 m** **(ii)** **42°**

Recommendations

- More practice in constructing lines, angles and triangles.
- The concept of significant figures should be reinforced.
- More problems on practical applications of trigonometry.

Question 7

The question tested the candidates' ability to:

- determine the distance on a map, given the scale
- convert units of length within the SI system
- calculate the area of two triangles within a quadrilateral
- determine the length of one side of a quadrilateral given the perimeter of the quadrilateral and the length of three sides.

The question was attempted by 79 per cent of the candidates, less than 1 per cent of whom scored the maximum available mark. The mean score was 1.9 out of 10.

In part (a), performance was generally poor. Very few candidates were able to convert 20 km to centimetres and then obtain the distance on the map. Instead, candidates multiplied 500 000 by 20 km. In part (b) (i) most candidates were able to identify the right-angled triangle within the quadrilateral and so they used the formula ½ base × height to determine the area of Δ KJM. However, in part (b) (ii), most candidates could not obtain the area of Δ KLM. Some candidates used Pythagoras' theorem while others used different trigonomtric ratios instead of just subtracting the area of Δ KLM from the area of the quadrilateral. In part (b) (iii), most candidates were able to obtain the length of the three sides and then subtracted the results from the perimeter of the quadrilateral.

Solutions

(a) **4 cms**

(b) **(i)** **13.5 cm^2** **(ii)** **12.76 cms** **(iii)** **7.5 cms**

Recommendations

(i) More emphasis should be placed on map work.

(ii) Candidates should be exposed to more practical work on finding the area and perimeter of complex or composite shapes.

Question 8

The question tested candidates' ability to:

- define translation in plane as vectors, written as column matrices and recognise them when specified
- state the relationship between an object and its image when it undergoes translation in a plane

- state the relationship between an object and its image in a plane under a given enlargement in that plane

The question was attempted by 75 per cent of the candidates. No candidate scored the maximum available mark. The mean score was 1.1 out of 10.

Candidates were able to correctly apply the formula for area of triangle. In part (a), some candidates wrote the coordinates by stating the *y* before the *x*.

They could not have identify the type of transformation in (a) (ii). Instead, they drew a replica of Δ PQR in different quadrants. Most candidates were unable to identify the centre of enlargement. Even when the centre of the enlargement was identified, it was not correctly used to do the required enlargement.

Solutions

(a) (i) P (6, – 4) Q (10, – 4) R (6, –2)

(ii) Translation = $\begin{pmatrix} -4 \\ 8 \end{pmatrix}$ **or 4 left and 8 up**

(b) (i) C = (0, 2) (ii) P" (4, 6) Q" (12, 6) R" (4, 10)

(c) 4 cm^2 $\Rightarrow$ triangle PQR 16 cm^2 $\Rightarrow$ triangle P"Q"R"

Recommendations

(i) More practice involving plotting points and reading coordinates.

(ii) Practice drawing objects and images using various types of transformations.

Question 9

The question tested candidates' ability to:

- recognise the gradient of a line as the ratio of the vertical rise to the horizontal shift
- find by calculation the gradient of graphs of linear functions
- determine the equation of a line given the graph of the line
- use the functional notation *f(x)* for given domains
- draw, read and interpret graphs of
  - (a) $f(x) = a + bx + cx^2$
  - (b) simple non-linear functions given a table of values.

The question was attempted by 67 per cent of the candidates, less than 1 per cent of whom scored the maximum available mark. The mean score was 2.10 out of 10.

Generally candidates performed unsatisfactorily. They experienced difficulty in all parts of the question. In part (a) (i), many candidates were unable to calculate or estimate the gradient of the line ℓ. They did not use the formula $m = \dfrac{y_2 - y_1}{x_2 - x_1}$.

In part (a) (ii), most candidates were unable to state the equation of the line correctly, while in part (b) (i), most candidates were unable to calculate the correct values in the table and hence did not draw the desired quadratic curve.

In part (b) (ii), most candidates only plotted the three points which were given in the table. Many candidates did not draw a smooth curve, instead they connected the points with straight lines. In part (b) (iii), most candidates were unable to state the points of intersection (1, 2) and (–3, –6). They did not extend the line ℓ to meet the curve *f(x)*.

<u>Solutions</u>

(a) **(i)** **2** **(ii)** $\boldsymbol{y = 2x}$

(b) **(i)**

x	**–3**	**–2**	**–1**	**0**	**1**	**2**
fx	**–6**	**–1**	**2**	**3**	**2**	**–1**

(c) **(1, 2) and (–3, –6)**

<u>Recommendations</u>

- Students need more practice in curve sketching and how to differentiate between graphs of linear and non-linear functions.

- Teachers should do more problems in estimating and calculating gradients from given linear functions.

<u>Question 10</u>

The question tested the candidates’ ability to:

- solve problems involving averages

- draw and use pie charts

- determine experimental and theoretical probabilities of simple events

The question was attempted by 83 per cent of the candidates, 3 per cent of whom scored the maximum available mark. The mean score was 2.29 out of 10.

In part (a), candidates were able to accurately compute the total runs scored. In part (b), most candidates were able to arrive at the correct solution to (i) and (ii).

However, candidates were unable to arrive at the correct solution for runs scored in the fourth match and had difficulty expressing probability as a proper fraction. In addition, candidates did not subtract from 120 the sum of children who swam and who played soccer and did not express the answer as a proper fraction.

Solutions

(a)	**(i)**	**165**	**(ii)**	**79**		
(b)	**(i)**	**50**	**(ii)**	**138°**	**(iii)**	$\mathbf{\frac{24}{120}}$ **or** $\mathbf{\frac{1}{5}}$ **or 0.2**

Recommendations

(i) Students need to be exposed to more real life applications in order to reinforce the concept of average.

(ii) Teachers need to reinforce the probability (1).

CARIBBEAN EXAMINATIONS COUNCIL

REPORT ON CANDIDATES' WORK IN THE
CARIBBEAN SECONDARY EDUCATION CERTIFICATE
JANUARY 2009

MATHEMATICS

MATHEMATICS

GENERAL PROFICIENCY EXAMINATIONS

JANUARY 2009

GENERAL COMMENTS

The General Proficiency Mathematics examination is offered in January and May/June each year. The Basic Proficiency examination is offered in May/June only.

There was a candidate entry of approximately 15 300 in January 2009. Forty-nine per cent of the candidates achieved Grades I-III. The mean score for the examination was 84.54 out of 180 marks.

DETAILED COMMENTS

Paper 01- Multiple Choice

Paper 01 consists of 60 multiple choice items. This year, twenty-eight candidates each earned the maximum available mark of 60. Sixty-eight per cent of the candidates scored 30 marks or more.

Paper 02 - Essay

Paper 02 consisted of two sections. Section I comprised eight compulsory questions totalling 90 marks. Section II comprised six optional questions: two each from Relation, Functions and Graphs; Trigonometry and Geometry and Vectors and Matrices. Candidates were required to answer any two questions from this section. Each question in this section was worth 15 marks.

This year, four candidates each earned the maximum available mark of 120 on Paper 02. There were five candidates who each scored 119 marks. Approximately twenty-eight per cent of the candidates earned at least half of the maximum mark on this paper.

Compulsory Section

Question 1

This question tested candidates' ability to:

- perform basic operations with fractions
- solve problems related to currency conversion
- calculate compound interest over two periods

The question was attempted by 99.7 per cent of the candidates, 7 per cent of whom earned the maximum available mark. The mean mark was 6.66 out of 11.

Candidates demonstrated good proficiency in inverting and multiplying when dividing one improper fraction by another and calculating the exchange rate from BDS$ to EC$.

Candidates showed weak performance in finding the lowest common multiple, using it correctly to subtract fractions and using the algorithm for subtracting fractions. Some incorrect procedures were:

$$\frac{7}{3}-\frac{5}{6}=\frac{2}{3} \quad \text{and} \quad \frac{(4+7) \quad - \quad (2+5)}{12}$$

Candidates also experienced difficulty calculating the compound interest for 2 years.

Solutions:

(a) $\frac{5}{2}$

(b) (i) EC $1.35 **(ii) BDS $320.00**

(c) $27 933.60

<u>Recommendations</u>

The use of the calculator is a necessary skill in learning mathematics. Teachers are encouraged to teach students how to use the calculator, as well as the requisite skills necessary for determining the accuracy of the calculation such as estimation and rounding.

<u>Question 2</u>

This question tested candidates' ability to:

- write the difference between two algebraic fractions as a single fraction
- evaluate the result of a binary operation on two integers
- factorize an expression of the form ax + bx + ay + by
- use algebraic symbols to represent information
- use linear equations to solve a worded problem

The question was attempted by 99 per cent of the candidates, 9 per cent of whom earned the maximum available mark. The mean mark was 6.07 out of 12.

Candidates demonstrated good proficiency in choosing the appropriate common denominator in part (a) and recognizing that $5 * 2 = 5^2 - 2$ in the binary operation. Candidates were also able to express most of the statements as algebraic expressions.

However, some of the weaker candidates had difficulty following the algorithm for subtracting fractions. Errors were also seen in completing the factorization, writing the expression for '3 cm shorter than the first piece' and using the trial and error strategy to solve for the value of x.

Solutions:

(a) $\frac{m}{3n}$

(b) 23

(c) (3 + x) (x – 2y)

(d) (i) x; x – 3; 2x **(ii) x + x – 3 + 2x** **(iii) x = 6**

<u>Recommendations</u>

In the teaching of fractions, teachers need to focus on the algorithms for adding and subtracting common fractions. Emphasis should also be placed on translating verbal statements into mathematical symbols and solving simple equations in one unknown by using approaches other than trial and error.

<u>Question 3</u>

This question tested candidates' ability to:

- construct and use Venn diagrams
- solve problems involving the use of Venn diagrams
- use Pythagoras' theorem to find the side of a right-angled triangle

The question was attempted by 99 per cent of the candidates, 11 per cent of whom earned the maximum available mark. The mean mark was 6.38 out of 12.

Candidates demonstrated strengths in reproducing the Venn diagram, correctly interpreting the worded information, illustrating it on the Venn diagram and finding the total of the subsets.

In part (b), many candidates correctly chose Pythagoras' theorem to find the length of the third side of a right angled triangle but had difficulty applying the theorem correctly. Choosing the appropriate trigonometric ratio for finding the length MK also proved challenging.

Solutions:

(a) (iii) x = 12

(b) (i) MK = 5m **(ii) JK = 7m**

Recommendations

Students need more practice in the practical application of Pythagoras theorem. Exposure to the development of the theorem by determining the areas of the squares whose lengths are the sides of a right-angled triangle may be beneficial.

Question 4

This question tested candidates' ability to:

- write the coordinates of two points located on the x and y axes respectively
- determine the gradient of a line segment
- find the equation of a line given its graphical representation
- find the equation of a line which passes through a given point and is perpendicular to a given line.

The question was attempted by 94 per cent of the candidates, 5 per cent of whom earned the maximum available mark. The mean mark was 3.30 out of 11.

Although most candidates were able to write the coordinates of P, the point on the x – axis, candidates had difficulty writing the coordinates of Q, the point of the line on the y-axis. However, many of the candidates were able to calculate the gradient of the line segment; use their value of m and the coordinates of a point to find the value of c in the equation y = mx + c and recognize that for two perpendicular lines the product of their gradients is -1.

Candidates had difficulty recognizing that for a given value of x, the corresponding value of y could be found by substituting into the equation of the line.

Solutions:

(a) P (0, 3) Q (-2, 0)

(b) (i) $m = \frac{3}{2}$ **(ii)** $y = \frac{3}{2}x + 3$

(c) t = -9

(d) $y = \frac{-2}{3}x + 6$

Recommendations

Students should be encouraged to use graphical representations of linear equations to answer questions related to gradients, intercepts and equations of lines.

Question 5

This question tested candidates' ability to:

- calculate the volume of a right prism given the length of its edges
- calculate the surface area of the right prism
- solve worded problems based on the volume and dimensions of prisms

The question was attempted by 88 per cent of the candidates, 8 per cent of whom earned the maximum mark. The mean mark was 3.75 out of 10.

Candidates demonstrated proficiency in calculating the volume of the prism, the area of at least one face of the prism and dividing by 6 to find the volume of a small box.

However, many candidates did not recognize that the net of the prism had six faces and also had difficulty distinguishing between the volume and the area. In some cases, although the calculations were done accurately, the appropriate units of volume and area were not used.

Solutions:

(a) $7\,200\text{ cm}^3$

(b) $2\,776\text{ cm}^2$

(c) (i) $1\,200\text{ cm}^3$

(ii) $60\text{ cm}^2 = 6\text{cm} \times 10\text{cm}$ **or** $15\text{cm} \times 4\text{cm}$ **or** $20\text{cm} \times 3\text{cm}$

Recommendations

In teaching the concept of solids, teachers are urged to ensure that the students are familiar with the properties of the solids in relation to the nets of the solids, the dimensions, number of vertices, faces and edges.

Question 6

This question tested candidates' ability to:

- use a ruler and a pair of compasses to construct a rectangle
- identify and describe transformations given object and image
- find the centre of enlargement given the object and image
- determine the scale factor of an enlargement

The question was attempted by 92 per cent of the candidates, 2 per cent of whom earned the maximum available mark. The mean mark was 4.75 out of 12.

The majority of the candidates were able to construct or draw angles of 90^0, measure the sides of the rectangle accurately and complete the rectangle.

In part (b), candidates displayed weakness in determining the vector representing the translation. Many candidates also had difficulty locating the centre of enlargement and determining the scale factor of the enlargement.

Solutions:

(b) (i) $\begin{pmatrix} 4 \\ 4 \end{pmatrix}$

(ii) b) G (-5,0); c) scale factor of enlargement is 2

Recommendations

Teachers are encouraged to include practical work and authentic activities when teaching transformations. In addition, transformational geometry should be taught with and without graph paper to ensure that the basic concepts and procedures are understood.

Question 7

This question tested candidates' ability to:

- complete the cumulative frequency column from a grouped frequency distribution
- draw a cumulative frequency graph
- use the cumulative frequency curve to solve problems
- compute simple probability

The question was attempted by 93 per cent of the candidates, 2 per cent of whom earned the maximum available mark. The mean mark was 5.81 out of 12.

Candidates displayed proficiency in completing the cumulative frequency column and using given scales to draw the cumulative frequency curve.

However, some candidates did not label the axes correctly. In addition, a number of candidates connected the points plotted with straight lines instead of drawing a smooth curve to represent the ogive. Candidates also had difficulty determining the number who passed when the pass mark was given.

Solutions:

(a)

Mark	**Cumulative Frequency**
31 – 40	**30**
41 – 50	**46**
51 – 60	**58**
61 – 70	**66**

(b)(ii) Assumption: No student obtained a mark less than 0.5 or 1

(c) 30 students

(d) $\frac{16}{70}$

Recommendations

Students should be given more practice in the drawing of non-linear graphs, in order to perfect the skill of drawing the ogive and other curves. Emphasis should also be placed on interpreting the graphs to obtain useful information.

Question 8

This question tested candidates' ability to:

- draw the next diagram in the sequence of diagrams
- recognize number patterns
- calculate unknown terms in a number sequence
- state the formula for the n^{th} term in a sequence

The question was attempted by 94 per cent of the candidates, 7 per cent of whom earned the maximum available mark. The mean mark was 5.91 out of 10.

Candidates demonstrated strength in constructing the fourth diagram in the sequence and determining the missing number of dots and line segments by following the pattern in the table.

Candidates displayed weakness in determining the number of dots and lines segments for a pattern not represented in the table and stating the rule connecting the number of line segments to the number of dots.

Solutions:

	Dots	**Pattern**	**Line Segments**
(b) (i)	**62**	**$2 \times 62 - 4$**	**12**
(ii)	**92**	**$2 \times 92 - 4$**	**180**

(c) (i) 20 (ii) 42 (iii) l = 2d - 4

Recommendations

Students would benefit from exposure to problem solving involving the use of number patterns. These could be presented through projects where students work in groups or individually, exploring interesting real life situations which require the use of number patterns in making generalizations.

Question 9

This question tested candidates' ability to:

- change the subject of the formula
- complete the square
- find roots of a quadratic equation
- determine the domain interval over which a quadratic function is less
- than or equal to zero
- determine the minimum value of a quadratic function and state the
- value of x for which this minimum occurs

The question was attempted by 28 per cent of the candidates, 3 per cent of whom earned the maximum mark. The mean mark was 4.50 out of 15.

Candidates demonstrated strength in determining the roots of the quadratic equation and stating the minimum value of the quadratic function from the completed square. However, some candidates were unable to manipulate the equation to change the subject and complete the square. Very few candidates correctly stated the interval for which the function was negative or zero.

Solutions:

(a) $t = \frac{gp^2}{4} - r$

(b) (i) $\mathbf{f(x) = 2(x-1)^2 - 15}$ **(ii) x = -1.74, 3.74**

(iii) f(x) < 0 for -1.74 < x < 3.74]

(iv) Minimum value of f(x) is -15

(v) Minimum occurs when x = 1

Recommendations

The concept of changing the subject of a formula should be taught by placing emphasis on the inverse operation when transposing a quantity from one side of the equation to the other. Examples should include formulae with squares and square roots. Teachers must pay particular attention to the strategies used to teach the concept of completing the square. In addition, students should fully understand the importance of each component of the resulting expression.

Question 10

This question tested candidates' ability to:

- use function notation to determine the value of f(x) for a given value of x
- find the inverse of a function
- use the algebra of composite functions to prove an identity
- complete a distance/time graph from given information
- use a distance/time graph to solve problems

The question was attempted by 63 per cent of the candidates, 3 per cent of whom earned the maximum mark. The mean mark was 5.67 out of 15.

In part (a), candidates were able to calculate the value of the function and prove that the two given composite functions were equal. However, many were unable to determine the inverse of the function. While the majority of the candidates were able to determine values from the graph, errors were made in calculating average speed and the time taken to complete the journey. This was primarily as a result of candidates not using consistent units in the calculations.

Solutions:

(a) (i) $f(6) = 3$ (ii) $f^{-1}(x) = x + 3$

(b) (i) 20 minutes (ii) 150 km/h (iii) 50 minutes

Recommendations

Students should be exposed to a wide range of problems involving the use of travel graphs. Further, the importance of using the appropriate units should be emphasized.

Question 11

This question tested candidates' ability to:

- complete a table of values for $y = \frac{1}{2} \tan x$ for a given domain
- draw the graph of $y = \frac{1}{2} \tan x$ for a given domain
- use theorems in circle geometry to calculate the measure of angles

The question was attempted by 13 per cent of the candidates, 2 per cent of whom earned the maximum mark. The mean mark was 4.23 out of 15.

The performance in part (a) of this question was fair. The majority of the candidates were able to plot the given points on the recommended scale and determine the coordinates of the missing points. However, many of the candidates could not determine the estimate as required.

Generally, the candidates were unable to determine the measure of the unknown angles. In some cases, incorrect assumptions were made about the properties of the polygons in the circle and hence the answers and accompanying reasons were incorrect.

Solutions:

(a) (i)

	x	**y**
	20^0	**0.18**
	40^0	**0.42**

(iii) For $y = 0.7$, $x = 55^0$

(b) (i) 90^0 (ii)132^0 (iii) 66^0 (iv) 114^0

Recommendations

Teachers should provide opportunities for more work on constructing and interpreting scales on graphs. Greater attention should be given to the interpretation of graphs than to the mechanics of drawing them. Theorems related to the geometry of the circle should be verified by the accurate construction of diagrams.

Question 12

This question tested candidates' ability to:

- find the length of the side of a triangle using the cosine formula
- find the measure of the size of an angle in a triangle using the sine formula
- find the area of a triangle given two sides and the included angle
- show lines of latitude and longitude on the circle representing the earth
- calculate the distance between two places on the earth measured along a circle of longitude
- determine the radius of a circle of latitude

The question was attempted by 21 per cent of the candidates, 2 per cent of whom earned the maximum mark. The mean mark was 2.89 out of 15.

The performance on the question was generally weak. In part (a), the majority of candidates correctly applied the sine rule to determine the measure of angle UVW. However, very few candidates correctly calculated the length UW or the area of triangle TUW. In part (b), candidates were able to correctly label the circles of latitude and longitude on the diagram but experienced difficulty calculating the required radius and distance.

Solutions:

(a) **(i)** **9.17 m** **(ii)** **32.4^0** **(iii)** **34.6 m^2**

(b) **(ii)a)** **5560 km** **b)** **5990 km**

Recommendations

Students should be exposed to a range of exercises involving the trigonometric ratios, where they will be required to select the appropriate ratio: sine, cosine or tangent. Practical and authentic activities should be incorporated, where possible, as well as accurate scale drawings to reinforce concepts.

Question 13

This question tested candidates' ability to:

- write coordinates of points as position vectors
- write the displacement of one point from another as a column vector
- state the condition for two vectors to be parallel
- determine the magnitude of a vector
- determine the values of two variables given two equal vectors
- use a vector method to establish that a given quadrilateral is a parallelogram

The question was attempted by 20 per cent of the candidates, 2 per cent of whom earned the maximum available mark. The mean mark was 3.78 out of 15.

Candidates demonstrated strengths in writing the co-ordinates of the given points as position vectors and adding vectors with numerical values. However, the majority of the candidates experienced challenges
routing vectors, proving that the vectors QR and OP are parallel, finding the magnitude of a column vector and proving that the given shape was a parallelogram.

Solutions:

(a) (i) $\begin{pmatrix} 3 \\ 2 \end{pmatrix}$ **(ii)** $\begin{pmatrix} -1 \\ 3 \end{pmatrix}$

(b) (i) $\begin{pmatrix} 9 \\ 6 \end{pmatrix}$ **(iii)** $\sqrt{74}$

(c) (i) $\begin{pmatrix} a+1 \\ b-3 \end{pmatrix}$ **(ii)** **a = 2; b = 5**

Recommendations

The concept of vectors should be taught using concrete examples. Students should be challenged to express a given route using a variety of paths and vectors.

Question 14

This question tested candidates' ability to:

- multiply 2 × 2 matrices
- multiply a matrix by a scalar
- calculate the determinant of a 2 × 2 matrix
- find the inverse of a non-singular 2 × 2 matrix
- use matrices to transform geometrical shapes
- use the matrix method to solve a system of linear equations

The question was attempted by 32 per cent of the candidates, 1 per cent of whom earned the maximum available mark. The mean mark was 5.13 out of 15.

In this question, candidates demonstrated competence in multiplying a matrix by a scalar, writing a pair of simultaneous equations in matrix form and inverting a given 2 × 2 matrix. The areas of weak performance included multiplying two matrices and stating the effect of a matrix transformation on an object.

Solutions:

(a) $\begin{pmatrix} 15 & 39 \\ 12 & 33 \end{pmatrix}$

(b) (i) **V produces an enlargement, scale factor 2, centre of enlargement (0,0)**

(ii) $\begin{pmatrix} -2 & 0 \\ 0 & 2 \end{pmatrix}$

(iii) **A' (-2,4)** **B' (-2,2)** **C' (-4,2)**

(c) **(i)** $\begin{pmatrix} 11 & 6 \\ 9 & 5 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 6 \\ 7 \end{pmatrix}$

(ii) $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 5 & -6 \\ -9 & 11 \end{pmatrix} \begin{pmatrix} 6 \\ 7 \end{pmatrix}$

Recommendations

Teachers should emphasize the conditions for multiplication of matrices, noting that not all matrices can be multiplied. The use of matrices in transformations needs to be reinforced.

CARIBBEAN EXAMINATIONS COUNCIL

REPORT ON CANDIDATES' WORK IN THE SECONDARY EDUCATION CERTIFICATE EXAMINATION

JANUARY 2010

MATHEMATICS
GENERAL PROFICIENCY EXAMINATION

GENERAL COMMENTS

The General Proficiency Mathematics examination is offered in January and May/June each year. In January 2010, approximately 16,900 candidates registered for the examination. Approximately 45 per cent of the candidates achieved Grades I–III. The mean score for this examination was 79.5 out of 180 marks.

DETAILED COMMENTS

Paper 01 – Multiple Choice

Paper 01 consisted of 60 multiple-choice items. This year, 21 candidates each earned the maximum available score of 60 and approximately 68 per cent of the candidates scored 30 marks or more.

Paper 02 – Essay

Paper 02 comprised two sections. Section I consisted of eight compulsory questions totaling 90 marks. Section II consisted of six optional questions: two each from Relations, Functions and Graphs; Trigonometry and Geometry; and Vectors and Matrices. Candidates were required to answer any two questions from this section. Each question in this section was worth 15 marks.

This year, three candidates each earned the maximum available mark of 120 on Paper 02 and approximately 21 per cent of the candidates earned at least 60 marks on this paper.

Compulsory Section

<u>Question 1</u>

This question tested candidates' ability to:

- add, multiply and divide decimals
- calculate annual salary, commission and total income
- calculate a percentage of a sum of money
- solve problems involving ratio and proportion

The question was attempted by 99 per cent of the candidates, 16 per cent of whom earned the maximum available mark. The mean mark was 6.8 out of 11.

The majority of candidates scored full marks on the computation with decimals in Part (a). The errors observed included:

(a) $.7^2 = \ .7 + \ .7 = 17.4$
(b) $.7^2 = \ \ +.7^2 = \ .49$
(c) $.7^2 = \sqrt{8.7} = 2.949.$

Candidates who performed well on Part (a) also did well on Part (b). Those who encountered difficulties did not know the terms: 'fixed salary for the year', 'commission' or 'total income for the year'. Some common responses were:

i. Fixed salary for the year $= \frac{720\,000}{3140}$
ii. Fixed salary for the year$= 2\%\ of\ 3140$
iii. Fixed salary for the year$= 3140 \times 2 \times 12$
iv. Commission for the year$= 2\%\ of\ 37680$
v. Commission for the year$= \frac{2}{100} \times 720\,000 \times 12$

Part (c) of the question proved to be the most inaccessible. Many candidates established that 1 cup of mix would make 4 pancakes, but could not proceed to determine how many cups of mix would make 12 pancakes. A wide variety of strategies were attempted, several of which were appropriate, but generally, candidates did not correctly follow through on the path selected.

Solutions:

(a) 79.14
(b) (i) \$37680 (ii) \$14400 (iii) \$52080
(c) (i) 3 cups pancake mix (ii) 30 pancakes.

Recommendations

Reinforce understanding of the concepts of fixed salary, commission, yearly salary and total salary. Candidates need to understand the proper use of ratio and proportion. Attention should also be given to the skill of squaring a quantity.

Question 2

This question tested candidates' ability to:

- substitute numbers for algebraic symbols in simple algebraic expressions
- perform the four basic operations on directed numbers
- use the distributive property to simplify algebraic expressions
- use the laws of indices to simplify algebraic expressions with integral indices
- solve simple in-equations in one variable and interpret the result

The question was attempted by 99 per cent of the candidates, 2 per cent of whom earned the maximum available mark. The mean mark was 6.0 out of 12.

Most candidates substituted the values correctly in Part (a) but a substantial number lacked the numeracy skills to complete the simplification correctly. Common errors included:

(i) $--4 = \quad -4 = 4$
(ii) $\frac{36+-4}{8--4} = 36 - \frac{4}{8} + 4 = 39.5$ (Improper use of the calculator)

In (b) (i), most candidates were able to apply the distributive law to remove at least one pair of brackets. However, grouping like terms proved difficult for some who proceeded to rearrange terms without paying attention to their signs. A common occurrence was

$$3x - 3y + 4x + \quad y = (3x + 4x) - (3y + \quad y) = 7x - 11y$$

In (b) (ii), candidates generally knew how to evaluate $\frac{4\times3}{6}$ to obtain 2. However, many of the candidates multiplied the indices instead of adding them, and divided instead of subtracting. As a result, $x^2 \times x^4$ became x^8, and in some cases, $\frac{x^6}{x^3}$ was evaluated as x^3.

In Part (c), candidates frequently transposed incorrectly and even when they transposed correctly, they encountered difficulties with the use of directed numbers. A common occurrence was: $x - 3x = 2x$. A major problem arose at the point of solving $-2x < -4$. In most cases, candidates divided by -2 and did not change the sign of the inequality. The weaker candidates proceeded to solve the in-equation by solving two equations: $x - 3 = 0$ and $3x - 7 = 0$.

Solutions:

(a) $\frac{32}{12}$

(b) (i) $7x + 5y$, (ii) $2x^3$

(c) (i) $x > 2$, (ii) $x = 3$

Recommendations

Attention should be given to the mastery of the basic operations on directed numbers; to the collection of like-terms in algebraic expressions; to the algebra which give rise to the rules governing the manipulation of indices; to the process of solving simple linear in-equations in one variable and to the interpretation of the results obtained.

Question 3

This question tested candidates' ability to:

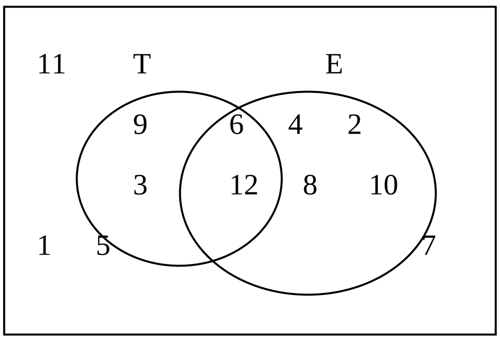
- list from a given set of numbers, multiples of 3 and even numbers
- list subsets of a given set
- determine elements in the intersection, union and complement of a given set
- draw a Venn diagram to represent sets
- use compasses, ruler and pencil to construct an equilateral triangle
- use compasses, ruler and pencil to construct a kite
- measure the size of a named angle

The question was attempted by 99 per cent of the candidates, 4 per cent of whom earned the maximum available mark. The mean mark was 6.8 out of 12. Generally, candidates found the question to be accessible. They were able to construct the Venn diagram and to correctly fill in the elements in the subsets. Some, however, did not appreciate that the elements of a subset are limited by the universal set, and so there were instances where the even numbers and the multiples of 3 went beyond the maximum of 12 as indicated in the universal set. A large number of candidates listed 1 as a multiple of 3. Several candidates confused the use of the symbols $\cap$ *and* $\cup$. The members of the set $T \cup E$ were often written for the members of $T \cap E$. In addition, it was common to find the same numeral written in different subsets of the Venn diagram. Identifying members of the set $(T \cup E)'$ posed a severe challenge for candidates.

Candidates were generally able to draw the equilateral triangle ABC. They however, frequently did so without displaying any construction lines. Some were not familiar with the shape of a kite and therefore located the point D in inappropriate places on the diagram. Many candidates selected angles other than $D\hat{A}C$ to measure; not taking into consideration that the middle letter gives an indication where the angle is formed. Also, there were those who stated the measure of the angle in cm rather than in degrees.

Solutions:

(a) (i)



(a) (ii) $T \cap E = \{6, 12\}$ $(T \cup E)' = \{1, 5, 7, 11\}$

(b) (iii) 53 ± 1

Recommendations

The basic concepts of sets and the vocabulary and symbols associated with them need to be reinforced through practice exercises. Candidates must give attention to the instructions on the construction of geometrical shapes regarding the instruments to be used. Construction lines are necessary when directed to use a pair of compasses.

Question 4

This question tested candidates' ability to:

- find the third side of a right-angled triangle
- find the length of the opposite side in a right angled triangle when the hypotenuse and the measure of the angle are given
- measure and state the length of a given line
- find the actual distance on the ground, given a distance on a map and the scale of that map
- calculate the speed, given distance and time for a journey
- convert $ms^{-1}\ to\ kmh^{-1}$

The question was attempted by 95 per cent of the candidates, 2 per cent of whom earned the maximum available mark. The mean mark was 3.4 out of 11.

A considerable number of candidates correctly applied Pythagoras' theorem to find the value of x. Although some candidates selected the appropriate trigonometric ratio (sine) to find the measure of $K\hat{L}N$, many encountered difficulties in applying the ratio to arrive at the correct solution.

Candidates encountered considerable difficulty with Part (b). Some tried counting squares to find the length of SF instead of simply measuring the length of the line with a ruler. They generally knew that to convert cm to m they should divide by 100, but they often did not use the scale to multiply the length of the line SF, to find the actual distance. Many candidates did not associate 1 cm on map with 1250 cm on land. The major difficulty in the question was attempting to convert from $ms^{-1}\ to\ kmh^{-1}$, and very few candidates did so successfully.

Solutions:

(a) (i) $x = 10\ cm$ (ii) $\theta = 30$

(b) (i) 7.7 $cm \leq SF \leq$ .0 cm on map (ii) 96.3 $cm \leq SF \leq 100\ cm$ on land

(iii) a) $9.9\frac{m}{s} \leq speed \leq 10.3\frac{m}{s}$

b) $35.64\frac{km}{h} \leq speed \leq 37.1\ km/h$

Recommendations

Give attention to making accurate measurements using a ruler. Practise using scales to represent actual measurements in your environment. Use maps and charts to assist in this regard.

Question 5

This question tested candidates' ability to:

- determine the equation of a straight line given the gradient and a point on the line
- plot points on a scale
- reflect a triangle in the line y = 2
- describe the single transformation which moves a triangle to its image in the plane

The question was attempted by 84 per cent of the candidates, 3 per cent of whom earned the maximum available mark. The mean mark was 5.5 out of 12.

Generally, candidates knew that the equation of the line would be in the form of $y = mx + c$ but did not know what to substitute into the formula nor how to find c. In some cases where they found a value for c, they were still unable to write the equation of the line.

Candidates were able to draw the coordinate axes using a given scale, to plot the given points and draw the two triangles whose vertices were given. However, drawing the line $y = 2$ was a source of difficulty for many candidates. Some drew the line $x = 2$, while others drew lines not parallel to either of the coordinate axes. Candidates were able to reflect triangle ABC in a line in the plane, but even when a line was drawn to represent $y = 2$, the triangle was seldom reflected in this line. Not many candidates knew how to describe the transformation that moved ΔABC on to $\Delta A^{//}B^{//}C^{//}$. In some cases, when it was correctly named as a translation, they were unable to write the vector correctly or to state in words the properties of the vector.

<u>Solutions:</u>

(a) $y = \frac{3}{5}x - \frac{7}{5}$

(b) (v) Translation by the vector $\begin{pmatrix} -9 \\ 1 \end{pmatrix}$

<u>Recommendations</u>

Explore linear relationships associated with authentic tasks. Make use of the graphical approach for giving semi-concreteness to the equation of a straight line. Use real objects to demonstrate transformations in the plane. Give attention to equations of lines in the form $y = k, or\ x = k,$ where k is a constant.

<u>Question 6</u>

This question tested candidates' ability to:

- complete a grouped frequency distribution table from raw data
- draw a histogram using data from a grouped frequency table
- calculate the probability of a random event
- select the most appropriate average to be used in a given situation

The question was attempted by 95 per cent of the candidates, less than 1 per cent of whom earned the maximum available mark. The mean mark was 4.6 out of 11.

A significant proportion of candidates did not complete the frequency table. Candidates were generally able to scale the axes for drawing the histogram and knew that the bars should be joined, but invariably failed to use the boundaries for this purpose.

In Part (c), candidates demonstrated an understanding of what 26 or more meant. However, the attempt at writing the probability proved challenging. Ratios such as $\frac{26}{6}$ and $\frac{3}{26}$ were common.

Candidates generally did not recognize that the median and the mode were types of averages. As a consequence, they could give no mathematical reason for the choice of a measure of central tendency for the given situation.

<u>Solutions:</u>
(a)

Distance (km)	21-25	26-30	31-35	36-40
No. of students	7	3	2	1

(c) $p(x > 25) = \frac{6}{26}$

Recommendations

It is a good habit to use a system of tallying to count data in situations as these. Familiarity with the difference in purpose of a bar chart and a histogram for representing data is essential. Pay attention to the fact that in a histogram, the bars are joined at the boundaries. Review the use of the mean, median and mode as measures of central tendency and when it is most appropriate to use any of these to represent the data set.

Question 7

This question tested candidates' ability to read and interpret the graph of a quadratic function to determine the

- value of $f(x)$ when $x = 0$
- values of x when $f(x) = 0$
- coordinates of the maximum point on the graph
- equation of the axis of symmetry
- values of x for a given value of $f(x)$
- interval in which x lies for a given range of $f(x)$

The question was attempted by 72 per cent of the candidates, 1 per cent of whom earned the maximum available mark. The mean mark was 2.1 out of 11.

Candidates knew where the x-intercepts were on the graph but were unable to properly interpret the scales and to write the correct values of x. They also knew where the maximum point was located but the same problem of reading the scales prevented them from adequately representing it. Candidates found writing the equation of the axis of symmetry to be the most inaccessible part of the question. Some attempted to complete the square for a quadratic expression whose real number coefficients they did not know. Writing the interval of the domain for which $f(x) > 5$ proved difficult for candidates, most of whom copied the expression: $a < x < b$, and substituted numbers for a and b not related to the values of x found in Part (v) of the question.

Solutions:

(i) At $x = 0, f(x) = 3$
(ii) For $f(x) = 0, x = -3 \; and \; 0.5$
(iii) Maximum point $(-1.3, 6.1)$
(iv) Axis of symmetry: $x = -1.3$
(v) For $f(x) = 5, x = -2 \; and - 0.5$
(vi) $f(x) > 5 \; for - 2 < x < -0.5$

Recommendations

While candidates work at mastering the skill of drawing graphs, thought must also be given to the finer points of interpreting the graphs already drawn. Attention should be paid to reading scales which are other than one-to-one.

Question 8

This question tested candidates' ability to:

- recognize and extend a pattern in a sequence of diagrams
- calculate unknown terms in two number sequences
- derive a formula connecting the variables in two number sequences

- use a derived formula connecting two variables to calculate the value of one variable when a value for the other is given

The question was attempted by 92 per cent of the candidates, 7 per cent of whom earned the maximum available mark. The mean mark was 5.2 out of 10.

Candidates were generally able to identify the pattern in each number sequence and to determine the values of *x* and *y*. They however encountered difficulties at the point where counting was not a useful method of solving the problem. Calculating the value of *z* needed some degree of generalization and candidates struggled at this point. They also failed to appropriately relate S to n in an equation, but were frequently able to derive the number of hexagons which can be made from 76 sticks. Many candidates employed the method of trial and error at this stage.

Solutions:

(a) (i) $x = 21$ (ii) $y = 26$ (iii) $101\ sticks$
(b) $S = 5n + 1$
(c) $n = 15$

Recommendations

There are many real-life situations that give rise to simple number patterns. Find examples of these in your environment and investigate the number sequences associated with them. Always look for ways of making generalizations from the patterns you observe.

Optional Section

Question 9

This question tested candidates' ability to:

- change the subject of a formula
- solve an equation involving a squared term
- complete the square
- solve a quadratic equation
- use information from a quadratic function stated algebraically to find the minimum point and intercepts on the graph

The question was attempted by 28 per cent of the candidates, 4 per cent of whom earned the maximum available mark. The mean mark was 4.6 out of 15.

Candidates encountered difficulties in attempting to make *v* the subject of the formula. They were, however, proficient at substituting the values of *E* and *M* into their version of the equation in an attempt to find *v*. Two frequent expressions for *v* were: $\frac{\sqrt{2E}}{M}\ and\ \sqrt{\frac{E}{2M}}$.

Completing the square was problematic for candidates. Making 3 a common factor was often done in an incomplete manner: $3(x^2 - 8x + 2)$ was often seen. When the formulae were used, some candidates misquoted them, and it was usual to see: $h = \frac{b}{2a}$; $k = \frac{b^2-4ac}{4a}$ instead of $h = -\frac{b}{2a}$; $k = \frac{4ac-b^2}{4a}$.

Even when candidates completed the square and solved the quadratic equation for values of *x*, they did not associate the results of these with the minimum point and the x-intercepts of the graph of the function.

Solutions:

(a) (i) $v = \sqrt{\frac{2E}{M}}$ (ii) $v = 2.63\ to\ 3\ s.f.$

(b) (i) $g(x) = 3\left(x - \frac{4}{3}\right)^2 - \frac{10}{3}$ (ii) x= 0.28, 2.39

(iii) a) 2 b) 2.39 c) $\left(\frac{4}{3}, \frac{-10}{3}\right)$

Recommendations

Approach the topic of changing the subject of the formula through modelling, for instance, using a balance. Modelling is also appropriate for developing an understanding of quadratic functions. In addition, there are many real-life situations which can be modelled using quadratic functions.

Question 10

This question tested candidates' ability to:

- translate verbal statements into algebraic inequalities
- draw graphs to represent inequalities in two variables
- use linear programming techniques to solve problems involving two variables

The question was attempted by 37 per cent of the candidates, 3 per cent of whom earned the maximum available mark. The mean mark was 4.7 out of 15.

The majority of candidates were able to derive the inequalities from the statements given. They were also able to write the profit function, choose the correct scales for the graph, draw the line $x + y = 20$, substitute coordinates of points in the region representing the solution set into the profit function and to identify the maximum profit. They, however, experienced difficulty with drawing the line $15x + 30y = 450$, correctly identifying the region representing the solution set and with writing the coordinates of points which were located on either the x-axis or the y-axis. For example, the point (20, 0) on the x-axis was often written as (0, 20).

Solutions:

(a) (i) $x + y \leq 20$ (ii) $15x + 30y \leq 450$

(b) (iii) The vertices of the shaded region are: $(0,0), (0, 15), (10, 10), (20, 0)$

(c) (i) $P = \quad x + 20y$ (ii) $P_{max} = \$300$

Recommendation

Practice in locating the region which represents the solution set of a system of inequalities is essential.

Question 11

This question tested candidates' ability to:

- solve problems involving bearings
- use the cosine rule for finding the length of a line segment
- draw a diagram showing angles of elevation
- use the tangent ratio to solve problems involving angles of elevation

The question was attempted by 21 per cent of the candidates, 4 per cent of whom earned the maximum available mark. The mean mark was 4.9 out of 15.

Some candidates experienced difficulty in showing that $P\hat{R}Q = 126^{\circ}$. They failed to establish the relationship between the bearing of P and of Q from R. Several candidates did not recognize that the cosine rule was applicable for determining the length of PQ, and some of those who chose the rule were unable to carry it through correctly because of weaknesses in their use of integers.

Candidates completed the diagram in Part (b) with high proficiency. A few, however, inserted the angles of elevation in the wrong places. In their attempt at calculating the lengths of KL and LM, candidates selected the appropriate trigonometric ratio, but some encountered difficulty when transposing the tangent function. Several candidates also attempted to find LM by directly applying the tangent ratio instead of first finding KM and then subtracting the length KL.

Solutions:

(a) (ii) $PQ = 128\ m$

(b) (ii) a) $KL = 26.1\ m$ b) $LM = 14.1\ m$

Recommendation

Problem-based learning offers a powerful tool for students' understanding of three-dimensional geometry. It brings relevance to the task and the resulting motivation allows for improved learning.

Question 12

This question tested candidates' ability to:

- apply circle theorems to find unknown angles
- find the area of a triangle given two sides and the included angle
- find the area of a sector of a circle
- find the area of a segment of a circle

The question was attempted by 20 per cent of the candidates, 2 per cent of whom earned the maximum available mark. The mean mark was 2.9 out of 15.

Most candidates recognized that the angle at the centre was twice the angle at the circumference of the circle. However, they were not familiar with the properties of a cyclic quadrilateral; instead, they judged that triangle DEF was isosceles and that DGHE was a trapezium, and as a consequence were not able to complete the question.

To find the area of triangle GCH, most candidates appropriately chose to use the formula:

$A = \frac{1}{2}ab\sin C$. However, some candidates attempted to find the height of the triangle and the length of the chord GH by using trigonometric ratios in a bid to use the formula: $A = \frac{1}{2}bh$.

For the area of the sector, some candidates used the formula: $A = \frac{1}{2}r^2\theta$, but the angle was not converted to radians and this led to incorrect results.

Most of the candidates did not recognize that the area of the segment could be found by taking the difference between the area of the sector and the area of the triangle.

Solutions:

(a) (i) $G\hat{F}H = 44$ (angle at centre = twice angle at circumference)

(ii) $G\widehat{D}E = 54$ (opposite angle in cyclic quadrilateral)

(iii) $D\hat{E}F = 82$ (3rd angle $of\ \Delta\ DEF$)

(b) (i) cm^2 (ii) 12.3 cm^2 (iii) 4.3 cm^2

Recommendations

Candidates need to recognize that as is stated in the questions, the diagrams are *not drawn to scale*. It is necessary, therefore, to apply the theorems and geometric properties of circles and polygons irrespective of what the diagrams appear to show.

Question 13

This question tested candidates' ability to:

- write the coordinates of a point as a position vector
- add two position vectors
- determine the midpoint of a line segment using a vector method
- solve problems in geometry using vector methods

The question was attempted by 18 per cent of the candidates, less than 1 per cent of whom earned the maximum available mark. The mean mark was 3.6 out of 15.

Candidates succeeded in writing the position vector for the point B. Adding the two position vectors was generally well done although some candidates encountered difficulties with basic operations on directed numbers.

Candidates who used coordinate geometry to determine the coordinates of the midpoint of AB did so successfully. Those who used a vector method were less successful since they generally found $\frac{1}{2}\overrightarrow{AB}$ and stopped at that point.

Interpreting the ratios was a point of difficulty for several candidates. $\overrightarrow{HF}$ was commonly written as $3u$. Candidates were familiar with the correct vector routes but they often substituted values incorrectly.

Candidates struggled with establishing the co-linearity of points O, G and H. They failed to recognize that OH and OG were parallel even after expressing one as a scalar multiple of the other.

Solutions:

(a) (i) a) $\overrightarrow{OB} = \binom{-1}{4}$ $\overrightarrow{OA} + \overrightarrow{OB} = \binom{4}{4}$ (ii) $x = 2,\ y = 2$

(b) (i) a) $\overrightarrow{HF} = 2\boldsymbol{u}$; b) $\overrightarrow{MF} = \boldsymbol{v} + 3\boldsymbol{u}$; c) $\overrightarrow{OH} = 2\boldsymbol{v} + \boldsymbol{u}$;

(iii) $\overrightarrow{OH}\ is\ a\ scalar\ multiple\ of\ \overrightarrow{OG};\ OH||OG; O\ is\ a\ common\ point.$

Recommendations

In an effort to make the study of vectors more interesting, authentic situations where the knowledge of vectors can be applied should be included in the instructional process.

Question 14

This question tested candidates' ability to:

- square a 2×2 matrix
- subtract 2×2 matrices
- use the properties of a singular matrix to solve a problem in algebra
- derive the transforming matrices given the object and the image
- determine the image of a point resulting from a combination of transformations

The question was attempted by 43 per cent of the candidates, less than 1 per cent of whom earned the maximum available mark. The mean mark was 2.6 out of 15.

Candidates knew how to subtract matrices but experienced difficulty computing N^2. Even some candidates who expressed N^2 as $N \times N$, made mistakes in the multiplication. To compute N^2, candidates simply squared the elements in the matrix.

For the singular matrix, although candidates proceeded to find the determinant, they invariably did not equate the determinant to zero. Some candidates evaluated $x \times x$ as $2x$.

Generally, candidates did not know how to set up the equation to show how a rotation matrix transforms the position vector of a point into the position vector of its image. Further, they did not demonstrate the knowledge of the order of matrices for multiplication.

In Part (d), candidates simply added the coordinates of the two points given and gave those as the elements of the translation vector.

In Part (e) there was confusion as to which operation to perform first.

Solutions:

(a) $L - N^2 = \begin{pmatrix} 2 & -1 \\ 1 & 0 \end{pmatrix}$

(b) $x = \pm 6$

(c) $p = -1, \quad q = 1$

(d) $r = -4, \; s = -2$

(e) $(-9, 6)$

Recommendations

A graphical approach to the teaching and learning of operations on matrices should help candidates to make sense of the abstract nature of this topic.

C A R I B B E A N E X A M I N A T I O N S C O U N C I L

REPORT ON CANDIDATES' WORK IN THE SECONDARY EDUCATION CERTIFICATE EXAMINATION

MAY/JUNE 2010

MATHEMATICS
GENERAL PROFICIENCY

GENERAL COMMENTS

The General Proficiency Mathematics examination is offered in January and May/June each year. This was the first examination of the revised Mathematics syllabus effective for the May/June 2010 examinations.

There was a candidate entry of approximately 88,400 in May/June 2010. Forty-one per cent of the candidates achieved Grades I to III. The mean score for the examination was 76.59 out of 180 marks.

DETAILED COMMENTS

Paper 01 – Multiple Choice

Paper 01 consisted of 60 multiple choice items. This year, 54 candidates each earned the maximum available mark of 60. Fifty-seven per cent of the candidates scored 30 marks or more.

Paper 02 – Structured Questions

Paper 02 consisted of two sections. Section I comprised eight compulsory questions totalling 90 marks. Section II comprised three optional questions: one each from (i) Algebra and Relations, Functions and Graphs; (ii) Measurement, Geometry and Trigonometry and (iii) Vectors and Matrices. Candidates were required to answer any two questions from this section. Each question in this section was worth 15 marks.

This year, 20 candidates each earned the maximum available mark of 120 on Paper 02. Approximately 24 per cent of the candidates earned at least half of the maximum mark on this paper.

Compulsory Section

Question 1

This question tested candidates' ability to

- subtract, multiply and divide fractions
- multiply, subtract and divide decimal numbers
- express a number correct to a given number of significant figures
- find the unit price, given the cost price and the number of items purchased
- determine the profit and profit percent, given the selling price and cost price

The question was attempted by 99 per cent of the candidates, 9.4 per cent of whom earned the maximum available mark. The mean mark was 7.0 out of 11.

Responses to this question were generally good. The majority of candidates were able to accurately perform the computations required with the use of a calculator.

In performing subtraction on fractions, some candidates failed to use a common denominator and incorrectly simplified $\frac{3}{2} - \frac{2}{5}$ as $\frac{1}{3}$ or $\frac{1}{-3}$.

In performing the division, some candidates mistakenly interpreted $\frac{11}{10} \div \frac{33}{10}$ as $\frac{33}{10} \div \frac{11}{10}$ and thus inverted the quotient instead of the divisor.

Squaring 2.5 also posed difficulties for some candidates, for example, 2.5^2 was interpreted as, 2.5×2 or $2^2 + 5^2$.

In computing $2.5^2 - \frac{2.89}{17}$ some candidates did not recognize that $\frac{2.89}{17}$ was a separate term and incorrectly subtracted 2.89 from 2.5^2, then divided by 17.

Expressing 6.08 correct to two significant figures also posed some difficulties for candidates. Common incorrect answers were 6.0, 60.8 and 61. In addition, a few candidates confused significant figures with scientific notation.

In Part (b), a few candidates were unable to differentiate between profit and profit percent. Surprisingly, a significant number of candidates could not express the profit as a percentage of the cost price. Some candidates did not multiply by 100 while others expressed the profit as a percentage of the selling price.

<u>Solutions:</u>

(a)	**(i)** $\mathbf{\frac{1}{3}}$	**(ii)**	**6.1**		
(b)	**(i) \$12.80**	**(ii)**	**\$2 998.50**	**(iii) \$1 078.50**	**(iv) 56%**

Recommendations

Teachers should allow students to estimate the result of a computation prior to performing computations using calculators. In this way, they can determine if their answers are reasonable and make adjustments to their procedures.

Basic concepts in computation of fractions, percentages and decimals should be reviewed using conceptual rather than procedural approaches. Students should also be encouraged to investigate the use of calculators in performing multi-step computations. They should be allowed to check their results using different orders and verify the correct order based on the context of the problem.

In teaching approximations, a clear distinction must be made between significant figures, decimal places and standard form. The use of each type in real life situations must also be emphasized.

Question 2

This question tested candidates' ability to

- substitute numbers for algebraic symbols in simple algebraic expressions
- perform the four basic operations on directed numbers
- convert verbal phrases to algebraic expressions
- solve a pair of simultaneous linear equations, algebraically
- apply the distributive law to factorize or expand algebraic expressions
- factorize quadratic expressions

The question was attempted by 99.6 per cent of the candidates, 2 per cent of whom earned the maximum available mark. The mean mark was 4.6 out of 12.

Candidates performed reasonably well on substituting the numbers in the algebraic expressions. However, a small percentage had difficulty using directed numbers. Common errors were observed in simplifying $2^2 - (-3)^2$, which was often written as $2^2 - -3^2$ and equated to 4 + 9 =13.

In Part (b), a large number of candidates were unable to successfully use symbols to express a phrase as an algebraic expression. Further, they did not know when to use brackets and incorrectly wrote $7x + y$ instead of $7(x+y)$ in Part (b) (i).

Responses to Part (b)(ii) were generally below the required standard. Candidates either omitted this part or made unsuccessful attempts. Common incorrect responses included xy and $x < y$.

Many candidates displayed competence in solving simultaneous equations. Both the elimination and substitution methods were used, with the majority choosing the method of elimination. However, too many errors were made in simplifying algebraic terms, irrespective of the method used. Where the substitution method was used, many candidates were unable to express one variable in terms of the other. Quite a number of candidates arrived at the correct answers seemingly by trial and error and did not show working. A few candidates chose a matrix method but this approach was also fraught with errors and correct solutions were seldom obtained.

In factorizing the given expressions, the method of common factors and the difference of two squares was generally known by candidates. However, quite a number of them incorrectly interpreted $4y^2 - z^2$ as $4(y^2 - z^2)$; $(4y)^2 - z^2$ or $(2y - z)^2$.

Grouping terms to factorize also posed challenges for many candidates who did not strategize to obtain a second pair of common factors. For example, they often wrote:

$2ax - 2ay - bx + by = 2a(x - y) - b(x + y)$ and could go no further.

Some candidates correctly factorized the first step as $2a(x - y) - b(x - y)$ but failed to complete the factorization.

In (d)(iii), candidates also had difficulty factorizing the quadratic expression $3x^2 + 10x - 8$ and many ended up with incorrect factors such as $(3x - 2)(x - 4)$; $(3x + 2)(x - 4)$ or $(3x + 4)(x - 2)$. This suggests that they had no idea how to check their results to obtain the original expression.

Solutions:

(a) **(i)** **-2** **(ii)** **-5**

(b) **(i)** $\boldsymbol{7(x+y)}$ **(ii)** $\boldsymbol{y(y+1)}$

(c) $\boldsymbol{x = 3,\ y = 1}$

(d) **(i)** $\boldsymbol{(2y-z)(2y+z)}$ **(ii)** $\boldsymbol{(x-y)(2a-b)}$ **(iii)** $\boldsymbol{(x+4)(3x-2)}$

Recommendations

Teachers need to emphasize the importance of brackets when making substitutions. When indices are used in substitutions, they should note that the index must apply to what is inside the bracket only.

In solving simultaneous linear equations, teachers should also encourage students to determine the most efficient strategy to use. Both elimination and substitution methods should be taught and students must develop strategies to decide on which method is better in a given situation.

Teachers should also pay close attention to mathematical vocabulary so that students are familiar with basic terminology such as *solve*, *simplify* and *factorize*.

Question 3

This question tested candidates' ability to

- use a Venn diagram to solve practical problems involving two sets
- determine elements in the intersection, union and complement of a set
- solve a simple equation
- calculate the area of a compound shape
- use Pythagoras' theorem to find one side of a right-angled triangle
- solve geometric problems using properties of congruent triangles

The question was attempted by 99.2 per cent of the candidates, 3 per cent of whom earned the maximum available mark. The mean mark was 4.4 out of 12.

Performance was generally unsatisfactory with candidates having extremely poor responses to Part (b) in particular. In Part (a), the majority of candidates were able to identify the elements in the intersection, but only a small percentage of candidates could determine the elements in $A \cap B'$ and $B \cap A'$ correctly.

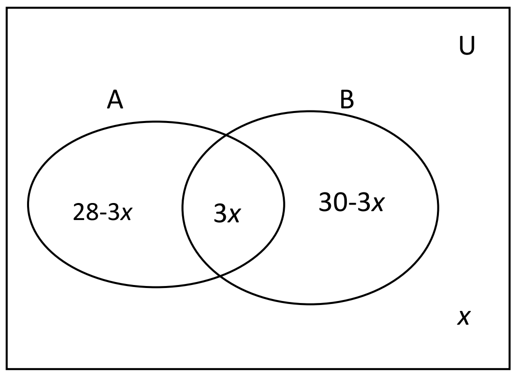
Although some candidates correctly placed $3x$ in the intersection, they subtracted x in their expressions to obtain (28 – x) and (30 – x) for $A \cap B'$ and $A' \cap B$ respectively. As a result, they obtained $x = -9$ but were unable to conclude that one cannot have a negative number of tourists. Far too many candidates produced an equation when asked for an expression.

In Part b (i) candidates generally applied Pythagoras' Theorem but many used it incorrectly, primarily using the incorrect side as the hypotenuse, instead of DE. Those who did not recognize that Pythagoras could be used applied methods such as the cosine rule or trigonometric ratios, which could not produce the intended solution.

Quite a few candidates assumed that ABCD was a square and gave the area as 36 cm^2 instead of 30 cm^2. Weaker candidates chose the formula for perimeter in computing the area. However, the majority of candidates knew that they had to add two separate areas to determine the area of the compound shape ABCDE.

Solutions:

(a) (i)



(a)	**(ii) 58 - 2*x***	**(iii) *x* = 9**	
(b)	**(i) EF = 3 cm**	**(ii) DF = 4 cm**	**(iii) 42 cm^2**

Recommendations

Teachers should teach students to verify that the information recorded in their Venn diagram accurately represents the given data. This can be done by summing up the elements in the subsets and checking to see that the total for the universal set is obtained. Students should also devise strategies to check for unreasonableness of answers and make the adjustments where necessary.

In using Pythagoras' Theorem, students should first study the information given and then decide by inspection whether subtraction or addition is required. When diagrams are not drawn to scale, students should not assume that lengths look similar and are therefore equal.

Concepts of area and perimeter should be taught simultaneously and formulae should be introduced through guided investigative approaches.

Question 4

This question tested candidates' ability to

- solve problems involving direct variation
- construct a triangle given two angles and one side
- draw and measure line segments accurately
- construct angles of 60^o and 90^o
- measure the size of a given angle using a protractor

The question was attempted by 94.8 per cent of the candidates, 8 per cent of whom earned the maximum available mark. The mean mark was 4.4 out of 11.

Generally, performance was unsatisfactory in both parts of this question. In Part (a), although candidates were able to substitute correctly to determine the value of *k*, a significant number of them had difficulty transposing correctly after substituting and obtained incorrect values.

Candidates generally drew and measured the line segments accurately. However, the weaker candidates measured starting at one rather than zero. The construction of angles posed a problem for many candidates. They had more success in constructing 60^o than 90^o. It was also evident that some candidates used a pair of compasses in drawing the figure but erased their arcs afterwards thinking that this was the correct procedure. A small number of candidates also had problems labelling their diagram.

Solutions:

(a) **(i)** $k = 0.5$ **(ii)** $y = 450$
(b) **(ii)** **(a)** **EF = 12 ± 0.1 cm** **(b)** $\angle EFG = 30^o \pm 1$

Recommendations

Candidates need to be reminded that they must use the required instruments when attempting questions on the construction of figures. They must also show all construction lines and refrain from answering these questions on graph paper.

Question 5

This question tested candidates' ability to

- interpret and use functional notations such as $f(x)$ and $gf(x)$
- derive the inverse of a simple function
- determine the scale used on the axis of a graph
- use a graph to determine the value of one variable, given the equation
- state the range of a function from its graph

The question was attempted by 83.7 per cent of the candidates, 3 per cent of whom earned the maximum available mark. The mean mark was 4.0 out of 12.

Generally, performance on this question was unsatisfactory. A significant number of candidates did not attempt this question or scored zero.

In Part (a), candidates were able to substitute the value of x into the function $f(x)$ and many obtained the correct answer for $f(4)$. However, they were not as successful in evaluating $gf\,(4)$. Many candidates could not interpret this notation and proceeded to evaluate $g(4)$ instead of using the result obtained for $f(4)$. Those candidates who went the route of first finding $gf(x)$ experienced difficulties in squaring $(2x - 5)$ and invariably ended up with an incorrect expression.

In determining $f^{-1}(x)$, many candidates knew that they had to interchange the variables, but far too many were unable to correctly carry out the other steps. A major difficulty arose when they had to transpose a term with a negative coefficient.

In Part (b), candidates generally knew that the scale for the x-axis was in the ratio 1:2 but few were able to write down the scale using the correct format. They were also able to use the graph to determine the value of y when $x = -1.5$ and to state at least one of the values of x for which $y = 0$. However, far too many candidates failed to follow instructions and used calculations instead of the graph in determining the unknown values.

Although candidates were able to state the range of values of y, many were unable to write the answer in the form $a \le y \le b$, as requested.

<u>Solutions:</u>

(a) **(i) (a) 3** **(b) 12** **(ii)** $\boldsymbol{f^{-1}(x) = \frac{x+5}{2}}$

(b) **(i) 2 cm represents 1 unit** **(ii)** $\boldsymbol{y = -3.8 \pm 0.1}$

(iii) $\boldsymbol{x = -3, 1}$ **(iv)** $\boldsymbol{-4 \le y \le 5}$

Recommendations

Teachers need to emphasize the role of language in teaching functional notation. In particular, students need to understand the meaning of $f(x)$ and $gf(x)$. With respect to the latter, they must allow students to make connections between notation in geometry (when representing combined transformations) and notation in algebra (for composite functions).

Students also need to understand the language used in interpreting graphs. Terms such as *range* and phrases such as *the value of x for which y = k* are not fully understood.

Question 6

This question tested candidates' ability to

- use appropriate theorems to determine the measure of given angles
- state the centre, angle and direction of the rotation, given a triangle and its image after a rotation
- state the geometric relationships between an object and its image after a rotation
- state its image after a translation by a given column vector, given a point

The question was attempted by 94.8 per cent of the candidates, 1 per cent of whom earned the maximum available mark. The mean mark was 3.3 out of 11.

This question was poorly done. Although some candidates were able to determine the measure of angles *x* and *y*, few were able to give correct reasons for their answers. A small number of candidates were familiar with alternate angles but co-interior angles were rarely mentioned.

In Part (b), describing the rotation proved particularly challenging for candidates. Some of them stated the centre correctly but they used informal language when describing the direction. Responses such as to the left, westward and south east were often given. It was evident that candidates did not know how to state a geometrical relationship between a triangle and its image. Many omitted this part of the question while others merely described the triangles. Candidates also had difficulty writing the image of the point, L, in coordinate form.

Solutions:

(a) (i) 54° (ii) 65°

(b) (i) (a)(0, 0) (b) and (c) 90°, anticlockwise OR 270^0, clockwise

(ii) Congruent or same size and shape (iii) (2, 1)

Recommendations

Students should have opportunities to express their ideas and to communicate effectively, orally and in writing, in the classroom. These experiences are necessary to develop mathematical vocabulary and proficiency in communication, not only in mathematics but in their daily experiences. Allowing students to orally state reasons for their answers can be a useful classroom strategy to assist them in improving their vocabulary and communication skills.

Question 7

This question tested candidates' ability to

- complete a frequency table from a set of raw scores
- determine the class boundaries for a given class interval
- construct a histogram from a set of data using given scales
- interpret data from a frequency distribution
- determine the probability of an event using data from a frequency table

The question was attempted by 71.9 per cent of the candidates, 4 per cent of whom earned the maximum available mark. The mean mark was 5.9 out of 11.

The performance on this question was generally good. The majority of candidates displayed a competence in completing the frequency table. However, a significant number of candidates were unable to state the lower boundary for the interval 20 – 29. Consequently, they did not use the class boundaries in drawing the histogram, but used the lower limits instead.

The majority of candidates used the correct scales and plotted the frequencies correctly. However, some of the weaker candidates drew a bar graph instead of a frequency polygon.

While many candidates were able to determine the number of students who threw the ball a distance of 50 metres or more, a significant number could not determine the probability.

Solutions:

(a)

Distance (m)	**Frequency**
20 – 29	**3**
20 – 39	**5**
40 – 49	**8**
50 – 59	**6**
60 – 69	**2**

(b) **19.5**

(d) **(i)** **8** **(ii)** $\frac{8}{24}$ **or** $\frac{1}{3}$

Recommendations

Teachers must ensure that students can differentiate between the statistical graphs and relate these differences to the type of data that is being represented.

When calculating probability, students should be reminded that results cannot be a whole number and this should be reinforced through reference to the probability scale.

Question 8

This question tested candidates' ability to

- recognize and extend a pattern in a sequence of diagrams
- calculate unknown terms in number sequences
- derive a formula connecting the variables in given sequences

The question was attempted by 91.6 per cent of the candidates, 16 per cent of whom earned the maximum available mark. The mean mark was 6.7 out of 10.

Performance on this question was quite good with the majority of candidates scoring more than 6 marks. Candidates were generally able to extend the pattern by drawing the fourth figure in the sequence and to identify the pattern in each number sequence. A small number of candidates had difficulty moving from the 5th to the 15th sequence. The major challenge for them was generalizing the formulae for the number sequence.

Solutions:

	Figure	**Area of Figure**	**Perimeter of Figure**
(i)	**4**	**16**	**22**
(ii)	**5**	**25**	**28**
(iii)	**15**	**225**	**88**
(iv)	**n**	$\mathbf{n^2}$	**6n - 2**

Recommendation

Teachers need to utilize real life situations that give rise to simple patterns, be it shapes or numbers. They also need to give students opportunities to investigate different ways of making generalizations from the patterns they observe.

Optional Section

Question 9

This question tested candidates' ability to

- interpret a speed-time graph
- write inequalities to represent given constraints
- use a given scale to represent three given inequalities on a graph
- use linear programming techniques

The question was attempted by 27.5 per cent of the candidates, 1 per cent of whom earned the maximum available mark. The mean mark was 4.1 out of 15.

Candidates demonstrated competence in using the graph to determine the maximum speed and the interval for which the speed was constant. However, only a few candidates were able to correctly evaluate the area under the curve to obtain the distance travelled. Many candidates knew the intervals for the different stages of the journey but were unable to express it in the desired format.

In Part (b), candidates were generally able to translate verbal statements into algebraic inequalities and used the given scale in drawing their graphs. However, a significant number of candidates were unsuccessful in drawing the lines $y = x$, $y + x = 12$ and $y = 3$. In many cases, the line x = 3 was drawn for y = 3. Furthermore, they were unable to successfully show the regions satisfied by the inequalities. Consequently, only the very able candidates were successful in obtaining the required region satisfying all inequalities.

The minimum values of x and y proved to be a difficult task for most candidates.

Solutions:

(a) **(i) (a) 12 m/s** **(b) 4 sec** **(c) 102 m**

(ii) (a) $\mathbf{0 \leq t \leq 6}$ **(b)** $\mathbf{10 \leq t \leq 13}$ **(c)** $\mathbf{6 \leq t \leq 10}$

(b) **(i)** $\mathbf{x + y \leq 12}$ **(iv)** $\mathbf{x = 3}$ $\mathbf{y = 3}$

Recommendations

Emphasis must be placed on the use of the mathematical notation, $a \leq x \leq b$, when describing an interval and students should also be able to distinguish between the equation of lines of the form $y = k$ and $x = k$. Students should be encouraged to use points to test their solutions when solving inequalities in two variables.

<u>Question 10</u>

This question tested candidates' ability to

- solve geometric problems using the properties of circles and circle theorems
- solve practical problems involving heights and distances in three dimensional situations
- use the cosine rule in the solution of problems involving non-right-angle triangles
- use trigonometric ratios in the solution of right-angled triangles
- solve practical problems in 3-D situations involving angles of elevation

The question was attempted by 36.8 per cent of the candidates, 2 per cent of whom earned the maximum available mark. The mean mark was 2.3 out of 15.

Many candidates were able to correctly state the size of at least one angle. However, they were not able to state the reasons for their answers. A major weakness observed was that candidates made incorrect assumptions, for example, they assumed that triangle TPR was isosceles, that PR bisected angle TPR and some candidates even used the line TR as a diameter.

In Part (b), the cosine rule was stated correctly by the majority of candidates and substitution of the correct values was also seen. However, many candidates had difficulty following through to the end successfully.

Some of the weaker candidates used incorrect or inefficient strategies in calculating the required lengths. For example, Pythagoras' Theorem was used to calculate the length EG although the triangle was not right-angled. Where trigonometric ratios could have been used to calculate unknown sides of right-angled triangles, many applied the sine rule which resulted in lengthy calculations.

Only the very able candidates calculated the angle of elevation correctly.

<u>Solutions:</u>

(a) (i) 46° **(ii) 56°** **(iii) 124°**

(b) (i) 8.57 m **(ii) 12.2 m** **(iii) 46.9°**

Recommendations

When teaching Circle Geometry, teachers must remind students that assumptions cannot be made about the size of an angle or the lengths of sides since the figures are not drawn to scale. In preparing students for solving problems in trigonometry, clear distinctions must be made between strategies for solving right-angled triangles and non-right-angled triangles. Attention must also be paid to the efficiency of the strategies.

Question 11

This question tested candidates' ability to

- perform matrix multiplication
- evaluate the determinant of a 2×2 matrix
- obtain the inverse of a non-singular matrix
- given a matrix equation in x and y, use a matrix method to solve for x and y
- combine vectors and write expressions for given vectors
- determine the geometrical relationship between two vectors

The question was attempted by 21.3 per cent of the candidates, 1 per cent of whom earned the maximum available mark. The mean mark was 3.2 out of 15.

The performance on this question was generally poor with only a few candidates showing mastery in the skills tested. A significant number of candidates were unable to correctly multiply two matrices. Instead of multiplying row by column, many candidates multiplied the corresponding elements. Further, a large number of candidates did not recognize that the inverse of matrix B was matrix A and proceeded to compute the inverse. Those who chose this method often encountered further problems in calculating the determinant.

Many candidates did not use the inverse obtained in Part (ii) to solve for x and y in Part (iii). In addition, writing $\begin{pmatrix} x \\ y \end{pmatrix}$ as a product of two matrices was particularly challenging. Some candidats used the incorrect order by placing the 2×1 matrix on the left of the 2×2 matrix.

The vector component was equally challenging. Although many students placed M and N correctly on JK and JL respectively, they were not able to conceptualize 'one third' thereby placing M and N at random positions. Further, many of the candidates were unable to use the triangular law of vectors to determine the resultant vectors. Although there were correct attempts at stating routes, candidates had difficulty with the directions, often omitting the negative signs.

Many candidates could not state the relationship between MN and KL. There was a tendency to use words like 'collinear' and 'parallel' without referring to the actual findings. Those candidates who drew the diagram were able to determine that the vectors were parallel. However, they were not able to state the relationship between the vectors KL and MN in terms of their lengths.

Solutions

(a) **(i) AB =** $\begin{pmatrix} \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} \end{pmatrix}$ **(ii)** $\begin{pmatrix} \mathbf{1} & \mathbf{2} \\ \mathbf{2} & \mathbf{5} \end{pmatrix}$ **(iii)** $\begin{pmatrix} \boldsymbol{x} \\ \boldsymbol{y} \end{pmatrix} = \begin{pmatrix} \mathbf{1} & \mathbf{2} \\ \mathbf{2} & \mathbf{5} \end{pmatrix} \begin{pmatrix} \mathbf{2} \\ \mathbf{3} \end{pmatrix}$ **(iv)** $\boldsymbol{x} = \mathbf{8}$, $\boldsymbol{y} = \mathbf{19}$

(b) **(ii)** **a) 3*u*** **(b)** **-*u* + *v*** **(c)** **-3*u* + 3*v***

(iii) **KL is parallel to MN**
KL = 3MN

Recommendations

Emphasis should be placed on the relationship between a matrix, its inverse and the identity matrix. Students must see the connection between solving matrix equations and solving simple equations in terms of the use of an identity element.

Greater emphasis should be placed on a matrix as a system in which order is important. The use of real life examples where matrices are used to represent authentic situations should enable students to appreciate the meaning of operations on matrices.

Prior to the teaching of vectors, basic concepts in fractions and geometry must be reviewed. Vocabulary associated with vectors can be reinforced through the use of practical examples in locating points on a line segment or describing relationships between line segments.

C A R I B B E A N   E X A M I N A T I O N S   C O U N C I L

REPORT ON CANDIDATES' WORK IN THE SECONDARY EDUCATION CERTIFICATE EXAMINATION

JANUARY 2011

MATHEMATICS
GENERAL PROFICIENCY EXAMINATION

GENERAL COMMENTS

The General Proficiency Mathematics examination is offered in January and May/June each year. This was the first January examination for the revised Mathematics syllabus effective from May/June 2010.

There was a candidate entry of approximately 13,760 in January 2011. Thirty-seven per cent of the candidates earned Grades I–III. The mean score for the examination was 74.97 out of 180 marks.

DETAILED COMMENTS

Paper 01 – Multiple Choice

Paper 01 consisted of 60 multiple-choice items. This year, 24 candidates each earned the maximum available score of 60. Sixty-two per cent of the candidates scored 30 marks or more.

Paper 02 – Structured Questions

Paper 02 consisted of two sections. Section I comprised eight compulsory questions totalling 90 marks. Section II comprised three optional questions one each from (i) Algebra, Relation, Functions and Graphs; (ii) Measurement, Trigonometry and Geometry and (iii) Vectors and Matrices. Candidates were required to answer any two questions from this section. Each question in this section was worth 15 marks.

This year, no candidate earned the maximum available mark of 120 on Paper 02. There were five candidates who each scored 119 marks. Approximately 16 per cent of the candidates earned at least half of the maximum marks on this paper.

Compulsory Section

Question 1

This question tested candidates' ability to

- square, add and multiply decimal numbers
- subtract and divide fractions involving mixed numbers
- solve problems involving wages and overtime

The question was attempted by 99 per cent of the candidates, 15.5 per cent of whom earned the maximum available mark. The mean mark was 6.98 out of 11.

In general, candidates provided satisfactory responses to this question. However, there were a number of incorrect strategies employed in an attempt to solve the problem. These included for Part (a):

(i) $5.8^2 + 1.02) \times 2.5 = 5.8 \times 5.8 + 1.02 \times 2.5 = 36.19$ (Incorrect application of the distributive property)

(ii) $5.8^2 + 1.02) \times 2.5 = \ 5.8 \times 2 + 1.02) \times 2.5 = \ 11.6 + 1.02) \times 2.5 = 31.55$ (Multiplying by 2 instead of squaring)

(iii) $5.8^2 + 1.02) \times 2.5 = \ 5.64 + 1.02) \times 2.5$ (Squaring 0.8 and then adding 5)

(iv) $5.8^2 + 1.02) \times 2.5 = \ 25.64 + 1.02) \times 2.5$ (Squaring 5 and 0.8 separately and adding the result)

In Part (b), most candidates were able to calculate the basic weekly wage for one employee, but very few candidates were able to calculate the overtime wages and the number of hours worked overtime.

Solutions:

(a)	**(i) 86.65**	**(ii)** $\frac{2}{21}$		
(b)	**(i) \$380**	**(ii) \$85.50**	**(iii) \$684**	**(iv) 48 hours**

Recommendations

Teachers should ensure that students master the use of the scientific calculator in performing basic arithmetic operations on rational numbers. Attention should also be given to the order in which operations are performed.

Problems related to wages and overtime should be approached from a conceptual rather than an algorithmic viewpoint. In addition, a distinction should be made between rates and wages.

Question 2

This question tested candidates' ability to
- simplify algebraic fractions
- factorize algebraic expressions involving HCF
- change the subject of the formula
- solve worded problems involving simple linear equations

The question was attempted by 99 per cent of the candidates, 6.51 per cent of whom earned the maximum available mark. The mean mark was 4.40 out of 12.

The performance of candidates on this question was generally unsatisfactory. In Part (a), candidates were generally able to simplify the algebraic fractions, although there was a high incidence of $\frac{6x-5x}{15} = \frac{1}{15}$.

For Part (b), the majority of candidates were able to obtain a common factor even when they could not follow through to obtain the second factor. In some cases, candidates attempted to group and wrote $a^2b + 2ab = a(a+b) + 2(a+b) = (a+2)(a+b)$.

For Part (c), a large proportion of candidates attempted to transpose *r* before clearing the fraction, so $q = \frac{p^2-r}{t}$ became $q + r = \frac{p^2}{t}$. In general, candidates were unable to transpose *t* and to find *p* from p^2.

The primary difficulty experienced in Part (d) was writing an expression for the total number of donuts sold. It was common to see $5 \times 2x + 3$ for the number sold in 5 large boxes instead of $5(2x+3)$. A considerable number of candidates used trial and error to arrive at the number of donuts in each type of box.

<u>Solutions</u>:

(a) (i) $\boldsymbol{\frac{x}{15}}$

(b) $\boldsymbol{ab(a+b)}$

(c) $\boldsymbol{p = \sqrt{qt+r}}$

(d) (i) $\boldsymbol{8x + 5(2x+3)}$

(ii) a) $\boldsymbol{number\ in\ small\ box = 10 \quad b)\ number\ in\ large\ box = 23}$

Recommendations

When factorizing, students should practise dividing each of the expressions separately by the common factor.

Teachers should provide students with adequate examples of finding the inverse operations, as well as the operation associated with each quantity or variable to be transposed.

<u>Question 3</u>

This question tested candidates' ability to

- apply the laws of indices to simplify expressions
- calculate the volume of a right rectangular prism
- convert from litres to cubic centimetres
- solve problems involving volume

The question was attempted by 99.3 per cent of the candidates, 2.2 per cent of whom earned the maximum available mark. The mean mark was 3.61 out of 11.

Candidates performed satisfactorily when applying the laws of indices for multiplication, although there were a few candidates who did not multiply the coefficients.

The formula for calculating the volume of the rectangular prism was widely known and correctly applied. However, some candidates omitted the units of volume from their calculations. In general, candidates did not consider the number of cartons to be a discrete quantity. In addition, a significant number of candidates could not distinguish between diameter and radius in their attempt to calculate the volume of the cylinder, and as a consequence, the diameter was substituted in place of the radius.

Solutions:

(a) $\mathbf{14p^7q^4}$

(b) (i) $\mathbf{240\ cm^3}$ **(ii)** $\mathbf{13\ cartons}$ **(iii)** $\mathbf{12.2\ cm}$

Recommendations

Students should be reminded of the importance of writing the units of any measurement derived from calculations. Further, the difference between discrete and continuous variables should be emphasized.

Question 4

This question tested candidates' ability to

- identify numbers which are prime and numbers which are odd
- draw Venn diagrams to show the relationship between sets of numbers
- construct an angle of 60 degrees using ruler and compasses only
- use a protractor to measure an angle
- construct a triangle and a parallelogram

The question was attempted by 98.3 per cent of the candidates, 0.94 per cent of whom earned the maximum available mark. The mean mark was 5.10 out of 12.

Candidates were proficient at listing the odd numbers between 4 and 12. They, however, were generally unaware that 1 is not a prime number and that 2 is a prime number. Some candidates did not understand that the universal set included the sets of prime numbers and the set of odd numbers under consideration, and proceeded to draw a third set to represent the universal set.

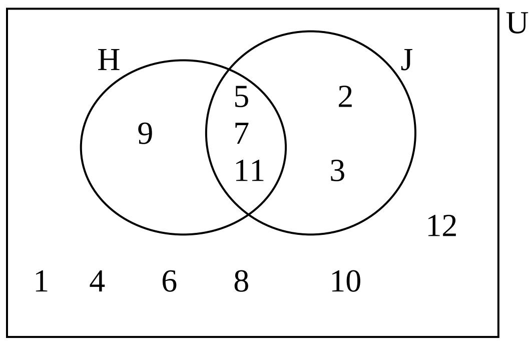
Most candidates were able to draw lines of length 9 cm and 7 cm. However, they experienced difficulty with determining the measure of $M\hat{N}L$ and some candidates gave the length of LM instead.

Although the majority of candidates who attempted this part of the question demonstrated knowledge of the properties of a parallelogram, many of them failed to arrive at the position of K by construction.

<u>Solutions</u>:

(a) (i) $\boldsymbol{H = \{5, 7, 9, 11\}}$ **(ii)** $\boldsymbol{J = \{2, 3, 5, 7, 11\}}$

iii)



(b) (ii) $\angle \boldsymbol{MNL = 48 \pm 1}$

Recommendations

Teaching the historical background to the development of our decimal number system should provide candidates with greater motivation to study the system of numbers, identify their various subsets and understand the relationships between the subsets.

Construction of polygons using geometrical tools must be practised if students are to be proficient at constructing the simple geometrical figures including angles of 30, 45, 60, 90 and 120 degrees.

<u>Question 5</u>

This question tested candidates' ability to

- determine the gradient of a line from a given equation
- obtain the equation of the line which is perpendicular to a given line
- solve problems involving a mapping and a given function

The question was attempted by 80.6 per cent of the candidates, 4.42 per cent of whom earned the maximum available mark. The mean mark was 2.87 out of 11.

In Part (a), candidates had difficulty determining the gradient of the line, even though a number of candidates attempted to rearrange the equation to the form $y = mx + c$. Further, candidates were unfamiliar with the concept that the product of the gradients of perpendicular lines is -1.

Candidates demonstrated good proficiency in using the given function and known mappings to calculate the values of k and $f(3)$ using a variety of strategies including trial and error.

Solutions:

(a) **(i)** $\mathbf{\frac{2}{3}}$ $\quad$ $(\boldsymbol{ii})\ \boldsymbol{y = \frac{-3}{2}x + 13}$

(b) $(\boldsymbol{i})\ \boldsymbol{k = 5}$ $\quad$ $(\boldsymbol{ii})\ \boldsymbol{f(3) = 4}$ $\quad$ $(\boldsymbol{iii})\ \boldsymbol{x = 10}$

Recommendations

Changing the subject of the formula is an essential skill in algebra and as such should be given appropriate attention.

Students should approach the concept of Coordinate Geometry by actually drawing graphs and be allowed to discover the relationships between parallel and perpendicular lines from this graphical approach.

Question 6

This question tested candidates' ability to

- read and interpret data presented on a line graph
- calculate the mean of a data set
- interpret the trend presented on a line graph

The question was attempted by 98.8 per cent of the candidates, 17 per cent of whom earned the maximum available mark. The mean mark was 7.94 out of 11.

Candidates performed satisfactorily on this question. They showed good proficiency at reading the values from the graph, determining the sales for the month of June and calculating the mean. However, there were some candidates who stated the median instead of the mean and others who divided the total sales by 2 instead of 5 to calculate the mean.

Candidates had difficulty stating the two consecutive months in which the largest decrease in sales occurred.

Solutions:

(i)

Month	**Jan**	**Feb**	**Mar**	**Apr**	**May**
Sales in Thousands $	**38**	**35**	**27**	**15**	**10**

(ii) **March and April** **(iii)** **$ 25000** **(iv)** **$ 25000**

Recommendations

The different measures of central tendency should be thoroughly reviewed by students and they should be encouraged to describe the trends and patterns observed on charts and statistical diagrams.

Question 7

This question tested candidates' ability to

- read and write coordinates from the Cartesian plane
- describe the single transformation which maps one triangle onto another in the plane
- enlarge a triangle given the centre of enlargement and the scale factor
- determine the area of an image given the area of the object
- describe the relationships between the object and the image under an enlargement

The question was attempted by 90.9 per cent of the candidates, less than one per cent of whom earned the maximum available mark. The mean mark was 2.09 out of 12.

Candidates performed unsatisfactorily on this question. They demonstrated some competence in reading the coordinates of R and R' and correctly stated that the transformation which maps triangle RST onto triangle $R'S'T'$ is a reflection. They, however, could not go on to completely describe the reflection.

Candidates recognized that a triangle and its image under a reflection are similar, but arriving at a second geometrical relationship proved difficult.

Very few candidates used the correct point (0, 2) as the centre of enlargement and many of them proceeded to use a matrix method which presumed that (0, 0) was the centre.

Most candidates could not relate the area of the object to its image based on the scale factor.

Solutions:

(i) $\boldsymbol{R(2,4)}$ $\boldsymbol{and}$ $\boldsymbol{R'}$ $\boldsymbol{(2,0)}$ **(ii) Reflection in the line** $\boldsymbol{y=2}$

(iii) b) $\boldsymbol{Area = 36\ square\ units}$

Recommendations

Students are encouraged to practise enlarging objects from any point in the plane using different scale factors. It is also useful to employ concrete objects in the teaching and learning of transformations, paying attention to the relationships between object and image in each case.

Further, it might be beneficial to integrate the teaching of transformations with the properties of similar and congruent figures.

Question 8

This question tested candidates' ability to

- determine the possible dimensions of a rectangle given its area
- determine the dimensions of a rectangle with the maximum area for a given perimeter

The question was attempted by 90 per cent of the candidates, 2.41 per cent of whom earned the maximum available mark. The mean mark was 5.35 out of 10.

Candidates performed satisfactorily on this question. They generally knew how to draw the rectangles for the given areas and how to complete the table to show the dimensions of the given rectangles. They were also able to determine the dimensions of length and width which would produce a certain perimeter even though, in several cases, it was not the desired perimeter of 24 units.

The major challenge to candidates was determining the dimensions of a rectangle that would give the maximum area for a perimeter of 36 cm. Candidates generally did not recognize that a square is a special rectangle.

Solutions:

Rectangle	**Length**	**Width**	**Area (square units)**	**Perimeter (units)**
A	**10**	**2**	**20**	**24**
B	**9**	**3**	**27**	**24**
C	**8**	**4**	**32**	**24**
D	**6**	**6**	**36**	**24**
E	**9**	**9**	**81**	**36**

Recommendations

Teachers should emphasize the properties of plane shapes with reference to similarities and differences between them. The fact that a square is a rectangle should also be reinforced.

Students should be provided with practice exercises involving drawing figures and calculating the respective areas and perimeters.

Optional Section

Question 9

This question tested candidates' ability to

- calculate the value of a function for a given value of its domain
- determine the inverse of a rational function
- write an expression for the composite of two functions
- use the method of completing the square to write an expression for a quadratic function expressed as $ax^2 + bx + c$
- determine the maximum value, the axis of symmetry and the x-intercepts of a quadratic function

The question was attempted by 73 per cent of the candidates, less than one per cent of whom earned the maximum available mark. The mean mark was 3.01 out of 15.

Candidates correctly substituted 5 into the function $f(x)$ but were sometimes unable to simplify to obtain the answer of $\frac{3}{5}$. Finding the inverse of the rational function proved difficult primarily because candidates lacked the algebraic skills to make y the subject of the formula after having interchanged x and y in the equation. They were also generally unable to write the composite function in the correct order.

In attempting to write $ax^2 + bx + c$ in the form $k - a(x + h)^2$, the formulae $h = \frac{-b}{2a}$ and $k = \frac{4ac - b^2}{2a}$ were widely used to good effect. However, among those who chose to complete the square, difficulties were encountered when dealing with the negative sign. Many candidates extracted the -1 coefficient of x^2 but were unable to correctly reinsert it. As a consequence, the expressions $-10 - 4(x + 3)^2$ and $-10 - 1(x - 3)^2$ resulted.

Having arrived at an expression in the form required, candidates in general could not identify the maximum value nor state the equation of the axis of symmetry. They, for the most part, attempted the solution of the quadratic equation by applying the quadratic formula but experienced challenges inserting the correct sign for each root of the equation.

Solutions:

(a) **(i)** $\frac{3}{5}$ **(ii) a)** $f^{-1}(x) = \frac{7}{2-x}$ **b)** $gf(x) = \sqrt{\frac{5x-7}{x}}$

(b) **(i)** $10 - (x + 3)^2$ **(ii) a)** 10 **b)** $x = -3$ **(iii)** $x = -6.16$ *or* $x = 0.16$

Recommendations

Candidates should be apprised of the purpose for writing the quadratic expression in the form suggested. Attention should be paid to the maximum or minimum value of such a function and the value of the variable for which this occurs.

Mastery of algebra is an essential skill for all students of mathematics at this level. Candidates therefore need to develop competence in a wide range of knowledge and skills in algebra.

Question 10

This question tested candidates' ability to

- use circle theorems and properties of a circle to determine the measure of an angle
- illustrate information on bearings
- apply the sine and cosine rules to calculate angles and distances

The question was attempted by 48.7 per cent of the candidates, 1.09 per cent of whom earned the maximum available mark. The mean mark was 3.09 out of 15.

Part (a) was poorly done. Although most candidates deduced that triangle OFG is isosceles, and as a result were able to arrive at the correct value for angle OGF, they did not recognize that GFED was a cyclic quadrilateral and as a consequence could not apply the cyclic quadrilateral theorem.

In Part (b), very few candidates were able to generate an accurate diagram from the information given. Those who drew correct diagrams recognized when to use the cosine and the sine formulae for finding the length of JL and the measure of angle JLK. Many of them, however, proceeded to incorrectly apply both formulae. Most candidates did not know which angle represented the bearing of J from L.

Solutions:

(a) (i) 31°   (ii) 56°   (iii) 124°
(b) (ii) a) 144   b) 174.15 km   c) 245.7°

Recommendations

Teachers should engage students in more practice on problems associated with circle theorems. Candidates need to approach the topic of bearings from a practical standpoint. Applications of the sine and cosine formulae need to be reinforced.

Question 11

This question tested candidates' ability to

- obtain the 2×2 matrix which transforms two points in the plane to their images
- use a matrix to obtain the image of a given point in the plane
- describe the transformation defined by a given matrix
- derive a displacement vector
- prove that three given points are collinear

The question was attempted by 39.3 per cent of the candidates, 0.13 per cent of whom earned the maximum available marks. The mean mark was 1.55 out of 15.

In Part (a), candidates demonstrated a high level of proficiency in writing the coordinates of a point as a position vector, but thereafter, they experienced considerable difficulty. They were unable to set up a system of equations which could be solved to produce the components of the matrix. Further, many candidates could not obtain the image of a point under the transformation described by the matrix. Describing the transformation, M, proved difficult.

In Part (b), candidates failed to obtain the vectors OR and RS. In addition, proving that P, R and S are collinear was outside of their level of competence.

Solutions:

(a) (i) $\begin{pmatrix} \mathbf{0} & \mathbf{1} \\ \mathbf{-1} & \mathbf{0} \end{pmatrix}$ **(ii)** $\boldsymbol{Z = \;\; -1, 5)}$ **(iii) Clockwise rotation of** $\mathbf{90}$ **about the origin, O.**

$\boldsymbol{b)}$ **(i) a)** $\overrightarrow{\boldsymbol{OP}} = \begin{pmatrix} \mathbf{2} \\ \mathbf{7} \end{pmatrix}$ **b)** $\overrightarrow{\boldsymbol{OR}} = \begin{pmatrix} \mathbf{6} \\ \mathbf{4} \end{pmatrix}$

$\mathbf{ii)}$ **a)** $\overrightarrow{\boldsymbol{RS}} = \begin{pmatrix} \mathbf{8} \\ \mathbf{-6} \end{pmatrix}$

Recommendations

Teachers should engage students in problems associated with transformations using matrices, position and displacement vectors and proving that three points are collinear.

CARIBBEAN EXAMINATIONS COUNCIL

REPORT ON CANDIDATES' WORK IN THE SECONDARY EDUCATION CERTIFICATE EXAMINATION

MAY/JUNE 2011

MATHEMATICS
GENERAL PROFICIENCY EXAMINATION

GENERAL COMMENTS

The General Proficiency Mathematics examination is offered in January and May/June each year. There was a candidate entry of approximately 90,000 in May/June 2011. Thirty-five per cent of the candidates earned Grades I–III. The mean score for the examination was 71.43 out of 180 marks.

DETAILED COMMENTS

Paper 01 – Multiple-Choice

Paper 01 consisted of 60 multiple choice items. This year, 180 candidates earned the maximum available score of 60. The mean mark for this paper was 32.32 out of 60 marks.

Paper 02 – Structured Questions

Paper 02 comprised two sections. Section I consisted of eight compulsory questions totalling 90 marks. Section II consisted of three optional questions: one each from Algebra, Relations, Functions and Graphs; Measurement, Trigonometry and Geometry; and Vectors and Matrices. Candidates were required to answer any two questions from this section. Each question in this section was worth 15 marks.

Thirty-seven candidates earned the maximum available score of 120 marks on Paper 02. Approximately 19 per cent of the candidates earned at least half of the maximum mark on this paper.

Compulsory Section

Question 1

This question tested candidates' ability to

- add, subtract, multiply and divide fractions involving mixed numbers and decimals
- express a value to a given number of significant figures
- express one quantity as a fraction of another
- solve problems involving invoices and shopping bills

The question was attempted by 99 per cent of the candidates, 18.8 per cent of whom earned the maximum available mark. The mean mark was 7.45 out of 11.

The responses to this question were generally good. Most candidates demonstrated a high level of proficiency in computing the common denominator and one correct numerator when adding fractions although some of them could not convert a mixed number to an improper fraction. The candidates demonstrated competence with the the application of the order of operations although many could not divide one fraction by another. For example, $\frac{27}{8} \div \frac{9}{2}$ was often written as $\frac{8}{27} \times \frac{9}{2}$ or $\frac{9}{2} \times \frac{8}{27}$. Several candidates interpreted $\sqrt{0.0256}$ as 0.0256^2.

About ten per cent of the candidates made no attempt at calculating the per cent VAT which is an indication that they could not interpret the relevant information given in the table. For those who attempted this part of the question, the major error was dividing the amount paid as VAT by the total amount of the bill rather than by the sub-total before VAT.

Solutions

(a) (i) $\frac{3}{4}$ (ii) 0.83

(b) \$15.60; \$13.20; 6; 12%

Recommendations

Teachers should provide more opportunities for students to practise solving problems with operations involving fractions. They should encourage and teach the use of the calculator in simplifying expressions related to computations with fractions, squares and square roots.

Question 2

This question tested candidates' ability to

- simplify simple algebraic fractions
- perform binary operations
- factorize algebraic expressions
- solve problems involving direct variation

The question was attempted by 98 per cent of the candidates, 6.6 per cent of whom earned the maximum available mark. The mean mark was 4.63 out of 12.

Performance on this question was generally unsatisfactory. Although a large percentage of candidates were able to apply the LCM to the algebraic fraction, they were unable to follow through with expanding, collecting and simplifying the terms. Substituting the numbers into the binary operation and factorizing the expression $xy^3 + x^2y$ were well done; but many candidates were unable to successfully complete the task of factorizing by grouping the expression $2mh - 2nh - 3mk + 3nk$.

Solutions

(a) (i) $\frac{7x - 5}{12}$

(b) 25

(c) (i) $xy(y^2 + x)$ (ii) $(2h - 3k)(m - n)$

(d) $a = 30$; $b = 8$

Recommendations

Teachers should ensure that the algebra of directed numbers is adequately mastered by students. The basic principles of algebra including understanding algebraic terms, expanding brackets, factorizing and collecting terms should be reinforced and tested regularly.

Question 3

This question tested candidates' ability to

- determine elements in the intersection, union and complement of sets
- solve problems involving the use of Venn diagrams
- draw and measure angles and line segments accurately using appropriate geometrical instruments
- construct lines, angles and polygons using appropriate geometrical instruments

The question was attempted by 97 per cent of the candidates, 9.4 per cent of whom earned the maximum available mark. The mean mark was 5.21 out of 11.

The performance of candidates on this question was generally satisfactory. Candidates easily identified the number of students who studied neither Art nor Music, and the number who studied Music only. Nevertheless, many of them encountered difficulty identifying the universal set as representing all the elements in the Venn diagram. Hence, the 4 students who studied neither Art nor Music were excluded from the sum of the elements. In some cases, candidates formulated the equation to be solved for *x* but were unable to solve the equation.

In Part (b), candidates were able to draw and measure the required straight lines, but demonstrated little proficiency in using the protractor to measure the required angles. Moreover, they seemed not to recognize that 125° is an obtuse angle.

Solutions

(a) (i) 4 (ii) 9 (iii) 14

(b) (ii) GH = 8.5 ± 0.2 cm.

Recommendations

Teachers should provide students with more practice in writing and solving algebraic equations in one unknown; they should encourage the use of authentic situations that would involve displaying information in a Venn diagram. The correct use of all geometrical instruments must be taught and reinforced.

Question 4

This question tested candidates' ability to

- solve a simple linear inequation
- determine the length and perimeter of a square
- determine the radius and area of a circle

The question was attempted by 92 per cent of the candidates, 2.1 per cent of whom earned the maximum available mark. The mean mark was 2.54 out of 10.

Generally, the performance of candidates on this question was unsatisfactory. In Part (a), few candidates were able to get the sign of the inequality correct when dividing by the negative coefficient of x. This posed some difficulty in answering Part (a) (ii) especially if $x < -2$ was offered as a solution for Part (a) (i).

A few candidates were able to determine the perimeter of the square from the length calculated for the side of a square. Similarly, the area of the circle was easily computed from the radius.

Solutions

(a) (i) $x > -2$ (ii) $x = -1$

(b) (i) a) 11 cm b) 44 cm.

(ii) a) 7 cm b) 154 cm^2.

Recommendations

Candidates need to be exposed to more practical sessions on transposition and on the division by a negative value across an inequality sign. In addition, they would benefit from more practical work in aspects of measurement such as area and perimeter.

<u>Question 5</u>

This question tested candidates' ability to

- identify the relationship between an object and its image after an enlargement
- use Pythagoras' Theorem to solve problems
- use trigonometric ratios in the solution of right-angled triangles in the physical world
- calculate the area of a triangle

The question was attempted by 83 per cent of the candidates, 4.2 per cent of whom scored the maximum available mark. The mean mark was 2.72 out of 12.

The performance of candidates on this question was unsatisfactory. A large number of candidates were unable to determine the scale factor given the dimensions of the two similar triangles. However, they were able to multiply the value of the scale factor by the length of OM to calculate the length of its image.

Many of the candidates had some knowledge of Pythagoras' Theorem and when to use it, although some candidates did not apply it correctly. Most candidates realized that a trigonometric ratio was to be used to calculate the length QS and the measure of the angle θ, but were unable to identify the correct ratio in these two instances. Some candidates used the sine and cosine rules and invariably applied them incorrectly. In Part (b) (iii), the majority of candidates attempted to find the area of the ΔPQR using ½ base x height, but were unable to find the length of PR. Since this meant using trigonometric ratios to find PS and then SR and adding, many candidates did not carry through the calculations to the end. Candidates resorted to using the dimensions given on the diagram, for example: Area = ½ x 12.6 × 8.4 and Area = ½ × 12.6 x QS. Heron's formula was also used with little success in most cases. Candidates who attempted to use the formula, *Area of triangle = ½absinC*, could not determine the value of angle PQR and used the angle 15˚ instead.

Solutions

(a) (i) k = 2 (ii) 10 cm (iii) 20 cm

(b) (i) 3.26 m (ii) 67.2° (iii) 32.4 m^2

Recommendations

Students should be engaged in activities to help them determine the most efficient strategies for finding the solutions to problems. Mathematical terms should be used consistently when teaching a topic and in solving problems. Further, instruction in the use of trigonometric ratios should include authentic tasks where possible.

Question 6

This question tested candidates' ability to

- derive a composite function
- derive the inverse of a function
- determine the intercept of the graph of a linear function
- determine the gradient and equation of a straight line

The question was attempted by 90 per cent of the candidates, 4.3 per cent of whom earned the maximum available mark. The mean mark was 4.56 out of 12.

The majority of candidates faced significant challenges while attempting to simplify the algebraic fractions in Part (a). They knew that they needed to interchange x and y in the equation to obtain the inverse of the function $f(x)$, but having done this, they were unable to make y the subject of the equation.

Some candidates demonstrated a limited understanding of the concept of gradient, using the ratio $\Delta x \div \Delta y$ instead of $\Delta y \div \Delta x$. In addition, many candidates were unable to determine the equation of a line, resulting in many calculating the length of the line or giving a range of values between which the length of the line lies.

Solutions

(a) (i) $g\left(\frac{1}{2}\right) = -\frac{1}{2}$ (ii) $2x + 2$ (iii) $f^{-1}(x) = \frac{x-8}{6}$

(b) (i) A (-2, 3), B(4, 6) (ii) $\frac{1}{2}$ (iii) $y = \frac{1}{2}x + 4$

Recommendations

Teachers should give special attention to Mathematical terminology and symbolism since many concepts, definitions and symbols appear to be misinterpreted by students. Special attention should also be given to simplifying algebraic expressions and transposing equations.

Question 7

This question tested candidates' ability to

- complete a cumulative frequency table for grouped data
- draw a cumulative frequency graph
- estimate the median of a data set by using a cumulative frequency graph
- determine simple probability using the cumulative frequency graph

The question was attempted by 88 per cent of the candidates, 3.8 per cent of whom earned the maximum available mark. The mean mark was 4.18 out of 12.

Generally, the performance of candidates on this question was unsatisfactory. Some candidates demonstrated competence in correctly completing the cumulative frequency table. In addition, they were able to use the given scales correctly and to plot the vertical coordinates of the points on the graph, although many candidates plotted the frequency instead of the cumulative frequency. Some candidates attempted to calculate the median mass of the packages from the table rather than estimating it from the graph.

Solutions

(a)

Mass (kg)	**No. of Packages**	**Cumulative Frequency**
1–10	12	12
11–20	28	40
21–30	30	70
31–40	22	92
41–50	8	100

(c) (i) Median = 24 kg (ii) $\frac{80}{100}$

Recommendations

Teachers should expose students to the drawing and interpretation of cumulative frequency curves. Authentic tasks are a useful tool to use in this regard. Attention must be given to the calculation of simple probability in a variety of situations.

Question 8

This question tested candidates' ability to

- generate a term of a sequence
- derive a general rule given the terms of a sequence
- solve problems involving concepts in number theory

The question was attempted by 96 per cent of the candidates, 6.6 per cent of whom earned the maximum available mark. The mean mark was 6.03 out of 10.

The performance of candidates on this question was satisfactory. Almost all of the candidates who attempted this question were able to draw the fourth diagram in the sequence, calculate the number of sticks in the sixth diagram and the number of thumb tacks in the seventh diagram. In addition, most of the candidates were able to determine the pattern connecting the number of sticks and the number of thumb tacks, and hence complete the given table. However, some candidates were unable to use the order of operations to write the rule to correctly show the relationship between *t* and *s*.

Solutions

(b) (i) 24 (ii) 22

(c)

	No of Sticks ***s***	**Rule Connecting** ***t* and *s***	**No. of Thumb Tacks** ***t***
(i)	52	1 + (¾ × 52)	40
(ii)	72	1 + (¾ × 72)	55

(c) $t = 1 + (¾)\, s$

Recommendations

Teachers should expose students to more concrete experiences in discovering number patterns and sequences, including pictorial representations. They should give students more opportunities to practise forming generalizations from number patterns.

Optional Section

<u>Question 9</u>

This question tested candidates' ability to

- solve a pair of equations in two variables when one is linear and the other non-linear
- determine the maximum or minimum value of a quadratic function expressed in the form $a(x + h)^2 + k$
- interpret a speed-time graph to determine time, speed and distance

The question was attempted by 57 per cent of the candidates, one per cent of whom earned the maximum available mark. The mean mark was 2.44 out of 15.

In an attempt to solve the pair of simultaneous equations, most candidates recognized that some strategy was needed to eliminate one variable, but they lacked the skills needed to simplify the algebraic terms and to transpose these terms. Hence, candidates were unable to obtain an equation in one variable. The majority of the candidates could not complete the square to find the minimum value of the function. Many candidates used the formulae for *h* and *k*, which were very often not stated correctly and therefore produced incorrect answers.

In Part (b), candidates found it difficult to find the distance travelled from the speed-time graph. Several of them used the formula $d = s \times t$ instead of calculating the area under the curve. Many candidates interpreted the shape of the graph in the second stage to be a level road and did not recognize that the gradient was zero.

Solutions

(a) (1, 3) (– 3, 15)

(b) (i) $4(x – 1)^2 – 6$ (ii) Minimum value = – 6 (iii) Minimum at x = 1

(c) (i) 20 seconds (ii) gradient = 0, constant speed (iii) 210 m

Recommendations

Teachers should provide students with adequate practice in solving a pair of equations in two variables when one equation is quadratic and the other linear. Students also need more practice in expressing a quadratic function in the form $a(x + h)^2 + k$, identifying the minimum value of a given function and the value of x for which this minimum occurs.

Question 10

This question tested candidates' ability to

- use the properties of circles and circle theorems to determine the sizes of angles
- use the sine and cosine rules to solve problems involving triangles
- solve problems involving bearings

The question was attempted by 45 per cent of the candidates, one per cent of whom earned the maximum available mark. The mean mark was 3.38 out of 15.

Candidates generally knew that the measure of angle XYZ = 116^{o} but incorrectly stated the reason for this being so. Most of them were unable to interpret and apply the alternate segment theorem to correctly find the value of angle YXZ.

In Part (b), the majority of the candidates correctly calculated the value of x. However, candidates experienced difficulty calculating the distance RP and the bearing of R from P.

Solutions

(a) (i) 116° (ii) 23° (iii) 26°

(b) (i) 76° (ii) 299 km (iii) 218.5°

Recommendations

Teachers should use a systematic approach to provide students with sufficient exposure and practice in solving problems based on circles and circle theorems, the use of the sine rule and cosine rule to solve problems and solving practical problems involving bearings.

Question 11

This question tested candidates' ability to

- evaluate the determinant of a 2 × 2 matrix
- derive the inverse of a non-singular 2 × 2 matrix
- perform multiplication of matrices by a scalar
- determine a 2 × 2 matrix associated with a given transformation
- combine vectors
- use vectors to solve problems in geometry

The question was attempted by 48 per cent of the candidates, less than one per cent of whom earned the maximum available mark. The mean mark was 1.93 out of 15.

The performance of candidates on this question was generally unsatisfactory. Very few candidates knew how to determine, by matrix algebra, the transformation matrix which maps two given points onto given images. Several candidates tried to solve the problem by inspection. Many of the candidates did not have the correct order to perform the matrix multiplication. However, the most challenging part of the question was proving the three points to be collinear.

Solutions

(a) $\frac{1}{2}\begin{pmatrix} 4 & -5 \\ -2 & 3 \end{pmatrix}$

(b) (i) $a = 1$, $b = -1$ (ii) Clockwise Rotation of 90 degrees about (0,0)

(c) (i) a) $-b + a$

b) $\frac{1}{3}(-b + a)$

c) $\frac{1}{3}(b + 2a)$

Recommendations

Teachers should provide students with more guided practice mapping various 2 × 2 matrices with their associated transformations and solving problems involving vectors. The use of vectors to solve various problems in geometry should also be emphasized.

CARIBBEAN EXAMINATIONS COUNCIL

REPORT ON CANDIDATES' WORK IN THE SECONDARY EDUCATION CERTIFICATE EXAMINATION

JANUARY 2012

MATHEMATICS
GENERAL PROFICIENCY EXAMINATION

GENERAL COMMENTS

The General Proficiency Mathematics examination is offered in January and May/June each year. There was a candidate entry of approximately 14 200 in January 2012. Forty per cent of the candidates earned Grades I–III. The mean score for the examination was 77.14 out of 180 marks.

DETAILED COMMENTS

Paper 01 – Multiple Choice

Paper 01 consisted of 60 multiple-choice items. This year, 24 candidates each earned the maximum available score of 60. Sixty-one per cent of the candidates scored 30 marks or more.

Paper 02 – Structured Questions

Paper 02 consisted of two sections. Section I comprised eight compulsory questions totalling 90 marks. Section II comprised three optional questions: one each from (i) Algebra, Relation, Functions and Graphs; (ii) Measurement, Trigonometry and Geometry; and (iii) Vectors and Matrices. Candidates were required to answer any two questions from this section. Each question in this section was worth 15 marks.

This year, three candidates earned the maximum available mark of 120 on Paper 02. Approximately 21 per cent of the candidates earned at least half the maximum marks on this paper.

Compulsory Section

Question 1

This question tested candidates' ability to

- square and divide fractions involving mixed numbers
- determine the square root of and add decimal numbers
- write a decimal number in standard form
- solve problems involving wages and overtime.

The question was attempted by 99 per cent of the candidates, 9.6 per cent of whom earned the maximum available mark. The mean mark was 7.89 out of 12.

In general, candidates provided satisfactory responses to this question. However, some incorrect responses provided useful insights into some misconceptions held by candidates.

These included for Part (a):

(i) $\left(1\frac{3}{4}\right)^2 = 2\frac{6}{8}$ (Each digit is multiplied by 2 instead of squaring the numerator and denominator in the equivalent improper fraction)

(ii) $\left(1\frac{3}{4}\right)^2 = 1\frac{9}{16}$ (Each digit in the mixed number is squared)

In Part (b), while most candidates were able to calculate the basic weekly wage, a large proportion of candidates experienced difficulty in determining the overtime wage for one hour. Incorrect attempts included either the multiplication of the basic hourly wage by 2.5 or by 0.5 instead of by 1.5.

Solutions

(a) (i) $\frac{7}{8}$ (ii) 0.446, 4.46×10^{-1}

(b) (i) \$900 (ii) \$33.75 (iii) \$405 (iv) 16 hours

Recommendations

Teachers should provide students with opportunities to use the calculator to perform basic arithmetic operations on rational numbers. Attention should also be given to finding the square and square root of improper fractions and decimals.

<u>Question 2</u>

This question tested candidates' ability to

- solve a pair of simultaneous linear equations
- factorize quadratic functions involving the difference of squares and grouping
- solve worded problems involving simple linear equations.

The question was attempted by 99 per cent of the candidates, 2.5 per cent of whom earned the maximum available mark. The mean mark was 3.54 out of 12. The performance of candidates on this question was generally unsatisfactory.

In Part (a), which required candidates to solve simultaneous equations in two unknowns, a large proportion of candidates showed some proficiency on this objective but some were unable to correctly complete the process. Many of them made errors in attempting to eliminate one of the variables as they experienced some difficulty with directed numbers.

For Part (b), which involved the factorization of quadratic expressions, few candidates demonstrated an awareness of the difference of squares and, as a consequence, the factors were not obtained for $x^2 - 16$. Factorizing $2x^2 - 3x + 8x - 12$ was more competently done. Nevertheless, even when candidates were able to identify the common factors, they encountered difficulty with the signs.

In Part (c), few candidates were able to show that the total amount spent on the 28 tickets was $\$(15x + 420)$. Further, they did not combine the separate expressions $(28 - x) \times 15$ and $30x$ to obtain the stated result.

Solutions

(a) $x = 3, y = 2$
(b) (i) $(x - 4)(x + 4)$ (ii) $(2x - 3)(x + 4)$
(c) (i) a) $(28 - x)$ b) $30x$ c) $15(28 - x)$ (ii) $15x + 420$ (iii) 16

Recommendations

Teachers should engage students in translating worded phrases into algebraic expressions and equations. The difference between squares is a useful tool in factorization and needs special attention by teachers and students.

Question 3

This question tested candidates' ability to

- identify and list odd and prime numbers
- draw a Venn diagram to illustrate the relationship between two sets
- construct a triangle which included an angle of $45°$
- construct a perpendicular to a line from a point outside that line
- determine the measure of an angle.

The question was attempted by 99 per cent of the candidates, 1.4 per cent of whom earned the maximum available mark. The mean mark was 5.58 out of 12.

In Part (a), most candidates were able to list the odd numbers from the elements in the set but were however unable to identify the prime numbers. Generally, candidates recognized that the subsets are enclosed by the universal set and were able to represent on the Venn diagram the subsets and the members of $(A \cup B)^I$ correctly.

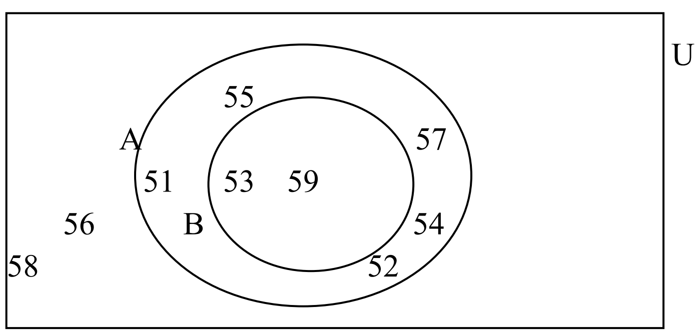
In Part (b), candidates were able to use the ruler to draw at least one line accurately and to complete and label the triangle CDE with the 45° angle at the point D. They, however, were unable to construct the 90° angle, and resorted to the use of either the set square or the protractor to draw the 90° angle, following which they used a pair of compasses to bisect that angle to obtain $\angle$CDE = 45°. However, most candidates simply used their protractor to draw the angle CDE = 45°.

Constructing the perpendicular from C to DE was outside of the level of competence for most candidates. Many of them resorted to constructing the perpendicular bisector of DE, which does not pass through C. A large proportion of candidates drew the perpendicular line from C to DE but located the point F other than at the point of intersection of the two lines.

Solutions

(a) (i) $A = \{51, 53, 55, 57, 59\}$ (ii) $B = \{53, 59\}$

(iii)



Recommendations

Teachers are advised to review sets of numbers such as prime, composite, odd, even, square, natural, integer, whole, rational, irrational and real, as well as the divisibility laws as they pertain to identifying odd and prime numbers. In addition, the appropriate representation of sets in Venn diagrams needs to be taught. Instruction in constructing polygons should also include the construction of angles and perpendicular lines.

Question 4

This question tested candidates' ability to

- determine the time interval between two times written in the 24-hour clock notation
- calculate distance in km given time in minutes and speed in km/h
- convert from litres to cubic centimetres
- calculate the height of a cuboid given the length and width of its base
- derive and solve a simple linear equation in one unknown using concepts of area and volume of a cuboid.

The question was attempted by 98 per cent of the candidates, 5.0 per cent of whom earned the maximum available mark. The mean mark was 3.91 out of 11.

Candidates were proficient at evaluating the time in minutes between 7:35 and 7:45 on the 24−hour clock. They however experienced difficulty in determining the time elapsed between 6:40 and 7:35, with the most common result being 95 minutes instead of 55 minutes.

The formula, $distance = speed \times time$, was well known but attention was not paid to the different units used in the question. The inconsistency with the use of units was again evident in Part (b). Here, candidates knew that $h = \frac{V}{A}$, but did not know how to convert litres to cubic centimetres. A frequent incorrect conversion was $4.8\,l = 480\ cm^3$. The greatest challenge candidates encountered was in expressing area in terms of a variable h, and in establishing the equation connecting the volume of the cuboid to 286.

Solutions

(a) (i) 10 minutes (ii) 55 minutes (iii) 49.5 km
(b) 16 cm
(c) (i) $4h\ cm^2$ (ii) $52h\ cm^3$ (iii) 5.5 cm

Recommendations

Teachers should give special attention to conversion of units. Emphasis should be placed on the value of a quantity based on its unit; for example: $4.8l > 300cm^3$. A common-sense approach to the calculation of time elapsed should be encouraged so that when Mathematics is applied, the result of the calculation fits the estimation.

Question 5

This question tested candidates' ability to

- use the properties of parallel lines, transversals, triangles and angles to determine the measure of angles
- state the coordinates of points on a graph
- fully describe a simple transformation given an object and its image
- determine the coordinates of the images of two points after a translation by a given vector.

The question was attempted by 92 per cent of the candidates, 3.1 per cent of whom earned the maximum available mark. The mean mark was 3.44 out of 10.

The performance of candidates on this item was generally unsatisfactory. In Part (a), candidates were able to recognize that the sum of the internal angles of a triangle is $180°$ and that alternate angles are equal. However, they were less familiar with recognizing the equality of corresponding angles. A glaring deficit in their responses was failure to give correct reasons to support their calculations.

In Part (b), candidates recognized the transformation as being a reflection but were unable to give a clear description using appropriate vocabulary. The greatest challenge was identifying the mirror line of the reflection.

Solutions

(a) (i) $72°$ (ii) $22°$ (iii) $72°$ (iv) $86°$
(b) (i) $P(2,1)$ $Q(4,3)$ (ii) Reflection in the X-axis (iii) $P^{//}(5,-5)$ $Q^{//}(7,-3)$

Recommendations

Teachers are advised to use the correct mathematical language when explaining concepts in geometry and encourage students to include reasons for the answers when performing calculations in geometry. Students should be taught to test the plausibility of their answers based on information given in the question (for example, $122°$ cannot be the measure of an acute angle). Teachers should give students adequate practice in performing transformations, and in stating the main features needed for describing a transformation.

Question 6

This question tested candidates' ability to

- complete a table of values for a quadratic function over a given domain
- plot points on a pair of axes and draw a smooth curve through the points
- use a quadratic graph to estimate the value of y for a given value of x
- state the equation of the line of symmetry associated with the graph of a quadratic function
- use a quadratic graph to estimate the minimum value of a quadratic function
- determine the roots of a quadratic equation from a quadratic graph.

The question was attempted by 92 per cent of the candidates, 2.7 per cent of whom earned the maximum available mark. The mean mark was 4.80 out of 11.

Candidates' performance on this item was generally satisfactory. The majority of them were able to calculate the missing values of y in the table of values, and even those who encountered difficulties in so doing, proceeded to correctly plot the points given to obtain the graph of the function. There were some challenges estimating, from the graph, the value of y which corresponds to a given value of x. Generally, candidates did not know how to illustrate the method used to obtain their solution by drawing lines on the graph. They demonstrated competence at reading the minimum value of the function, but were unable to determine the equation of the line of symmetry and the roots of the equation $x^2 - 2x - 3 = 0$ from the graph.

Solutions

(a) when $x = -1, y = 0$ and when $x = 2, y = -3$
(b) minimum value quadratic curve with roots at -1 and 3, y-intercept at -3, minimum point at (1, -4)
(c) $y = 2.25$
(d) (i) $x = 1$ (ii) $y = -4$ (iii) $x = -1, x = 3$

Recommendations

Teachers should provide candidates with practice in writing the equations of lines parallel to the coordinate axes. Students should be taught that curves are drawn free hand and not against straight edges. They should be exposed to writing equations and in-equations and to determine the solutions associated with each type.

Question 7

This question tested candidates' ability to

- complete a grouped frequency table to show mid-interval values and frequencies using information provided on a histogram
- identify the median class associated with a histogram
- determine from a histogram the number of items in the data set
- calculate an estimate of the mean of a grouped data set
- estimate what proportion of the data set is above a given value.

The question was attempted by 92 per cent of the candidates, 3.0 per cent of whom earned the maximum available mark. The mean mark was 4.27 out of 12.

Candidates performed satisfactorily on this question. A number of candidates were unfamiliar with the term 'modal class' and proceeded, in many instances, to state the highest frequency (25) instead of the interval (11–20) which had the highest frequency. When calculating the mean, many candidates divided by the number of intervals (5) or by 50 which is the largest limit in the table. A large number of candidates did not equate the total number of seedlings with the total frequency. For example, they added the upper class limits to obtain 150, or they summed the mid-interval values to obtain 127.5. Candidates in general did not use the sum of products to estimate the mean for the grouped frequency distribution. In many instances, the 5 mid-values were added and in several other cases the 5 frequencies were summed.

Solutions

(a)

Height in cm	**Mid-point**	**Frequency**
21–30	25.5	23
31–40	35.5	20
41–50	45.5	14

(b) (i) 11–20 (ii) 100 (iii) 24.2 cm (iv) $\frac{34}{100}$

Recommendations

Teachers need to emphasize the difference between the measures — mode, median and mean — and to distinguish between median class and modal class. They should use more practical examples to assist students in understanding the concept of probability. Further, greater emphasis should be placed on the extraction and interpretation of information from statistical graphs.

Question 8

This question tested candidates' ability to

- draw the fourth figure in a sequence of shapes given the first three figures in the sequence
- complete a table to show the terms in a sequence of numbers
- determine the term of a sequence
- derive the general equation connecting the value of the function with the term of the sequence.

The question was attempted by 95 per cent of the candidates, 10.1 per cent of whom earned the maximum available mark. The mean mark was 6.60 out of 10.

Generally, the performance of candidates on this question was satisfactory, with a large proportion of candidates recognizing the pattern of squares in the sequence. Various strategies were used to compute the number of straws used in the fourth figure. These included counting, use of formulae and following the pattern. However, some candidates were unable to deduce the correct formula and ended up with the incorrect number of straws used for the fourth figure. Trial and error was commonly used by candidates in an attempt to deduce the value of *n* for which the number of straws totalled 106. In addition, a large number of candidates omitted Part (d) of the question which required that they derive a formula to connect the number of the figure to the number of straws used.

Solutions

(b)

Figure	**Total Number of Straws**	
	Formula	**Number**
4	4(6) - 3	21
10	10(6) - 9	51

(c) 21

(d) $6n-(n-1) = 5n+1$

Recommendation

Teachers should assist students with writing formulae and the expressions derived from number sequences.

Optional Section

Question 9

This question tested candidates' ability to

- change the subject of the formula for a rational function
- determine the inverse of a rational function
- determine, for a rational function, the value in the domain that is mapped onto zero in the range
- solve problems using linear techniques.

The question was attempted by 49 per cent of the candidates, 1.0 per cent of whom earned the maximum available mark. The mean mark was 2.40 out of 15.

Candidates' performance on this question was generally unsatisfactory. While some candidates were able to correctly write down the coordinates of at least one vertex and to substitute their values in the profit function, most of the candidates did recognize the relationship between transposing the formula and finding the inverse of the function. In fact, a few got Part (a) (i) incorrect but were able to use the same technique required to find the inverse of the function. In Part a (iii), $f(0)$ was often seen instead of $f^{-1}(0)$. In Part (b), stating the inequalities proved to be problematic as seen from the following examples where instead of $x \geq 6$ and $x + y \leq 40$ respectively, many candidates wrote $x > 6$, $x = 6$, $x \leq 6$, $x < 6$ and $x + y = 40$, $x + y > 40$, $x + y < 40$, $x + y \geq 40$.

Solutions

(a) (i)$x = \frac{4y+3}{y-2}$ (ii) $f^{-1}(x) = \frac{4x+3}{x-2}$ (iii) $x = \frac{-3}{2}$

(b) (i) $x \geq 6,\ x + y \leq 40$ (ii) $(6, 2), (6, 34),\ (30, 10)$ (iii) $(30, 10)$

Recommendations

Teachers should place more emphasis on questions involving linear inequalities. The difference between the signs for different inequalities should be clearly explained when students are in junior school. When teaching the solution of linear equations, more examples on solving literal equations should be done so that students can see the relationship between solving such equations and transposing formulae.

Question 10

This question tested candidates' ability to

- calculate the angle subtended at the centre of a regular hexagon by one of its sides
- determine the area of a regular hexagon given the length of one side
- use trigonometric ratios and formulas to solve problems related to bearings and three-dimensional figures.

The question was attempted by 43 per cent of the candidates, 1.2 per cent of whom earned the maximum available mark. The mean mark was 2.89 out of 15.

Candidates' performance on this question was generally unsatisfactory. In Part (a), a large number of candidates were able to evaluate the measure of angle AOB and quite a few also knew that to find the area of the hexagon they needed to multiply the area of one triangle by 6. However, a number of candidates assumed that triangle AOB was right-angled and used OA as the height and AB as the base of the triangle. Part (b) proved more challenging for candidates as many of them could not identify the angle of elevation and the right angles in the three-dimensional drawing. It was common to see the angle PKM used as the angle of elevation instead of angle PKL. A large number of candidates knew that they could use the cosine formula for calculating the length of KM in triangle KLM. Nevertheless, even when they substituted correctly into the formula, they could not complete the response since they encountered difficulty with the use of directed numbers.

Solutions

(a) (i) 60° (ii) 65 cm^2
(b) (ii) a) 7.98 m b) 28.8 m c) 23°

Recommendations

Teachers should provide students with sufficient practice in calculating the areas of triangles using the formula $A = \frac{1}{2}ab\, SinC$. They should employ a more practical approach to teaching trigonometry involving three-dimensional situations, assisting students in differentiating between the perpendicular height and the slant height.

Question 11

This question tested candidates' ability to

- write the coordinates of points as position vectors
- derive a displacement vector
- solve problems in geometry using vectors
- add and multiply matrices
- find the inverse of a two by two matrix
- solve a matrix equation with two unknowns using the inverse method approach.

The question was attempted by 65 per cent of the candidates, 2.3 per cent of whom earned the maximum available mark. The mean mark was 4.33 out of 15.

While a number of candidates were able to write the coordinates of points as position vectors, determine displacement vectors and identify the correct route for summing vectors, they were unable to write the inverse of vectors whose additive inverses were required as part of the route. Ina addition, challenges were experienced with the multiplication of two matrices, inverting a matrix and using the inverse of a matrix to solve a system of linear equations. Many candidates resorted to re–writing the linear equations associated with the matrix equation and solving the system by the process of elimination or by substitution.

Solutions

(a) (i) a) $\overrightarrow{OA} = \begin{pmatrix} -1 \\ 3 \end{pmatrix}$ b) $\overrightarrow{OB} = \begin{pmatrix} 5 \\ 1 \end{pmatrix}$ c) $\overrightarrow{BA} = \begin{pmatrix} -6 \\ 2 \end{pmatrix}$

(ii) a) $\overrightarrow{BG} = \begin{pmatrix} -3 \\ 1 \end{pmatrix}$ b) $\overrightarrow{OG} = \begin{pmatrix} 2 \\ 2 \end{pmatrix}$

(b) (i) $\begin{pmatrix} 1 & 8 \\ 1 & 8 \end{pmatrix}$ (ii) $\begin{pmatrix} -3 & 13 \\ -1 & 11 \end{pmatrix}$

(c) (i) $\frac{1}{2}\begin{pmatrix} 1 & -2 \\ -1 & 4 \end{pmatrix}$ (ii) $x = 1;\ y = 2$

Recommendations

Teachers should utilize diagrams including the Cartesian plane in the teaching of vector algebra. They should emphasize that direction in vectors is important and ought to be stressed in addition to the magnitude. Further, they should provide students with more practice in matrix multiplication and using the matrix method to solve a pair of simultaneous equations in two unknowns.

C A R I B B E A N   E X A M I N A T I O N S   C O U N C I L

REPORT ON CANDIDATES' WORK IN THE CARIBBEAN SECONDARY EDUCATION CERTIFICATE EXAMINATION®

MAY/JUNE 2012

MATHEMATICS
GENERAL PROFICIENCY EXAMINATION

GENERAL COMMENTS

The General Proficiency Mathematics examination is offered in January and May/June each year. There was a candidate entry of approximately 95,000 in May/June 2012. Thirty-three per cent of the candidates earned Grades I–III. The mean score for the examination was 66.40 out of 180 marks.

DETAILED COMMENTS

Paper 01 — Multiple Choice

Paper 01 consisted of 60 multiple choice items. This year, 298 candidates each earned the maximum available score of 60 marks in the paper. The mean mark for this paper was 32.70 out of 60 marks.

Paper 02 — Structured Questions

Paper 02 comprised two sections. Section I consisted of eight compulsory questions for a total of 90 marks. Section II consisted of three optional questions: one each from Algebra, Relation, Functions and Graphs; Measurement, Trigonometry and Geometry; and Vectors and Matrices. Candidates were required to answer any two out of three questions from this section. Each question in this section was worth 15 marks.

This year, 13 candidates earned the maximum available score of 120 marks on the paper. Furthermore, based on the data collected, candidates were able to secure full marks in every question on the paper.

The mean mark for this paper was 33.64 out of 120 marks.

Compulsory Section

<u>Question 1</u>

This question tested candidates' ability to

- subtract and divide fractions involving mixed numbers
- express a fraction in its lowest terms
- determine percentage profit or loss given cost price and selling price
- determine cost price given selling price and percentage profit
- perform currency conversions using given rates
- solve problems involving currency conversions.

The question was attempted by 99.4 per cent of the candidates, 3.1 per cent of whom earned the maximum available mark. The mean mark was 4.60 out of 12.

In general, candidates provided fair responses but there were far too many instances of poor responses in all the areas tested. In Part (a), the majority of candidates knew that they had to calculate the Lowest Common Multiple (LCM) of the denominators but a significant number of candidates made careless errors obtaining the equivalent fractions. The following errors were noted:

- In converting the mixed number $2\frac{4}{5}$ to an improper fraction, some candidates wrote $\frac{24}{5}$ instead of $\frac{14}{5}$
- After simplifying the numerator and the denominator, far too many candidates divided their terms in the reverse order, using the numerator as the divisor instead of the denominator.

In addition, candidates did not state the fraction in lowest terms and some candidates incorrectly wrote $\frac{19}{21} = 1\frac{2}{21}$

For Part (b), the majority of candidates were able to determine the percentage loss as a percentage of the cost price but few were able to reverse this process and determine the cost price given the selling price and percentage profit. Some common errors in Part (i) were:

- Stating the loss instead of the percentage loss, a common error being \$55 – \$44 = 11% instead of \$11

- Computing the percentage loss using the selling price rather than the cost price.

In Part (ii), candidates merely subtracted the 25% from \$100 and obtained \$75 as the cost price. This was an extremely popular incorrect response, demonstrating that candidates did not know how to proceed to solve problems on percentages when the whole was not given.

For Part (c), candidates experienced great difficulty in setting up the required proportions, others made careless errors in simplifying decimal products and obtained unreasonable answers to the various parts.

Solutions

(a) $\frac{19}{21}$ (b) (i) 20% (ii) \$80

(c) (i) TT\$2.50 (ii) EC\$216 (iii) US\$96

Recommendations

When solving fraction problems involving mixed operations, teachers should encourage students to present their work in logical steps, simplifying each step as they proceed towards the solution. The use of the calculator in performing decimal computations should be emphasized. Students must also be taught estimation skills so that they can determine the reasonableness of their answers.

Question 2

This question tested candidates' ability to

- factorize algebraic expressions involving common factors, the difference of two squares and grouping
- solve a linear equation involving fractions
- solve a pair of simultaneous linear equations.

The question was attempted by 97.9 per cent of the candidates, 3.7 per cent of whom earned the maximum available mark. The mean mark was 3.79 out of 12.

Generally, performance on this question was weak, with candidates only demonstrating proficiency in a few of the basic concepts in algebra that were tested. Candidate's inability to perform simple operations with directed numbers was a general weakness throughout.

In Part (a), the inability to recognize the Highest Common Factor (HCF) in an algebraic expression was also a clear weakness. Even though candidates were able to find a common factor they were unable to determine the HCF. Candidates were unable to recognize the difference of two squares and some actually commented that the expression could not be factorized. Candidates also had difficulty in factoring the second group and ignored the negative signs, that is, obtaining $y(x-2y)$ instead of $-y(x+2y)$.

For Part (c), a variety of methods was used to solve the simultaneous equations including the use of Cramer's Rule. However, candidates experienced difficulty in identifying the correct operation (addition or subtraction) to eliminate one of the variables.

Solutions

(a) (i) $2x^2y\,(x + 3y)$ (ii) $(3x - 2)\,(3x + 2)$ (iii) $(4x - y(x + 2y)$

(b) $x = 9$ (c) $x = 4,\ y = 1$

Recommendations

Students must be taught to recognize the various types of algebraic expressions so that they can select the correct strategy for factorization. When using common factors they must also be able to recognize when an expression has been completely factorized. A consistent method of clearing fractions in an equation must be taught and these methods must be connected to arithmetical operations with fractions. Errors can also be avoided if students are taught to verify their solutions when solving simultaneous equations.

Question 3

This question tested candidates' ability to

- use a Venn diagram to solve practical problems involving two sets
- determine the number of elements in subsets involving two intersecting sets
- solve problems involving Venn diagrams using algebraic methods
- represent points on a plane given bearings
- solve problems involving bearings using Pythagoras' theorem and trigonometric ratios

The question was attempted by 97.6 per cent of the candidates, 4.3 per cent of whom earned the maximum available mark. The mean mark was 4.63 out of 12.

Performance was generally unsatisfactory; however, a significant number of candidates scored higher in Part (b) than in Part (a).

In Part (a), only a small percentage of candidates was able to obtain $(30 - 9x)$ for the number of students who play only tennis. Many candidates incorrectly obtained $y = 30 - x$ as the answer.

Candidates also had difficulty relating parts of a set to the whole set. In many instances, the expression given did not match the information offered by the candidates in their Venn diagram. In such cases, incorrect equations resulted in unrealistic solutions such as fractions and the candidates were unable to deduce that their answers were incorrect.

In Part (b), the majority of candidates produced correctly labelled diagrams showing the positions of Q and R. Although candidates generally recognized the use of Pythagoras' theorem to solve the triangle, they had problems applying it to the problem.

Candidates were also unable to apply the correct trigonometrical ratio for finding an angle. Many candidates made serious omissions in presenting their work, such as $tan = \frac{15}{20}$ and $\frac{20}{15}$. Some candidates applied the sine and cosine rules but very few arrived at a correct answer using these methods.

A significant number of candidates attempted to find the angle QRP instead of QPR, indicating that basic skills in naming angles are missing.

Solution

(a) (i) $y = 30 - 9x, \quad z = 4$ (ii) a) $x + 34$ b) $x = 2$

(b) (i) (ii) 25 km (iii) 53^0

Recommendations

In interpreting and constructing Venn diagrams, teachers need to ensure that students understand the requisite vocabulary. For example, they need to know how to interpret the word *only* when using Venn diagrams. When responding to questions in solving for algebraic unknowns, they must be guided to differentiate between an expression and an equation.

In solving right–angled triangles, candidates must ensure that their diagrams are labelled correctly with the known values clearly shown. This will facilitate problem solving and identification of the correct ratio to be used.

Question 4

This question tested candidates' ability to calculate

- the length of an arc of a circle
- the perimeter of a sector of a circle
- the area of a sector of a circle
- the volume of a prism using the area of the cross section
- the mass of a prism given its density.

The question was attempted by 81.4 per cent of the candidates, 2.3 per cent of whom earned the maximum available mark. The mean mark was 2.45 out of 10.

Responses to all parts of this question were generally poor, indicating that basic concepts in measurement are generally not understood by many candidates.

In Part (a), candidates had more difficulty calculating the arc length and perimeter of the sector than calculating the area. They were able to choose the appropriate formula for the area and perimeter but the majority used 90^0 instead of 270^0 when substituting for the angle subtended. In particular, candidates displayed poor understanding of the concept of perimeter and many did not include the two radii as part of the total perimeter. In substituting for area, some candidates interpreted πr^2 as $(\pi r)^2$.

Candidates failed to make a connection between Part (a) and Part (b) and did not recognize that the area of the sector was in fact the cross–sectional area of the tin. Hence, some candidates recalculated the area. They also experienced problems substituting in the formula $V = Ah$ while weaker candidates attempted to use $l \times w \times h$.

Candidates generally disregarded the units that should have been used in the various parts of the questions.

Solutions

(a) (i) 16.5 cm (ii) 23.5 cm (iii) 28.875 cm^2

(b) (i) 577.5 cm^3 (ii) 4216 kg

Recommendations

The poor performance on this topic suggests that teachers should revisit the teaching of basic concepts in measurement rather than focus on the use of formulae. Measurement language and vocabulary need to be addressed, in particular the meaning of cross section, height and base as they relate to solids. Candidates need to understand how to calculate the perimeter, area and volume of regular as well as irregular shapes. Close attention must be also paid to the units for measuring these attributes.

Question 5

This question tested candidates’ ability to

- use mathematical instruments to construct a triangle given two angles and a corresponding side
- measure the length of a line segment
- given two points, determine
    - the gradient of the line
    - the equation of the line
    - the midpoint of the line
    - the length of the line.

The question was attempted by 84.1 per cent of the candidates, 2.1 per cent of whom earned the maximum available mark. The mean mark was 3.33 out of 12.

The performance of candidates on this question was generally unsatisfactory. In Part (a), many candidates did not use the required instruments as no construction lines were shown. Constructing the 45^0 angle posed greater difficulty for candidates than constructing the 60^0 angle. A significant number of candidates labelled their diagram incorrectly and thus failed to measure the required line.

In Part (b), many candidates were successful in calculating the gradient of the line. Candidates wrote the correct formulae for the equation and length of a line but made careless errors in substituting and simplifying the values. Determining the equation of a line proved to be most challenging for the majority of candidates. After calculating the gradient and the y-intercept, many candidates could not write down the equation of the line. A large number of candidates plotted the given points on a graph and drew a straight line but they were unable to use the diagram to answer any of the questions.

Solutions

(a) (ii) $RQ = 5.9 \pm 0.1$ cm

(b) (i) $\frac{4}{3}$ (ii) $y = \frac{4}{3}x - 2$ (iii) (3, 2) (iv) 10

Recommendations

Teachers must encourage students to make a sketch of the diagram, prior to constructing. This will allow them to review the labelling and the given information before creating an accurate diagram.

Concepts in coordinate geometry should be well grounded through the use of drawings prior to developing the formulae. The teaching of algebraic techniques such as simplification and substitution should precede topics in coordinate geometry.

Question 6

This question tested candidates’ ability to

- locate the centre of enlargement given an object and its image
- state the scale factor and coordinates of the centre of enlargement
- determine the ratio of the area of the image to the area of the object
- draw a triangle given its coordinates
- describe fully the transformation which maps a given triangle onto its image.

The question was attempted by 80.8 per cent of the candidates, 1.1 per cent of whom earned the maximum available mark. The mean mark was 2.56 out of 11.

Overall, performance on this question was poor, with many candidates displaying limited knowledge of geometric transformations.

The areas of bad performance were in locating the centre of enlargement and in describing the single transformation that mapped triangle LMN onto triangle ABC.

Candidates showed little knowledge of the method to find the centre of enlargement. A large number connected the corresponding vertices of LMN and PQR but failed to extend the lines to a point of convergence. In many cases, they connected all vertices to the origin and stated that as the centre of enlargement.

Most candidates were able to correctly recognize the scale factor as 2 but it was often written as a ratio of 1:2 or as a column vector. Weaker candidates interpreted scale factor as the scale used on the axes on the graph. Also, the centre of enlargement was incorrectly written as (5,1) instead of (1,5) and, in a few cases, as a column vector.

Candidates showed limited knowledge of the relationship between the scale factor and the ratio of the areas, that is, k^2. Many took pains to calculate the areas using lengthy methods and wasted valuable time.
Some candidates were able to accurately plot the points A, B and C but there were several cases where the points were incorrectly labelled.

Part (e) of the question was very poorly done. Candidates frequently stated two consecutive transformations that were often incorrect. Many of those who recognized the rotation had difficulties in describing it. They seemed unable to distinguish clockwise from anticlockwise and in many cases if the correct angle of 90^0 was written, the centre of rotation was omitted. A common incorrect answer was that the single transformation was an enlargement as candidates confused Part (a) with Part (e).

Solutions

(b) Scale factor = 2, centre (1, 5)

(c) 4

(e) A rotation of 90^0 in an anticlockwise direction with centre (0,0).

Recommendations

The topic of transformations appears not to have been extensively taught; students were very unfamiliar with the concept. Teachers need to emphasize the characteristics that define each transformation as well as the properties of each one. The specific vocabulary associated with each transformation needs to be emphasized.

<u>Question 7</u>

This question tested candidates' ability to

- calculate cumulative frequencies from a frequency table
- draw a cumulative frequency curve (ogive) for a set of data
- use a cumulative frequency curve to estimate the median
- use a cumulative frequency curve to determine the probability of an event.

The question was attempted by 89.6 per cent of the candidates, 3.2 per cent of whom earned the maximum available mark. The mean mark was 4.19 out of 11.

The majority of candidates were competent in completing the cumulative frequency table and using the correct scales. However, a significant number of candidates did not draw the cumulative frequency curve using the cumulative frequencies but plotted their frequencies instead. Also, some candidates plotted their cumulative curve using the lower limits and midpoints instead of the upper–class boundaries. Some even interchanged the horizontal and vertical axes.

A significant number of candidates were unable to use their cumulative frequency curve to estimate the median or the number of persons 75 years or younger who visited the clinic. Often, candidates who drew the lines on the graph failed to state the determined values. Candidates were unable to calculate the probability correctly. While some read their graph correctly, they simply left their answers as whole numbers, displaying no knowledge of the concept of probability. Some candidates incorrectly used the sum of the cumulative frequencies as the total number of persons in the sample.

Solutions

(a) 35 and 47 (c) (i) 64 years (ii) $\frac{43}{50}$

Recommendations

Teachers must ensure that students construct and interpret cumulative frequency curves from real-world data so that they can develop a sound understanding and appreciation for the statistical concepts they will encounter. In representing statistical graphs, care must be taken to explain what variables are to be plotted when setting up the axes for different types of graphs.

<u>Question 8</u>

This question tested candidates' ability to

- recognize a spatial pattern in a given sequence of drawings
- continue a pattern by drawing a given shape in the sequence
- use a pattern to generate subsequent terms in a number sequence
- use a pattern to derive a rule for the n^{th} term in a number sequence.

The question was attempted by 93.2 per cent of the candidates, 3.2 per cent of whom earned the maximum available mark. The mean mark was 6.30 out of 10.

In general, candidates' responses ranged from satisfactory to quite good. Those who had difficulty could not interpret the table but ignored the breaks in the columns and assumed that it was continuous. Hence, they treated Part b (ii), as though it were Figure 5, continuing from Figure 4.

In Part (a), almost all of the candidates were able to complete a fourth figure in the sequence. However, in Part (b), some candidates had difficulty recognizing the pattern of square numbers and multiplied by two instead. Generating the n^{th} term for the sequence posed most challenges for candidates and many used numbers instead.

Solutions

(b) (i) 16, $2(4) + 1 = 9$ (ii) 10, $2(10) + 1 = 21$

(iii) 400, $2(20) + 1 = 41$ (iv) n^2, $2n + 1$

Recommendations

Teachers should continue to engage students in activities involving pattern recognition and continuation. Students should be encouraged to generate their own patterns using concrete objects or abstractly. The use of algebra as a tool to describe and generalize a rule should be emphasized.

Question 9

This question tested candidates' ability to

- solve a pair of simultaneous equations in which one is linear and the other is a quadratic
- deduce whether or not a given line is a tangent to a curve
- construct inequalities for given conditions
- determine the region satisfied by a system of three inequalities
- state the coordinates of the vertices of the region satisfied by inequalities
- determine the maximum profit.

The question was attempted by 45.7 per cent of the candidates, 1.0 per cent of whom earned the maximum available mark. The mean mark was 2.72 out of 15.

Generally, candidates' responses to this question were unsatisfactory. While many candidates knew that they had to eliminate one variable in solving the simultaneous equations and proceeded to do so, weak algebraic techniques prevented them from arriving at correct solutions. Simplifying the quadratic equation posed problems for the weaker candidates and quite a large number of them were unable to solve the equation. The majority of candidates were able to state that the line $y = 8 - x$ is a tangent to the curve but only a small minority was able to justify why this was so.

Part (b) had slightly better responses than Part (a). Writing the inequalities was the most difficult part of the question for candidates but they were successful in identifying the common region. Naming the vertices of the region and using the profit equation to determine the maximum profit was attempted by the more able candidates. However, there were far too many errors in reading the coordinates of the vertices of the region and some candidates stated the maximum without showing evidence of substitution of the other vertices in the profit equation.

Solutions

(a) (i) $x = -4, -4 \quad y = 12$

(ii) The line is a tangent to the curve because it touches the curve at one point only.

(b) (i) $y \geq \frac{1}{2}x, \; x \geq 2,$ (iii) (2, 1) (2, 10) (8, 4) (iv) $46

Recommendations

Translating worded phrases into inequalities needs to receive more attention. Students must recognize the difference between a worded expression that represents an equation and one that represents an inequality. They should be exposed to performing translations in both directions, words to symbols and symbols to words. In addition, they should also be taught how to verify their algebraic expressions by substituting numbers, for example, if *x* is at least 8, then *x* must be 8, 9, 10, and so on.

<u>Question 10</u>

This question tested candidates' ability to

- solve for unknown values in a triangle using the sine and cosine rule

- solve geometric problems using properties of

    - lines and angles
    - circles and circle theorems
    - congruent triangles
    - isosceles triangles.

The question was attempted by 47.1 per cent of the candidates, 1.3 per cent of whom earned the maximum available mark. The mean mark was 2.72 out of 15.

Performance on this question was generally unsatisfactory.

In Part (a), a, large number of candidates recognized the use of the sine and cosine rules but experienced difficulty applying them. For example, candidates made incorrect substitutions of values in these formulae and could not apply basic techniques to solve for the unknown. A common error in using the cosine rule is illustrated below:

$$49 = 64 + 100 - 160cosQ$$

$$49 = 164 - 160cosQ$$

After this step, candidates wrote

$49 = 4\ cosQ$ instead of $160cosQ = 164 - 49$

For Part (b), many candidates experienced difficulty obtaining OUZ, failing to recognize the properties of isosceles triangles. Circle theorems were also hardly recognized and some candidates used rather lengthy procedures to determine the measure of the unknown angles. Many candidates wrote answers only, omitting working or explanations to support their answers.

Recommendations

In preparing students to answer optional questions, teachers must ensure that they have the necessary foundation skills. In these questions the students must have mastered basic algebraic skills and basic concepts in geometry. In addition, students must be encouraged to give explanations, making reference to geometric theorems, when solving problems involving angle calculations.

Solutions

(a) (i) 6 cm (ii) 44^0

(b) (i) a) 35^0 b) 55^0 c) 20^0 (ii) a) ΔYOX b) $\Delta\ ZUX$

Question 11

This question tested candidates’ ability to

- combine vectors to determine resultants
- state the geometrical relationship between two vectors
- sketch the relative position of given points
- solve for an unknown variable in a matrix equation
- determine the inverse of a 2 x 2 matrix
- use a matrix method to solve simultaneous equations.

The question was attempted by 53.1 per cent of the candidates, 1.7 per cent of whom earned the maximum available mark. The mean mark was 3.28 out of 15.

Although this question was the most popular optional question, the responses were not encouraging.

In Part (a), candidates were able to add the vectors but experienced great difficulty describing the geometrical relationship and representing the points to show their relative positions.

In Part (b) (i), candidates experienced difficulty with matrix multiplication and solving matrix equations. They were more successful in finding the determinant of the matrix in Part (ii) but this was done separately as they failed to see the connection between Parts (i) and (ii). Only the more able candidates were able to use matrix methods to solve the simultaneous equations.

Solutions

(a) (i) a) $\begin{pmatrix} 3 \\ 2 \end{pmatrix}$ b) $\begin{pmatrix} 9 \\ -6 \end{pmatrix}$ (ii) BC = 3 BA

(b) (i) $a = 3,\ b = -2$ (ii) $-\frac{1}{2}\begin{pmatrix} -3 & 4 \\ -1 & 2 \end{pmatrix}$ (iii) $x = 4,\quad = -1$

Recommendations

Teachers must allow students to explore the geometry of vectors before introducing the matrix algebra. They must be able to identify routes for vectors using visual props and then proceed to writing equations. In solving equations using matrix methods, students must be encouraged to present their solution in stages so that full working is shown at each stage. In this way they can review their solutions and avoid carrying over computational errors.

CARIBBEAN EXAMINATIONS COUNCIL

REPORT ON CANDIDATES' WORK IN THE CARIBBEAN SECONDARY EDUCATION CERTIFICATE® EXAMINATION

JANUARY 2013

MATHEMATICS
GENERAL PROFICIENCY EXAMINATION

GENERAL COMMENTS

The General Proficiency Mathematics examination is offered in January and May/June each year. There was a candidate entry of approximately 12 900 in January 2013; 40 per cent of the candidates earned Grades I–III. The mean score for the examination was 78 out of 180 marks.

DETAILED COMMENTS

Paper 01– Multiple Choice

Paper 01 consisted of 60 multiple-choice items. This year, 29 candidates each earned the maximum available score of 60. Sixty-eight per cent of the candidates scored 30 marks or more.

Paper 02 – Structured Questions

Paper 02 consisted of two sections. Section I comprised eight compulsory questions totalling 90 marks. Section II comprised three optional questions: one each from (i) Algebra, Relations, Functions and Graphs; (ii) Measurement, Trigonometry and Geometry; and (iii) Vectors and Matrices. Candidates were required to answer any two questions from this section. Each question in this section was worth 15 marks.

This year, one candidate earned the maximum available mark of 120 on Paper 02. Approximately 20 per cent of the candidates earned at least 60 marks on this paper.

Compulsory Section

<u>Question 1</u>

This question tested candidates' ability to

- perform the four basic operations with decimals
- use the order of operations to do calculations with decimals
- convert from one currency to another
- find the cost price given the percentage sales tax.

The question was attempted by 99 per cent of the candidates, 4.3 per cent of whom earned the maximum available mark. The mean mark was 7.52 out of 11.

The majority of candidates produced satisfactory responses to this question. In general, they demonstrated competence in the use of the calculator to perform computations in the correct order. However, there were some misconceptions by some candidates as are reflected in the following methods and procedures.

In Part (a), instead of squaring, some candidates multiplied by 2 and $.3^2$ was evaluated as $2 \times .3$.

In Part (b), the hotel accommodation for 3 nights was misunderstood and the return airfare was also multiplied by 3. Few of the candidates associated the EC\$1610.00 charge, which included a 15% sales tax, with 115% of the cost price. Most of them proceeded to calculate 15% of EC\$ 610.00, and then to subtract the result from this amount to determine the cost price.

Solutions

(a) 9.257

(b) (i) US \$647 (ii) US \$596.30

(iii) Angie's since \$596.30 < \$647 (iv) \$1400

Recommendations

Teachers are advised to teach proportion using the unitary method. They should also use authentic tasks in the teaching of consumer arithmetic. Attention should be given to squaring and to writing the result of a computation exactly.

Question 2

This question tested candidates' ability to

- use the distributive property of multiplication over addition
- solve a linear equation in one unknown
- factorize quadratic functions involving difference of squares
- solve worded problems involving a pair of simultaneous equations.

The question was attempted by 99 per cent of the candidates, 10.6 per cent of whom earned the maximum available mark. The mean mark was 5.48 out of 12.

The performance of candidates on this question was satisfactory. In Part (a), while most candidates were able to correctly apply the distributive property, they were generally unable to simplify the equation and to correctly proceed to calculating the value of *p*.

In Part (b), candidates encountered difficulties in factorizing the quadratic expressions. Some popular but incorrect methods seen included
$25m^2 - \quad = (25m - 1)(25m + 1)$; $2n^2 - 3n - 20 = n(2n - 3) - 20$;
$2n^2 - 3n - 20 = (n + 4)(2n - 5)$.

In Part (c), candidates correctly translated words into symbols and wrote the pair of equations in x and y. The majority of candidates identified an appropriate strategy for solving the pair of equations such as elimination, substitution, matrices and trial and error. The most popular choice was elimination; but with this, several candidates added the equations and hence could not eliminate either variable.

Solutions

(a) $p = \frac{3}{2}$

(b) (i) $(5m - 1)(5m + 1)$ (ii) $(2n + 5)(n - 4)$

(c) (i) $5x + 12y = 61$; $10x + 13y = 89$ $30x$

(ii) a)5 g; b) 3 g

Recommendations

Teachers should emphasize the appropriate use of the distributive, associative and commutative properties and should emphasize the difference between an expression and an equation. Students should practise the factorization of quadratic expressions and solving simultaneous linear equations. The difference between squares is a useful tool in factorization and needs continued attention by teachers and students.

<u>Question 3</u>

This question tested candidates' ability to

- represent information in a Venn diagram
- formulate and solve a linear equation in one unknown using information from a Venn diagram
- solve simple geometrical problems using the properties of parallel lines and angles in isosceles triangles
- use the properties of two triangles, to explain why they are similar but not congruent.

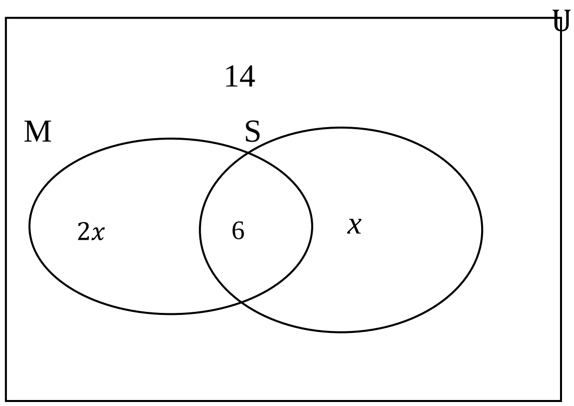
The question was attempted by 99 per cent of the candidates, 1.5 per cent of them earned the maximum available mark. The mean mark was 5.42 out of 12.

The performance of candidates on this question was moderate. In Part (a), most candidates were able to enter 6 in the intersection of the sets M and S, $2x$ in $M \cap S'$ and x in $S \cap M'$, but were less proficient in identifying the number of elements in $(M \cup S)'$. Many of them were unable to form an equation from information represented on the Venn diagram, and in several instances when they arrived at the equation, they lacked the algebraic skills to solve it.

In Part (b), candidates were able to correctly determine the measure of the angle formed at A. However, calculating angle AED proved more challenging, as well as providing a reason why the two triangles in question were similar but not congruent. A common incorrect response was that they were similar because they were two isosceles triangles.

Solutions

(a)(i)



(ii) $x = 10$

(b) (i) a) $B\hat{A}C = 72°$ b) $A\hat{E}D = 54°$

(ii) Similar since corresponding angles are equal

Not congruent because lengths of corresponding sides are unequal.

Recommendations

Teachers are expected to revise on an ongoing basis the content covered in Set Theory at the lower grades of the school. Students should be encouraged to provide reasons why plane figures could be similar but not congruent.

<u>Question 4</u>

This question tested candidates' ability to

- change the subject in a formula
- solve problems involving the inverse and the composite functions
- rearrange a linear equation to determine the gradient of a given line
- determine the equation of the perpendicular bisector of a given line

The question was attempted by 93 per cent of the candidates, 1.6 per cent of them earned the maximum available mark. The mean mark was 2.53 out of 12.

The performance of candidates on this question was generally weak. In Part (a) (i), they seldom attempted to collect terms in r, the variable that they were required to make the subject of the formula. Even when this was done, they did not proceed further since it was not recognized that by factorizing they could isolate r. Their attempt at making r the subject in $v = \pi r^2 h$ was equally poorly done.

In Part (b) (i) a), candidates could not find the inverse of the function and therefore could not evaluate $f^{-1}(19)$.

In Part (c) (i), candidates could not determine the gradient of a line when given its equation. Most candidates were unable to use coordinate geometry to solve the problem.

Solutions

(a) (i) $r = \frac{h}{(1-h)}$ (ii) $r = \sqrt{\left(\frac{V}{\pi h}\right)}$

(b) (i) $f^{-1}(19) = 7$ (ii) $gf(3) = 4$

(c) (i) gradient of GH $= -\frac{3}{2}$ (ii) Equation of JK is: $3y = 2x - 5$

Recommendations

Teachers should give special attention to the balancing nature of equations and should expose the students to a variety of questions requiring the change of the subject of a formula. Students need to be taught how to determine the gradient of a line when the equation of the line is given. Teachers should provide more practice for students in finding the inverse of a function.

Question 5

This question tested candidates' ability to

- measure and state the length of a line and the measure of an angle
- determine the bearing of one point from another
- solve problems related to scales and distances
- construct an angle of $120°$ using ruler and compasses only.

The question was attempted by 93 per cent of the candidates, 1.5 per cent of whom earned the maximum available mark. The mean mark was 5.25 out of 12.

In Part (a), candidates were able to correctly measure and state the length of RT and the measure of an angle although a number of them could not correctly identify the bearing. In Part (b), candidates demonstrated some proficiency in correctly applying the scale to determine the value of RM. However, the limited knowledge of bearings prohibited successful completion of the problem.

Solutions

(a) (i) 5.8 cm (ii) $65°$ (iii) 174 m

(b) (i) 10 cm (iii) $N\hat{R}M$

Recommendations

Teachers need to use mathematical language when teaching concepts in geometry. Efforts should also be made to teach related concepts at the same time instead of in isolation. Questions should be set using authentic situations to assist in the transfer of learning to everyday experiences.

Question 6

This question tested candidates' ability to

- determine the radius when the diameter is given
- determine the circumference of the cross section of a cylinder
- use information from the net of a cylinder to determine its curved surface area
- determine the depth of water in a cylinder when given the volume of water, in litres.

The question was attempted by 91 per cent of the candidates, 2.5 per cent of whom earned the maximum available mark. The mean mark was 3.35 out of 11.

In Part (a), candidates easily obtained the radius of the circle and the circumference of the cross section of the cylinder. In Part (b), when the net of the same cylinder was given, many candidates did not see the relationship between the two. As a consequence, they invariably arrived at the incorrect answer for the curved surface area of the cylinder.

In Part (c), many candidates did not convert litres to cubic centimeters as was required. They also had difficulty formulating an equation for the volume of water in the cylinder; and in situations where they formed the equation, many of them could not correctly make h the subject of the formula.

Solutions

(a) (i) 6 cm (ii) 37.68 cm
(b) $\text{a} = 37.68 \text{ cm}$; $\text{b} = 8 \text{ cm}$
(c) $\text{h} = 4.4 \text{ cm}$

Recommendations

Teachers should ensure that students are familiar with the nets of solids. Attention should also be given to the consistency of units within equations.

Question 7

This question tested candidates' ability to

- complete a grouped frequency table
- identify the modal class of a grouped frequency distribution
- identify the class interval in which a given score would lie
- calculate an estimate of the mean of grouped data
- estimate the proportion of the data set above a given value.

The question was attempted by 89 per cent of the candidates, 3.1 per cent of whom earned the maximum available mark. The mean mark was 3.66 out of 10.

Although candidates were generally familiar with the term *modal class*, they were unable to identify the correct interval in which a given score would lie.

When calculating the mean, many candidates divided by the sum of the mid-point values by 6 instead of dividing the sum of the values $f \times x$ by 100. Generally, candidates did not know why the value calculated for the mean was just an estimate rather than the exact value.

To determine the probability that a student chosen at random would score 40 or more, several candidates divided 40 by 100, while others attempted to divide the interval 40−49 by 100.

Solutions

(a) (i) modal class is $20-29$ (ii) 19.4 lies in the class $10-19$

(b) (i)

Score	**Mid-point $(\mathbf{x})$**	**Frequency $(\mathbf{f})$**	$\mathbf{f \times x}$
20−29	24.5	25	612.5
30−39	34.5	22	759
40−49	44.5	20	890
50−59	54.5	12	654

(b) (ii) Sample mean $= 31.4$

(c) The mean calculated is an estimate because the assumption was made that the scores in any interval are all equal to the mid-value of the class.

(d) Probability $= \frac{32}{100}$

Recommendation

Students need to practise all aspects of statistics to maximize their chances of solving questions of this nature.

Question 8

This question tested candidates' ability to

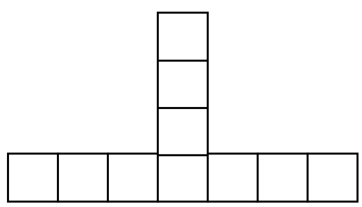
- draw the fourth diagram in a sequence of diagrams in which the first three diagrams in the sequence are given
- recognize and generate the terms of a sequence
- determine the term of the sequence for which the value of the function is given
- derive an equation connecting the value of the function with the term of the sequence.

The question was attempted by 97 per cent of the candidates, 3.9 per cent of whom earned the maximum available mark. The mean mark was 4.83 out of 10.

A large number of candidates recognized the pattern of squares in the sequence and were able to correctly draw the fourth diagram. They were able to identify the pattern of numbers within the table and used this to arrive at the numerical values that were missing from the table. However, the majority of candidates were unable to describe the pattern using algebraic symbols.

Solutions

(a)



(b) (i) $a = 10$ (ii) $b = 28$ (iii) $c = 14$

(c) $3n - 2$

Recommendation

Teachers should assist students with writing expressions and formulae derived from number sequences.

Optional Section

Question 9

This question tested candidates' ability to

- calculate values of *y* for given values of *x* in a reciprocal function
- plot points associated with a reciprocal function and draw a smooth curve through the points
- write a quadratic function in the form $a(x-h)^2+k$
- determine, for the quadratic function, its minimum value and the equation of its line of symmetry
- solve a quadratic equation.

The question was attempted by 71 per cent of the candidates, 2.9 per cent of whom earned the maximum available mark. The mean mark was 5.75 out of 15.

In Part (a) (i), candidates were able to correctly calculate the missing *y*-coordinates for the function $y = \frac{3}{x}$. However, many candidates found it difficult to use the scales given to plot points where the *x*-coordinates were in decimal form. They also had difficulty using uniform scales on both axes.

In Part (a) (ii), they were able to correctly represent the scales given on the graph.

In Part b (i), candidates were able to use an appropriate strategy to solve a quadratic equation. However, they experienced some difficulty obtaining correct values for *a*, *h* and *k* when they attempted to complete the square. Candidates also had difficulty identifying the minimum value from the expression for the completed square. However, the coordinates for the minimum point were often quoted correctly. Applying the correct coefficients to the quadratic formula and simplifying the expression was also problematic for candidates. As a result $\frac{-5 \pm \sqrt{5^2 - 4\times 3\times 1}}{2\times 3}$ was often seen, and $-5 \pm \sqrt{\frac{13}{6}}$ = -5 ± $\sqrt{2.16}$ or $5 \pm \sqrt{\frac{13}{6}}$ = 5 ± $\sqrt{2.16}$ was the penultimate stage of the simplification leading to incorrect solutions.

Solutions

(a) (i)

$x\ (sec)$	0.5	3	5
$y\ (^{m}/_{s})$	6	1	0.6

(b) (i) $3\left(x-\frac{5}{6}\right)^2 - \frac{13}{12}$ (ii) $f_{min} = -\frac{13}{12}\ when\ x = \frac{5}{6}$ (iii) $x = \ .43, 0.23$

Recommendations

Teachers should provide students with more practice questions involving fractional coordinates and solutions of quadratic equations with $b < 0$. They should also help students to recognize their weaknesses and encourage them not to choose questions for which they were not adequately prepared.

<u>Question 10</u>

This question tested candidates' ability to

- use circle theorems to evaluate the measure of three different angles
- use the cosine formula to solve for an unknown angle in a triangle
- determine the area of a triangle in which the measures of three sides are given
- use information derived from three-dimensional geometry to draw a given triangle showing the angle of elevation
- use trigonometric ratios to calculate an unknown height.

The question was attempted by 48 per cent of the candidates, less than one per cent of whom earned the maximum available mark. The mean mark was 2.49 out of 15.

In Part (a), most candidates correctly identified angle RMQ as a right angle and, as a result, were able to derive the measure of angle MRQ. Some attempted to solve the problem by using angles in a triangle with no attention given to the circle theorems. A large number of candidates identified triangle PMN as isosceles and proceeded to correctly calculate the measure of angle PMN.

In Part (b) (i), most candidates opted to use the cosine rule to calculate the measure of angle ABC. However, there were those who attempted to use the sine rule, and in some cases the sine ratio, ignoring the fact that triangle ABC was not right angled. Many of the candidates who attempted to use the cosine rule, could not follow through correctly. In most cases, they substituted into the formula incorrectly, and ended up calculating angle BAC instead. Also, in an attempt to determine the area of triangle ABC, the majority of them used the formula
$Area = \frac{1}{2}bh$ rather than $Area = \frac{1}{2}ab\ sinC$.
In Part (b) (ii), most candidates could not illustrate triangle TAB with the right angle at A. In most cases they proceeded to incorrectly use AB as the hypotenuse of the triangle rather than as the adjacent side.

Solutions

(a) (i) $M\hat{R}Q = 70°$ (ii) $P\hat{M}R = 20°$ $(iii)\ P\hat{M}N = 63°$
(b) (i) a) $A\hat{B}C = 137.1°$ b) $3849.5\ m^2$ $(ii)\ a)$ T b) $TA = 73.9\ m$

Recommendations

Teachers should provide students with sufficient practice in solving problems associated with circle theorems and in calculating areas of triangles using the formula $A = \frac{1}{2}ab\,SinC$. In addition, students need greater exposure to problems involving three-dimensional geometry.

Question 11

This question tested candidates' ability to

- determine the resultant of two or more vectors from a vector diagram
- solve for the object point given the coordinates of the image and the transforming matrix
- write the matrix which represents an enlargement
- determine the coordinates of the image which results when an object undergoes a combination of two transformations
- write a matrix to represent an authentic situation.

The question was attempted by 41 per cent of the candidates, less than one per cent of whom earned the maximum available mark. The mean mark was 1.77 out of 15.

In Part (a), while candidates were generally able to write a correct route for each vector sum, they were unable to correctly substitute into their expressions or to simplify by collecting like terms.

In Part (b), candidates did not recognize that they could use the inverse of J to obtain the coordinates of the object when given those of the image. In many instances, they incorrectly multiplied J by (5, 4).

In Part (c), candidates were not able to write the matrix to represent the enlargement and very few of them were able to obtain and write the combined transformation HJ in matrix form.

In Part (d), candidates appeared to be confused by the difference between a 3×2 *and a* 2×3 matrix and between a 1×3 *and a* 3×1 matrix. As a consequence, it was common to find incompatible matrices set up for multiplication.

Solutions

(a) (i) $\overrightarrow{\mathrm{MK}} = -\mathrm{u} + \mathrm{v}$ (ii) $\overrightarrow{\mathrm{SL}} = \frac{2}{3}\mathrm{u} + \frac{1}{3}\mathrm{v}$ (iii) $\overrightarrow{\mathrm{OS}} = \frac{1}{3}\mathrm{u} + \frac{2}{3}\mathrm{v}$

(b) $\mathrm{P}(4, -4)$

(c) (i) $\mathrm{H} = \begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix}$ (ii) $(21, 15)$

(d) (i) $\begin{pmatrix} 2 & 0 \\ 5 & 6 \\ 3 & 10 \end{pmatrix}$ (ii) $\begin{pmatrix} 40 & 55 & 120 \end{pmatrix}$ (iii) $\begin{pmatrix} 40 & 55 & 120 \end{pmatrix}\begin{pmatrix} 2 & 0 \\ 5 & 6 \\ 3 & 10 \end{pmatrix}$

Recommendations

Teachers should provide students with adequate practice in the multiplication of matrices with emphasis on the compatibility of matrices for multiplication. Attention should also be given to the use of the inverse matrix in determining the object coordinates when given the coordinates of the image under a given transformation.

C A R I B B E A N   E X A M I N A T I O N S   C O U N C I L

REPORT ON CANDIDATES' WORK IN THE CARIBBEAN SECONDARY EDUCATION CERTIFICATE® EXAMINATION

MAY/JUNE 2013

MATHEMATICS
GENERAL PROFICIENCY EXAMINATION

GENERAL COMMENTS

The General Proficiency Mathematics examination is offered in January and May/June each year. There was a candidate entry of approximately 92 400 in May/June 2013 and 30 per cent of the candidates earned Grades I–III. The mean score for the examination was 65 out of 180 marks.

DETAILED COMMENTS

Paper 01 — Multiple Choice

Paper 01 consisted of 60 multiple–choice items. It was designed to provide adequate coverage of the content with items taken from all sections of the syllabus. Approximately 74 per cent of the candidates earned acceptable grades on this paper; the mean score was 33 out of 60 marks. This year, 142 candidates earned the maximum available score of 60.

Paper 02 — Structured Questions

Paper 02 consisted of two sections. Section I comprised eight compulsory questions for a total of 90 marks. Section II comprised three optional questions: one each from Algebra, Relations, Functions and Graphs; Measurement, Trigonometry and Geometry; and Vectors and Matrices. Candidates were required to answer any two out of three questions from this section. Each question in this section was worth 15 marks. The mean score for this paper was 31 out of 120 marks.

Compulsory Section

<u>Question 1</u>

This question tested candidates' ability to

- add, subtract, multiply and divide fractions involving mixed numbers and decimals
- use the calculator to evaluate the square and the square root of rational numbers
- solve problems involving compound interest.

The question was attempted by 99 per cent of the candidates, 3.7 per cent of whom earned the maximum available mark. The mean mark was 5.28 out of 11.

In Part (a), candidates generally provided unsatisfactory responses. While they were able to follow through on the algorithm for subtracting the fractions, many of them were confused as to which of the improper fractions should be inverted when performing division. They demonstrated competence in using the calculator to compute the square and square root of rational numbers. However, two common errors were $(0.32)^2 = 0.32 + 0.32$ and $(0.32)^2 = 2 \times 0.32$.

In Part (b), many candidates did not attempt to calculate either the cost of the same quantity of juice or the volume of juice for the same cost; and in several instances when the correct approach was taken, they experienced difficulty comparing small values such as 0.012 cents and 0.0114 cents.

In Part (c), candidates were able to identify the interest at the end of one year as 8 per cent of the principal. However, those who used the compound interest formula, experienced difficulty in separating the interest from the aggregate.

Solutions

(a) **(i)** $\frac{11}{18}$ **(ii)** **1.3524**

(b) **1 ml of the 350 ml package costs 1.20 cents**
1 ml of the 450 ml package costs 1.14 cents
∴ the 350 ml package is the more cost — effective buy

(c) **(i)** **$768** **(ii)** **$6 000** **(iii)** **$480**

Recommendations

Teachers should provide students with opportunities to work with scientific calculators in performing basic arithmetic operations on rational numbers. They should also incorporate real-life situations into the mathematics lesson so that students can become competent applying the content to authentic situations.

<u>Question 2</u>

This question tested candidates' ability to

- factorize algebraic expressions involving the difference of squares and a quadratic function in three terms
- change the subject of the formula from Fahrenheit to Celsius
- evaluate an algebraic expression by substituting a real number value
- solve problems involving a simple linear equation in one unknown.

The question was attempted by 98 per cent of the candidates, less than 1 per cent of whom earned the maximum available mark. The mean mark was 2.33 out of 12.

The performance of candidates on this question was unsatisfactory. In Part (a), candidates were generally able to factor out $2x$ from $2x^2 - 8x$ but could not proceed further with the difference of two squares. Factorizing $3x^2 - 5x - 2$ proved even more challenging, as candidates could not determine the factors of $-6x^2$ which could be added to produce $-5x$.

In Part (b), candidates experienced difficulty with transposing the formula. A common incorrect strategy is given in the following example: $F - 32 = \frac{9}{5}C$ therefore $\frac{5}{9}F - 32 = C \; or \; \frac{F-160}{9} = C$.

In Part (c), many candidates were unable to translate verbal phrases into algebraic expressions or to formulate an equation.

Solutions

(a) **(i)** $\mathbf{2x\,(x-2)\,(x+2)}$ **(ii)** $\mathbf{(3x+1)\,(x-2)}$
(b) **(i)** $\boldsymbol{C = \frac{5}{9}(F-32)}$ **(ii)** $\mathbf{C = 45}$
(c) **(i)** **a)** $\mathbf{(500 - x)}$ **b)** $\mathbf{10(500 - x) + 6x}$ **(ii)** $\mathbf{x = 223}$

Recommendations

Teachers should reinforce the following skills:

(i) Selecting appropriate numerical factors from a quadratic expression
(ii) Clearing fractions in an equation
(iii) Differentiating between factorizing an expression and solving an equation

Question 3

This question tested candidates' ability to

- determine elements in intersections, unions and complements of sets
- solve problems involving the use of Venn diagrams
- calculate lengths of line segments using properties of similar triangles
- explain why two given triangles are similar.

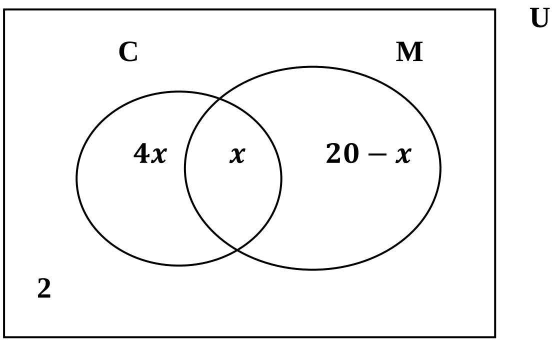
The question was attempted by 98 per cent of the candidates, 1.3 per cent of whom earned the maximum available mark. The mean mark was 4.19 out of 12.

In Part (a), most candidates identified the intersection of the two sets and were able to insert x in the correct region. A large number of candidates also knew the region which represented $C \cap M'$ and correctly inserted $4x$. However, a common incorrect response was to place $4x - x$ in this region. Most candidates incorrectly entered 20 instead of $20 - x$ in the region representing $C' \cap M$. In general, candidates were unable to formulate the correct expression for members in the universal set from information entered on their diagrams.

In Part (b) (i), while candidates correctly chose to apply Pythagoras' theorem, a large number of candidates could not correctly formulate it, and it was common to see $B^2 = 10^2 + 8^2$ instead of $BC^2 = 10^2 - 8^2$. In Part (b) (ii), many candidates wrote that the triangles were similar simply because they were both right–angled triangles.

Solutions

(a) **(i)**



(ii) **4*x* + 22** **(iii)** **8 students**

(b) **(i)** ***BC* = 6 cm** **(ii)** **The corresponding angles are equal.**

(iii) ***DE* = 3.6 m**

Recommendations

Students must be made aware that they should not assume that diagrams given on an examination paper are drawn according to scale. Teachers should encourage students to use mathematical jargon to explain similarity of plane geometrical figures. They need to assist students in differentiating between figures that are similar and those that are congruent. Attention should be given to the difference between an <u>algebraic expression</u> and an <u>algebraic equation</u>. Candidates need to give greater attention to solving problems involving the use of similar triangles.

Question 4

This question tested candidates' ability to

- use a ruler to measure the length of a line segment
- use a protractor to determine the measure of an angle
- determine by measurement the perimeter of a triangle
- determine the area of a triangle
- determine the gradient of a line, given the coordinates of two points on the line
- determine the coordinates of the midpoint of a line segment
- determine the equation of the perpendicular bisector of a line, given two points on the line.

The question was attempted by 87 per cent of the candidates, 1.6 per cent of whom earned the maximum available mark. The mean mark was 3.53 out of 12.

In Part (a), candidates demonstrated some proficiency in measuring the length of a line segment although there were those who stated their measured values with a constant error of 1 cm. A large number of candidates were unable to state the measure of angle EOD. Most candidates appeared to be familiar with the concept of perimeter even when their measurements were inaccurate. They generally applied the correct algorithm for finding the area of a triangle and a variety of formulae given in the rubrics were applied.

In Part (b), many candidates could not determine the gradient of a line that is perpendicular to a given line. In some situations, candidates who knew the gradient of the new line could not complete the solution to the problem because they did not use the coordinates of the midpoint of the given line. In most cases, the coordinates of one of the end points of the given line segment were substituted.

Solutions

(a)	**(i)**	**(4.9 ± 0.1) cm**	**(ii)**	**$36^o \pm 1^o$**	**(iii)**	**(18 ± 0.3) cm**	**(iv)**	**(12 ± 0.2) cm^2**
(b)	**(i)**	$-\frac{1}{2}$	**(ii)**	**(1,3)**	**(iii)**	**y = 2*x* + 1**		

Recommendations

Teachers should give greater attention to the use of the protractor in measuring angles. They should employ different methods in calculating the gradient of a line and the relationship between the gradient of a line and the gradient of the perpendicular bisector of the line should be reinforced.

Question 5

This question tested candidates' ability to

- write an equation to represent the direct proportion between two variables
- determine the value of the constant of proportionality
- determine the composite of a function
- determine the inverse of a function.

The question was attempted by 86 per cent of the candidates, 4.4 per cent of whom earned the maximum available mark. The mean mark was 3.00 out of 11.

The performance of candidates on this item was generally unsatisfactory. In Part (a), candidates were unable to derive the equation $A = kR^2$ from the information given. Some common incorrect representations were: $A = kR$; $R = \frac{\sqrt{A}}{k}$; $A = k + R^2$ and $A = (kR)^2$. In many situations where candidates obtained the correct equation, they encountered difficulty in transposing the equation to determine the value of k. For example: $36 = k \times 9 \therefore 36 - 9 = k$; $36 = k \times 9 \therefore \frac{9}{36} = k$.

In Part (b), candidates encountered difficulty in evaluating $f\,g\,(2)$. The majority of candidates calculated the value of $g\,(2)$, but many did not proceed to evaluate $f\,(13)$. Some popular incorrect responses were:

$f\,g\,(2) = f\,(2) + g\,(2)$; $f\,g(2) = f\,(2) \times g\,(2)$; $f\,g(x) = 4 \times \left(\frac{2x+1}{3}\right) + 5$ which actually is $g\,f\,(x)$; $f\,g\,(x) = \frac{2x(4x+5)+1}{3}$ which is an attempt at $f\,g\,(x)$ without replacing the x in $f\,(x)$.

Also, most candidates wrote the inverse function of $f\,(x)$ correctly but were unable to evaluate $f^{-1}\,(3)$.

Solutions

(a) (i) $A = kR^2$ (ii) $k = 4$ (iii) For $A = 196$, $R = 7$; $R = 5$, $A = 100$
(b) (i) 9 (ii) 4

Recommendations

Teachers are advised to encourage students to read the entire question carefully since some candidates appeared to focus only on the numerical aspects of the question and as a consequence missed important information. They should assist students in recognizing linkages between parts of a question. The use of flow charts might prove useful in helping students to better understand composition of functions and the determining the inverse.

<u>Question 6</u>

This question tested candidates' ability to

- convert from $km\,h^{-1}$ to $m\,s^{-1}$
- calculate the distance travelled, given speed and time
- describe completely a reflection in the plane
- draw the image of a triangle after undergoing a translation in the plane
- describe a transformation as the combination of two simple transformations.

The question was attempted by 87 per cent of the candidates, 1.1 per cent of whom earned the maximum available mark. The mean mark was 2.03 out of 11.

In Part (a), the candidates knew the formula relating distance, speed and time and were able to apply it correctly in the majority of cases, even when they could not convert from kmh^{-1} to ms^{-1}.

The performance on this question was generally unsatisfactory. Most candidates were able to identify the first transformation as a reflection but were unable to state the equation of the mirror line. They experienced difficulty with the use of the given translation vector and in many cases, when they correctly drew the image of LMN under the translation, they could not describe the combination of transformations that would map this image onto $L'M'N'$.

Solutions

(a) (i) 15 ms^{-1} (ii) 300 m

(b) (i) a reflection in the line $x = 7$

(iii) a reflection in the line $x = 7$ and a translation by the vector $\binom{0}{3}$

Recommendations

Teachers should place greater emphasis on the conversion of units within measurement. They should reinforce terminology used to describe translations, reflections, rotations and enlargements. Technology could be used to demonstrate to students the processes and effects of these basic geometrical transformations on plane figures.

Question 7

This question tested candidates' ability to

- complete the cumulative frequency in a grouped-frequency table
- draw a cumulative frequency graph, using given values in a table
- use a cumulative frequency graph to estimate the median of a data set
- use the cumulative frequency graph to estimate probabilities.

The question was attempted by 91 per cent of the candidates, 4.1 per cent of whom earned the maximum available mark. The mean mark was 4.57 out of 11.

Candidates performed moderately well on this question. The majority of them were able to complete the cumulative frequency column from the given frequency distribution and to use the given scales for the axes. Generally, they knew that a cumulative frequency curve should produce an ogive, although their attempts did not always produce a smooth curve and there were instances where candidates interchanged the axes when plotting the points. Drawing lines on the graph to show how the median was estimated and for the proportion of students who spent less than $23 posed a significant challenge for some of the candidates. It was common to see lines drawn vertically at $30 and horizontally at 18 or 19 students. This suggested that some candidates thought that the median was located at the midpoint of the amount of money spent. Further, there were candidates who correctly determined the number of students who spent less than $23 but did not proceed to state the probability.

Solutions

(a)

Amount Spent ($)	**No. of Students**	**Cumulative Frequency**
31 - 40	**11**	**30**
41 - 50	**8**	**38**
51 - 60	**2**	**40**

(c) **(i)** **$31** **(ii)** $\frac{12}{40}$

Recommendations

Teachers need to provide students with more practice in drawing cumulative frequency graphs and in using them to estimate measures of position and simple probabilities. Attention should be given to the plotting of points at the boundaries rather than at the limits of class intervals.

Question 8

This question tested candidates' ability to

- draw the fourth diagram in a sequence of diagrams in which the first three diagrams in the sequence are given
- complete a table to show the values in a sequence of numbers
- derive the general rule representing the patterns shown.

The question was attempted by 93 per cent of the candidates, 3.0 per cent of whom earned the maximum available mark. The mean mark was 4.08 out of 10.

Candidates' responses to this question revealed a number of weaknesses in their basic mathematical knowledge. There was a general disregard for the fact that each diagram after the first consisted of cubes linked together. Many candidates ignored the number 20 in the first column, which represented the 20^{th} diagram, and instead completed the table for the 5^{th} diagram. Very few candidates were able to write the algebraic expression for the nth diagram in the sequence and many opted for a description in words rather than symbols.

Solutions

(b) **(i)** $\boldsymbol{W = 36}$; $\boldsymbol{B = 20}$ **(ii)** $\boldsymbol{W = 164}$; $\boldsymbol{B = 84}$

(c) **(i)** $\boldsymbol{W = 8N + 4}$ **(ii)** $\boldsymbol{B = 4N + 4}$

Recommendations

Teachers should assist students with writing expressions and formulae derived from number sequences. They should devote time to the drawing of prisms on plain paper and graph sheets, pointing out cross-sections, unseen lines, faces, edges and vertices.

Optional Section

Question 9

This question tested candidates' ability to

- write linear inequalities to represent constraints presented in words
- draw graphs to represent linear inequalities in two unknowns
- write in words the relationship between two variables expressed as an inequality
- represent on a graph the region which represents the solution set of a system of inequalities
- complete the square of a quadratic function
- sketch a graph to represent a quadratic function.

The question was attempted by 56 per cent of the candidates, 1.1 per cent of whom earned the maximum available mark. The mean mark was 3.25 out of 15.

Candidates' performance on this question was unsatisfactory. While a large number of candidates were able to derive the inequality $\mathrm{x} + \mathrm{y} \leq 6$, and were proficient at drawing a line on the graph to represent $\mathrm{x} + \mathrm{y} = 6$, they experienced difficulty with drawing the line $y = 2$ and could not explain in words the meaning of $y \leq 2x$. When lines were drawn on the graph, many candidates labelled them using the inequality signs.

In Part (b), some candidates who attempted to write $3x^2 - 12x + 8$ in the form $a(x+h)^2 + k$, using the traditional method of completing the square, demonstrated little proficiency in this skill. Many of them attempted to factorize the expression instead. When asked to sketch the graph of $y = 3x^2 - 12x + 8$, many candidates resorted to constructing a table of values, plotting points, and drawing a curve through these points. Furthermore, some candidates, even having sketched the curve, could not determine the y-intercept on the sketch.

Solutions

(a) **(i)** $\boldsymbol{x + y \leq 6}$ **(ii)** $\boldsymbol{y \geq 2}$

(iii) $\boldsymbol{The\ number\ of\ mangoes\ must\ not\ exceed\ twice\ the\ number\ of\ oranges.}$

(b) **(i)** $\mathbf{3(}x\mathbf{-2)^2-4}$

Recommendations

Teachers should assist students in differentiating between sketching a graph and drawing a graph of a given function. They should expose students to the traditional method of completing the square. Some emphasis should be placed on determining the coordinates of minimum or maximum point from the result of completing the square.

Question 10

This question tested candidates' ability to

- use the properties of circles and circle theorems to determine the measures of angles
- use trigonometric ratios to solve problems related to bearings and angles of elevation in a three-dimensional figure.

The question was attempted by 44 per cent of the candidates, less than 1 per cent of whom earned the maximum available mark. The mean mark was 2.98 out of 15.

Candidates' performance on this question was generally unsatisfactory. There was evidence that candidates were exposed to the angle properties of triangles and circles but demonstrated misconceptions in their applications. For example, some candidates doubled the angle at the centre of the circle to obtain a value for the angle at the circumference.

Also, in Part (b) candidates encountered much difficulty in extracting plane shapes from the three-dimensional diagram, and even when they succeeded, they invariably labelled the right–angle incorrectly. Most of them could not identify the angle of elevation of T from S. Moreover, some candidates proceeded to use trigonometric rules instead of ratios in their attempt at solving the right–angled triangles.

Solutions

(a) **(i)** **55°** **(ii)** **100°** **(iii)** **50°** **(iv)** **15°**

(b) **(iii)** $\mathbf{65.5}\ \boldsymbol{m}$ **(iv)** **30°**

Recommendations

Teachers should employ a more practical approach in teaching trigonometry involving three-dimensional figures. They should assist students in making the determination as to when to use trigonometric ratios and when to use trigonometric formulae.

Question 11

This question tested candidates' ability to

- use vector geometry to determine the resultant of two or more vectors
- use vectors to solve problems in geometry
- evaluate the determinant of a 2 × 2 matrix
- obtain the inverse of a non-singular 2 × 2 matrix
- prove that the product of a matrix and its inverse is the identity matrix
- solve for four unknowns in a matrix equation.

The question was attempted by 45 per cent of the candidates, 1.1 per cent of whom earned the maximum available mark. The mean mark was 3.01 out of 15.

Candidates performed unsatisfactorily on this question. They were able to identify OB and PQ as being parallel even when their algebraic expressions for the vectors did not match their observation. In most instances, they were unable to arrive at the correct routes for the resultant vectors.

In Part (b), candidates encountered difficulty with finding the inverse of a two by two matrix, matrix multiplication and with solving the matrix equation. In the latter case, they resorted to writing two pairs of simultaneous equations and proceeded to solve them.

Solutions

(a) **(i)** **a)** $\overrightarrow{\boldsymbol{AB}} = -2\boldsymbol{a} + 2\boldsymbol{b}$ **b)** $\overrightarrow{\boldsymbol{PQ}} = \boldsymbol{b}$

(ii) $\boldsymbol{OB = 2PQ}$; $\boldsymbol{OB\ is\ parallel\ to\ PQ}$

(b) **(i)** $\frac{1}{2}\begin{pmatrix} 3 & -1 \\ -4 & 2 \end{pmatrix}$ **(iii)** $\boldsymbol{r = 1}$; $\boldsymbol{s = 2}$; $\boldsymbol{t = 0}$; $\boldsymbol{u = -3}$

Recommendations

Teachers should introduce vectors sufficiently early in the curriculum so that students have enough time to be familiar with the content. In teaching vector geometry, they should give attention to identification of a convenient route for determining the resultant vector, and to the skill of substituting and simplifying values for the route chosen.

Attention should also be given to the use of inverse matrices in the solution of systems of two linear equations.

CARIBBEAN EXAMINATIONS COUNCIL

REPORT ON CANDIDATES' WORK IN THE CARIBBEAN SECONDARY EDUCATION CERTIFICATE® EXAMINATION

JANUARY 2014

MATHEMATICS
GENERAL PROFICIENCY EXAMINATION

GENERAL COMMENTS

The General Proficiency Mathematics examination is offered in January and May/June each year. There was a candidate entry of approximately 11, 600 in January 2014 and 45 per cent of the candidates earned Grades I–III. The mean score for the examination was 74 out of 180 marks.

DETAILED COMMENTS

Paper 01 − Multiple Choice

Paper 01 consisted of 60 multiple-choice items. It was designed to provide adequate coverage of the content with items taken from all sections of the syllabus. Approximately 76 per cent of candidates earned acceptable grades on this paper; the mean score was 34 out of 60 marks. This year 16 candidates each earned the maximum available score of 60. Sixty per cent of the candidates scored 30 marks or more.

Paper 02 – Structured Questions

Paper 02 consisted of two sections. Section I comprised eight compulsory questions for a total of 90 marks. Section II comprised three optional questions: one each from Algebra, Relations, Functions and Graphs; Measurement, Trigonometry and Geometry; and Vectors and Matrices. Candidates were required to answer any two out of the three questions from Section II. Each question was worth 15 marks. The mean score for this paper was 41 out of 120 marks. This year, no candidate earned the maximum available mark of 120. Approximately 17 per cent of candidates earned at least 60 marks.

Section I: Compulsory Questions

<u>Question 1</u>

This question tested candidates' ability to

- perform the four basic operations on common fractions
- compute the square and positive square root of decimals
- divide decimals
- round off to a given number of decimal places
- solve problems involving cost price, taxes, profit and percentage profit.

The question was attempted by 99 per cent of candidates, 8.2 per cent of whom earned the maximum available mark. The mean mark was 7.03 out of 11.

The performance of candidates was satisfactory. In Part (a), some candidates knew the algorithms for subtracting, dividing and adding fractions although several erred by not following the order of operations. In Part (b), some candidates multiplied by 2 instead of squaring 1.31. Further, a number of candidates rounded off too early in their computations. In Part (c), some candidates made the error of adding money to the number of bracelets, while some obtained a percentage profit that was not logical in the context of the problem. Also, many candidates used an incorrect value for the cost price when calculating the percentage profit.

Solutions

(a) $2\frac{7}{8}$

(b) 2.40

(c) (i) $ 8 160 (ii) a) $3 200.25 b) 39%

Recommendations

Teachers should provide students with opportunities to use the calculator to perform basic arithmetic operations on rational numbers. Attention should also be given to squaring and finding the square root, converting improper fractions to decimals, and estimating the answer and the order in which operations should be performed.

<u>Question 2</u>

This question tested candidates' ability to

- solve a linear in-equation in one unknown and to graphically represent its solution set
- factorize quadratic functions by grouping and by difference of squares
- find the product of two binomial expressions
- solve a pair of simultaneous equations.

The question was attempted by 98 per cent of candidates, 4.8 per cent of whom earned the maximum available mark. The mean mark was 4.84 out of 12.

Candidate performance was unsatisfactory. In Part (a), candidates experienced difficulty working with the inequality sign. It was a common feature to see the $\leq$ sign incorrectly replaced by the $\geq$ sign or the $=$ sign. Further, a large number of candidates who obtained the solution $x \leq 4$ could not represent the solution set on a number line.

In Part (b), candidates were proficient at factorizing the difference of squares and while they chose the correct strategy of factorizing by grouping in Part (b) (i), they encountered difficulty finding the second of the two factors. In Part (c), candidates knew that they needed to use the distributive property but could not complete the process. Generally, candidates encountered difficulty with both the multiplication and addition of directed numbers.

Solutions

(a) (i) $x \leq 4$
(b) (i) $(x - 2y)(3 + a)$ (ii) $(p - 1)(p + 1)$
(c) $2k^2 - 7k + 6$

Recommendations

Teachers should focus students' attention on the algebra of directed numbers and on the use of the number line to represent the solution of linear inequalities.

<u>Question 3</u>

This question tested candidates' ability to

- draw a Venn diagram to illustrate the relationship between two sets
- determine the elements in the intersection, union and complement of two sets
- describe the relationship between two sets using set notation
- formulate and solve a linear equation in one unknown
- determine the area of a composite plane figure bounded by straight lines.

The question was attempted by 99 per cent of candidates, 2.2 per cent of whom earned the maximum available mark. The mean mark was 3.42 out of 10.

The performance of candidates was unsatisfactory. In Part (a) (i), while most candidates were able to draw the Venn diagram, entering the correct value in each subset provided a considerable challenge. In Part (a) (ii), most candidates were able to determine the number of students who studied Spanish. However, they were unable to write, in set notation, the relationship between F and S.

In Part (b), candidates were generally able to derive an expression for the length of the floor. They were nevertheless unable to formulate an expression for the area of the floor and to derive an equation which could be solved to determine the value of x.

Solutions

(a) (ii) $n(S \cap F^{/}) = 12$ (iii) $F \subset S$
(b) (i) $l = 3x + 5$ (ii) a) $x = 5$ b) Area of floor $= 95\ \mathrm{m}^2$

Recommendations

Teachers are advised to constantly review all concepts of set theory. Students should be taught to incorporate algebraic expressions in solving problems in measurement such as finding the perimeter and area.

<u>Question 4</u>

This question tested candidates' ability to

- determine the Cartesian equations of three different lines
- determine the gradient of a line
- identify, by shading, the region which represents the solution set of a linear equation
- derive a system of inequalities which represent a given solution set on a graph
- determine the equation of a line which passes through the origin and is perpendicular to a given line.

The question was attempted by 88 per cent of candidates, less than one per cent of whom earned the maximum available mark. The mean mark was 1.99 out of 11.

Candidate performance was unsatisfactory. In Part (a), candidates recognized the line with equation $y = 2$, but not the lines with equations $y = x$ and $y = x + 2$. In Part (d), candidates demonstrated little proficiency at writing a system of inequalities to represent the solution set shown on the graph. In cases where candidates were able to correctly obtain two of the three inequalities, $x \geq 0$ was invariably omitted. In Part (e), candidates were unable to apply the gradient relationship between perpendicular lines and the significance of a line passing through the origin to write the equation of the required line.

Solutions

(a) (i) $line\ 1: y = x + 2$ (ii) $line\ 2: y = x$ (iii) $line\ 3: y = 2$
(d) $y \leq 2;\quad y \geq x;\quad x \geq 0$
(e) $y = -x$

Recommendations

Teachers should reinforce the basic concepts of inequalities in the senior classes. The use of test points to check the validity of inequalities should also be emphasized.

Question 5

This question tested candidates' ability to

- construct, using ruler and compasses, a triangle and a kite
- use a protractor to determine the measure of an angle
- determine the area of a trapezium
- calculate the volume of a prism
- convert from kilograms to grams.

The question was attempted by 94 per cent of the candidates, 3.2 per cent of whom earned the maximum available mark. The mean mark was 4.15 out of 12. The performance of candidates was generally unsatisfactory. In Part (a), while candidates were able draw lines of given lengths, they lacked the skill of using a pair of compasses to construct a triangle whose three sides were given. Moreover, many candidates experienced difficulty with obtaining, through measurement, the size of one of the angles of the triangle drawn. Further, a large number of candidates could not complete the kite $BACD$ using the triangle BAC which was already constructed or drawn.

In Part (b), candidates performed below expectations at finding the area of a trapezium. However, they were able to use the area calculated to find the volume of the prism. Many candidates could not calculate the mass, in grams, of 1 cm^3 of the metal, as they experienced difficulty with unit conversions.

Solutions

(a) $A\hat{B}C = 53° \pm 1°$
(b) (i) $135\ cm^2$ (ii) $405\ cm^3$ (iii) $3.7\ g$

Recommendations

Teachers should provide students with more opportunities to use mathematical instruments to construct plane shapes. They should also insist that students state the units when giving the results of measurements.

Question 6

This question tested candidates' ability to

- determine the measure of angles using the properties of parallel lines and transversals
- state the coordinates of a point in the plane
- state the length of a line segment drawn in the plane
- describe a transformation in the plane given an object and its image
- determine the location of the image of an object after it has undergone a simple transformation.

The question was attempted by 94 per cent of the candidates, less than one per cent of whom earned the maximum available mark. The mean mark was 3.35 out of 12.

Candidate performance was unsatisfactory. In Part (a), candidates recognized that x and $28°$ were alternate angles and correctly solved that part of the problem. They also recognized that there are two equal angles in an isosceles triangle and used this knowledge to determine the value of y. However, they were less competent at determining the value of *z*. Some candidates who correctly stated the value of *z* could not give the reason for their answer.

In Part (b), candidates were able to state the coordinates of a given point on the graph. While a number of candidates correctly stated the length of $K'L'$ as 2 units, there was a large number who gave 3 units as the measure. In addition, many candidates who correctly described the given transformation as a reflection were unable to state the correct mirror line. Further, the majority of candidates was unable to correctly use the given translation vector to derive the image of a point in the plane, and in many instances when they proceeded to add the vectors, they did not give the response as coordinates.

Solutions

(a) (i) $x = 28°$ (ii) $y = 104°$ (iii) $z = 104°$
(b) (i) $J\left(-4, 1\right)$ (ii) $K'L' = 2$ units (iii) A reflection in the line $y = x$
(iv) $J''\ (1, -2)$; $K''\ (4, -2)$; $L''\ (4, 0)$

Recommendations

Teachers should encourage students to use mathematical terms to describe the relationship between angles formed when parallel lines are crossed by a transversal. Candidates should be drilled in the practice of stating the reason or reasons for answers derived from the geometry of plane figures. Attention should also be given to describing transformations in the plane.

<u>Question 7</u>

This question tested candidates' ability to

- complete a grouped-frequency table to show mid-interval values and frequencies
- determine the size of the sample from information given in a grouped frequency table
- identify class limits, class boundaries and class width
- construct a frequency polygon.

The question was attempted by 94 per cent of the candidates, one per cent of whom earned the maximum available mark. The mean mark was 5.50 out of 12.

Candidates demonstrated proficiency in completing the table to show the midpoints and the class intervals. In addition, they were able to determine, from the table, the number of seedlings in the sample. However, they were less competent at constructing the frequency polygon, finding the class width, and stating the class limits and boundaries. Some candidates experienced difficulty differentiating between the vertical and horizontal axes. Moreover, a significant number of candidates were unable to correctly use the given scales to plot the points associated with the polygon.

Solutions

(a) 85 seedlings

(b) (i) lower class limit is 8 (ii) upper class boundary is 12.5 (iii) class width is 5

(c)

Height (in cm)	Midpoint	Frequency
18–22	20	16
23–27	25	22
28–32	30	18
33–37	35	14
38–42	40	0

Recommendations

Teachers should approach the construction of histograms and frequency polygons separately. Emphasis should be placed on the definition of a polygon and the shared properties of line graphs and frequency polygons. The correct use of scales in plotting points on graphs is an important concept which should be emphasized in all related areas of mathematics.

<u>Question 8</u>

This question tested candidates' ability to

- draw the fourth figure in a sequence of shapes in which the first three figures in the sequence are given
- determine the terms of a sequence for which the values of the function are given
- derive algebraic expressions related to the general term of a sequence.

The question was attempted by 97 per cent of the candidates, 10 per cent of whom earned the maximum available mark. The mean mark was 6.53 out of 10.

The performance of candidates was good. They demonstrated competence in answering most of the questions. In Part (a), obtaining the fourth diagram in the sequence proved challenging; many candidates drew triangles instead of trapezia. In Part (b) (iv), when writing the general formulae based on the given sequences, candidates resorted to $\times 4$ and $\times 4 + 2$ or 4^n and $4^n + 2$ instead of $4n$ and $4n + 2$.

Solution

(b)

No. of Trapezoids	Triangles	Dots
4	16	18
10	40	42
16	64	66
n	4n	4n+2

Recommendations

Teachers should provide experiences for students translating verbal statements to algebraic representations and vice versa. Further, students need practice in exploring patterns in the real world and representing these patterns algebraically.

Section II: Optional Questions

<u>Question 9</u>

This question tested candidates' ability to

- substitute numbers for symbols expressed as functions and composite functions
- determine the inverse of a rational function
- derive the composite of two linear functions
- interpret information given on the graph of a quadratic function
- determine the roots of a quadratic equation, its intercept on the *y*-axis and the coordinates of its minimum point.

The question was attempted by 70 per cent of the candidates, 1.5 per cent of whom earned the maximum available mark. The mean mark was 3.62 out of 15.

Candidates performed unsatisfactorily. In Part (a), they were generally able to determine the image of a domain member of a function and the composite of functions. While they were aware that they needed to reverse the mapping to find the inverse function, they lacked the algebraic skills to follow through with the process. Further, the inability to correctly transpose to find the inverse was a common feature.

In Part (b), a significant number of candidates were unfamiliar with the term *roots* of a function and were unable to interpret information shown on the graph.

Solutions

(a) (i) $g(4) = 10$; $h\big(g(4)\big) = -2$

(ii) $h^{-1}(x) = \frac{10}{x+3}$; $g\big(g(x)\big) = 9x - 8$

(b) (i) $x = -1, 5$ (ii) $c = -5$; (iii) $x = 2,\ y = -9$

Recommendations

Teachers should provide students with more opportunities to transpose formulae. Attention needs to be given to the association between the solutions of a quadratic equation and the term *roots* of the function. The pictorial representation of 3-set mapping diagrams should be utilized when introducing composite functions.

<u>Question 10</u>

This question tested candidates' ability to

- use the properties of circles, parallel lines, transversals and circle theorems to determine the measure of angles
- use the cosine rule to determine the length of the side of a triangle
- calculate the area of a triangle given two sides and the included angle
- solve problems related to bearings.

The question was attempted by 46 per cent of candidates, less than one per cent of whom earned the maximum available mark. The mean mark was 2.35 out of 15.

Generally, candidates performed unsatisfactorily. In Part (a), several candidates demonstrated that they knew that the angle at the circumference in a semicircle is a right angle and used this fact to calculate angle $\angle$FAW. However, the majority of candidates attempting this question failed to identify relationships associated with transversals crossing parallel lines, such as co-interior and alternate angles in the given diagram.

In Part (b), candidates selected the cosine formula as the appropriate rule in calculating QR. They nevertheless were unable to use the angle and bearing given to calculate the bearing of R from P. Further, finding the intermediate angles required to compute the bearing was problematic for most candidates.

Solutions

(a) (i) $F\hat{A}W = 36°$ (ii) $S\widehat{K}F = 126°$ (iii) $A\hat{S}W = 64°$

(b) (i) a) $61.3\ km$ b) $3516.6\ km^2$ (ii) $95°$

Recommendations

Teachers should expose their students to higher order geometric problems which require the use of basic concepts such as alternate angles, co-interior angles and bearings.

Question 11

This question tested candidates' ability to

- invert a two by two matrix
- determine the coordinates of the pre-image of a point given a transformation in the form of a matrix and the coordinates of the image
- solve problems in geometry using vectors.

The question was attempted by 48 per cent of the candidates, less than one per cent of whom earned the maximum available mark. The mean mark was 3.35 out of 15.

Candidates performed unsatisfactorily on this question. While they were generally able to invert the two by two matrix, they could not apply this inverse to determine the coordinates of the point which was mapped onto a given point by the original matrix. In vector geometry, candidates were proficient in sketching the triangle and inserting the point L two-thirds the distance along MN. However, they encountered difficulty attempting to use vector algebra to solve the problem. The major challenge was expressing $\overrightarrow{ML}$ in terms of m and n.

Solutions

(a) (i) $T^{-1} = \frac{1}{7}\begin{pmatrix} 3 & 1 \\ -1 & 2 \end{pmatrix}$ (ii) $a = 3,\ b = 2$

(b) (ii) a) $\overrightarrow{\mathrm{MN}} = -\mathrm{m} + \mathrm{n}$ b) $\overrightarrow{\mathrm{ML}} = \frac{2}{3}(-\mathrm{m} + \mathrm{n})$

(b) (iii) $\overrightarrow{\mathrm{OL}} = \begin{pmatrix} 7 \\ 2 \end{pmatrix}$

Recommendations

Teachers should reinforce students’ knowledge of geometric concepts through the use of manipulatives and authentic tasks. They should consider revisiting ratios when teaching vector geometry. Some attention should be given to the application of the inverse matrix in the solution of problems involving linear equations in two unknowns.

C A R I B B E A N   E X A M I N A T I O N S   C O U N C I L

REPORT ON CANDIDATES’ WORK IN THE CARIBBEAN SECONDARY EDUCATION CERTIFICATE® EXAMINATION

MAY/JUNE 2014

MATHEMATICS
GENERAL PROFICIENCY EXAMINATION

GENERAL COMMENTS

The General Proficiency Mathematics examination is offered in January and May/June each year. There was a candidate entry of approximately 90,100 in May/June 2014 and 50 per cent of the candidates earned Grades I–III. The mean score for the examination was 75 out of 180 marks.

DETAILED COMMENTS

Paper 01 — Multiple Choice

Paper 01 consisted of 60 multiple–choice items. It was designed to provide adequate coverage of the content with items taken from all sections of the syllabus. Approximately 70 per cent of the candidates earned acceptable grades on this paper; the mean score was 33 out of 60 marks. This year, 271 candidates earned the maximum available score of 60.

Paper 02 — Structured Questions

Paper 02 consisted of two sections. Section I comprised eight compulsory questions for a total of 90 marks. Section II comprised three optional questions: one each from Algebra, Relations, Functions and Graphs; Measurement, Trigonometry and Geometry; and Vectors and Matrices. Candidates were required to answer any two of the three questions from this section. Each question in this section was worth 15 marks. The mean score for this paper was 42 out of 120 marks.

Compulsory Section

<u>Question 1</u>

This question tested candidates' ability to

- use the calculator to divide decimals
- evaluate the square and the square root of rational numbers
- write a rational number correct to three significant figures
- solve problems involving hire purchase.

The question was attempted by 99 per cent of the candidates, 12.9 per cent of whom earned the maximum available mark. The mean mark was 7.21 out of 12.

In Part (a), candidates generally provided satisfactory responses. Those who used the calculator were able to perform the computations but did not approximate the result of the third computation to three significant figures.

In Part (b), a large number of candidates experienced difficulty in deriving the number of buckets of gravel that should be mixed with 4 buckets of cement. Two popular incorrect responses to this part of the question were: $\frac{4}{11} \times 6$ and $\frac{6}{11} \times 4$. Many candidates attempted to use the proportion 1:4:6 and any of the numbers given in the question to compute the required values.

In Part (c), most candidates were able to calculate the hire purchase price by adding the down payment to the total paid by instalments. However, some candidates incorrectly interpreted hire purchase to mean simple interest, and hence produced the following result: $\text{HP} = \frac{\text{P} \times \text{R} \times \text{T}}{100} = \frac{350 \times 120 \times 10}{100}$. Another common error in the solution to this part of the question was: Amt saved = Cost price − Hire purchase price.

Solutions

(a) **(i)** **350** **(ii)** **2.55** **(iii)** **15.7**

(b) **(i)** **24** **(ii) a)** **5** **b)** **30**

(c) **(i)** **$1550** **(ii)** **$251**

Recommendations

Teachers should provide students with a wide range of real life problems which require the appropriate use of mathematical computations including ratio and proportion. Students are encouraged to use calculators to perform mathematical calculations.

Question 2

This question tested candidates' ability to

- simplify algebraic fractions
- translate worded problems into algebraic expressions
- write an algebraic equation from information given in a flow diagram
- solve a pair of linear equations in two unknowns.

The question was attempted by 99 per cent of the candidates, 6.9 per cent of whom earned the maximum available mark. The mean mark was 5.69 out of 12.

The performance of candidates on this question was satisfactory. In Part (a), while most candidates were able to apply the algorithm for adding two algebraic fractions, they generally encountered problems with the use of directed numbers and hence did not simplify the numerator correctly.

In Part (b), candidates experienced difficulty with obtaining the right side of each equation. Two common incorrect responses were: $(i)\ x + 4 = \frac{x}{0.5}$, and (ii) $x^2 - 6 = x^2 + 9$.

In Part (c), most candidates were able to translate the information given in the flow charts into algebraic expressions and formulae but encountered difficulty with changing the subject of the formula to find the value of x when the value of y is given.

In Part (d), the majority of candidates identified a suitable strategy for solving the pair of simultaneous equations. However, when elimination was the chosen strategy, candidates who equated the coefficients of y proceeded to subtract the two equations even when they had $+3y\ and - 3y$ with which to work. Hence, the solution could not be completed.

Solutions

(a) $\frac{7x-5}{12}$

(b) **(i)** $x + 4 = \frac{x}{2} + 10$ **(ii)** $x^2 - 6 = 2x + 9$

(c) **(i)** $3x + 5$ **(ii)** **17** **(iii)** **1** **(iv)** $x = \frac{y-5}{3}$

(d) $x = 3,\ y = 1$

Recommendations

Teachers should emphasize the correct application of the distributive property, reinforce the distinction between an expression and an equation, provide candidates with opportunities to refresh their use of directed numbers and clarify in the minds and students when to add or subtract equations in the process eliminating a variable from a pair of simultaneous equations.

<u>Question 3</u>

This question tested candidates' ability to

- determine the number of elements in a set of integers bounded by two given integers
- list the elements in a defined set
- draw Venn diagrams to represent the relationship among three sets
- use a ruler to measure the length of a line segment
- use a protractor to determine the measure of an angle
- construct an angle of 60°
- construct a perpendicular to a line from a point outside the line.

The question was attempted by 98 per cent of the candidates, 2.4 per cent of whom earned the maximum available mark. The mean mark was 5.49 out of 12.

In Part (a), candidates were generally able to list the even numbers in their universal set but encountered difficulty with identifying the multiples of three from the same set. Most candidates were able to identify the intersection of the two sets they listed even when they misinterpreted the word "between". In addition, they demonstrated good proficiency in representing $A \cap B$, $A \cap B'$ and $B \cap A'$ on the Venn diagram but showed little understanding of what should occupy the region: $(A \cup B)'$.

In Part (b), while candidates were generally able to draw lines accurately and to construct an angle of 60°, they nevertheless encountered difficulty with identifying the angle to be measured and constructing a perpendicular to a given line from a point outside the line.

Solutions

(a) (i) 14 (ii) $\boldsymbol{A = \{12, 14, 16, 18, 20, 22, 24\}}$ **(iii)** $\boldsymbol{B = \{12, 15, 18, 21, 24\}}$

(b) (ii) $\boldsymbol{44°}$

Recommendations

In addition to teaching the required content, teachers should also focus on the mathematical terms in respective subject areas. Many students may know the content but experience challenges when exposed to terms such as between, inclusive, exclusive, at most, at least, in the context of mathematics.

<u>Question 4</u>

This question tested candidates' ability to

- use the scale of a map to determine lengths
- use the scale of a map to determine areas on a map
- use the scale of a map to determine actual distances and areas on the land.

The question was attempted by 92 per cent of the candidates, 1.1 per cent of whom earned the maximum available mark. The mean mark was 3.13 out of 10.

While most candidates interpreted the scale correctly, they were generally unable to transfer this understanding to the concept of area. Moreover, of those candidates who correctly computed that $1\ cm = 0.00001\ km$, few were able to continue the process to calculate the land represented by 8 cm on the map.

In Part (b), there were errors in the measurement of the line segment LM with a large number of candidates stating the length as 7 cm. Similarly, in Part (c), determining the area of the forest reserve on the map proved difficult for many candidates. Even when they counted the number of squares correctly, they proceeded to square this value. Many candidates could not make the conversion from cm^2 to km^2 although they were given the conversion factor from kilometres to centimetres.

Solutions

(a) **(i)** **50 000 cm** **(ii)** $(\mathbf{50\ 000})^{\mathbf{2}}\ \boldsymbol{cm^2}$ **(iii)** $\mathbf{0.5}\ \boldsymbol{km}$

(b) **(i)** $\mathbf{8}\ \boldsymbol{cm}$ **(ii)** $\mathbf{4}\ \boldsymbol{km}$

(c) **(i)** $\mathbf{15}\ \boldsymbol{cm^2}$ **(ii)** $\mathbf{3.75}\ \boldsymbol{km^2}$

Recommendations

Students require additional practice in applying scales to the solution of problems related to distances and areas. It is suggested that authentic examples be developed using maps of the school and immediate community.

<u>Question 5</u>

This question tested candidates' ability to

- describe a translation in the plane
- draw the image of a triangle after undergoing an enlargement
- solve problems involving angles of elevation
- solve problems involving trigonometric ratios.

The question was attempted by 90 per cent of the candidates, 1.7 per cent of whom earned the maximum available mark. The mean mark was 2.19 out of 12.

The performance of candidates on this item was generally unsatisfactory. In Part (a), candidates demonstrated some competence in determining the image under an enlargement. However, many of them did not use the given centre of enlargement even though the correct identified the origin as the centre of enlargement. Moreover, they were generally able to describe in words the transformation that mapped triangle ABC onto triangle $A''B''C''$, although only few of them used the conventional vector notation.

In Part (b), candidates in general stated that the angle of elevation of T from P is $40°$ even though a small number chose the complement of that angle and stated the result as $50°$. Candidates, in an attempt to determine the length of FP, correctly chose the tangent ratio. However, to determine the measure of the angle of elevation of T from Q, some candidates used Pythagoras' theorem to find TP, and then proceeded to use the sine rule or the cosine rule to find the measure of angle TQP. The result was more easily obtained using the right-angled triangle TQF.

Solutions

(a) **(ii)** $\boldsymbol{M\ is\ a\ translation\ by\ the\ vector}\ \begin{pmatrix} \mathbf{4} \\ \mathbf{-5} \end{pmatrix}$

(b) **(i)** $\mathbf{40°}$ **(ii)** $\mathbf{95.3}\ \boldsymbol{m}$ **(iii)** $\mathbf{20.6°}$

Recommendations

Teachers should give attention to the description of transformations, ensuring that students are familiar with the language used to describe such transformations. Students should be guided as to when it is appropriate to use trigonometric ratios and when to use the more complex trigonometric formulae.

Question 6

This question tested candidates' ability to

- read coordinates of points plotted on a graph
- determine the gradient of a line drawn on a graph
- determine the equation of a line drawn on a graph
- determine the equation of a line which is parallel to a given line and which passes through the origin
- estimate the gradient of the tangent at a given point on the graph of a quadratic function.

The question was attempted by 80 per cent of the candidates, 2.5 per cent of whom earned the maximum available mark. The mean mark was 2.59 out of 11.

In Part (a), candidates were generally successful in determining the values of $x\ and\ y$ by taking readings from the graph. However, some candidates used the equation of the curve to derive these values. For example: $y = x^2\ so\ x = \sqrt{y} = \sqrt{9} = 3$ and $y = (-1)^2 = 1$.

In Part (b), most candidates used the formula: $m = \frac{\Delta y}{\Delta x}$ to calculate the gradient of MN. However, there were instances when they had correctly substituted into this formula and failed to determine the correct value of the gradient because of challenges with the subtraction of directed numbers. Moreover, when asked to write the equation of the line which passes through $(0, 0)$ and parallel to MN, some candidates did not recognize that the gradients were the same but proceeded to use the negative reciprocal of the gradient of MN as the gradient of the line whose equation was required.

In Part (c), candidates appeared not to be acquainted with concept of a tangent to a curve at a given point. Many of them simply drew a horizontal line at $(2, 4)$.

Solutions

(a) **(i)** $\boldsymbol{x = 3}$ **(ii)** $\boldsymbol{y = 1}$

(b) **(i)** **2** **(ii)** $\boldsymbol{y = 2x + 3}$ **(iii)** $\boldsymbol{y = 2x}$

(d) **The gradient of tangent at (2, 4) is 4**

Recommendations

Teachers should guide students through the concept of drawing the tangent to a curve at a point and to demonstrate some applications of such tangents. They should reinforce the distinction between the gradients of parallel and perpendicular lines.

Question 7

This question tested candidates' ability to

- complete a frequency table
- determine the mode of a given set of data
- estimate the mean of a frequency distribution
- determine the probability of a simple event from randomly sampling a data set.

The question was attempted by 95 per cent of the candidates, 4.8 per cent of whom earned the maximum available mark. The mean mark was 5.27 out of 11.

Candidates performed moderately on this question. The majority were able to complete the tally column and the associated frequency column from the raw data provided, as well as the column with $\boldsymbol{f} \times \boldsymbol{x}$. However, many candidates did not understand the significance of these values since they were not used to compute the mean of the data set. Moreover, when candidates correctly stated the formula for deriving the mean number of books per bag, many of them proceeded to divide the total number of books by a number different from their value of $\sum f$.

In Part (d), in determining the probability that a student chosen at random had less than 4 books in his/her bag, many candidates were able to correctly determine only one of the two quantities, that is, either the favourable outcomes or the possible outcomes. Several candidates quoted a value of 10 as the probability, which is a clear indication that they did not appreciate the possible range of a probability measure.

Solutions

(a)

$(\boldsymbol{x})$	**Tally**	**Frequency(f)**	$\boldsymbol{f} \times \boldsymbol{x}$
3	~~////~~	**5**	**15**
4	~~////~~ /	**6**	**24**
5	~~////~~ / /	**7**	**35**
6	////	**4**	**24**
7	///	**3**	**21**

(b) **mode = 5 text books**

(c) **Total number of books is 127** **(ii)** **mean = 4.2 books per bag**

(d) $\boldsymbol{P(text\ books} < 4) = \frac{\mathbf{10}}{\mathbf{30}} = \frac{\mathbf{1}}{\mathbf{3}}$

Recommendations

Teachers need to provide students with more practice in identifying the measures of central tendency of a data set, especially for data presented in a table. They should reinforce the convention for representation of tally. Emphasis should be given to the fact that a probability value lies between 0 and 1.

<u>Question 8</u>

This question tested candidates’ ability to

- draw the fourth diagram in a given sequence of diagrams
- complete a table to show the values in a sequence of numbers
- derive the general rule representing the patterns in a sequence.

The question was attempted by 95 per cent of the candidates, 6.2 per cent of whom earned the maximum available mark. The mean mark was 6.60 out of 10.

Candidates’ responses to this question were generally good. They were able to draw the fourth figure in the sequence of figures even though they did not always insert the correct number of dots on each side of the pentagon.

In Part (b) (i), most candidates were able to follow the pattern in the formula column and to determine the correct number of dots on each figure by counting even when they did not write the correct formula. Writing an algebraic expression for the number of dots associated with each term in the sequence proved challenging for most candidates.

Solutions

(b) **(i)** $\boldsymbol{formula: 5 \times 6 - 5; \quad n = 25}$ **(ii)** $\boldsymbol{formula: 5 \times 7 - 5; \quad n = 30}$

(c) $\boldsymbol{n = 5(f + 1) - 5 = 5f}$

(d) $\boldsymbol{f = 29}$

Recommendations

Teachers need to extend the teaching of patterns and sequences to include deriving the general algebraic formulae for these sequences.

Optional Section

<u>Question 9</u>

This question tested candidates’ ability to

- determine the value of x for which a rational function is undefined
- evaluate the composite of a function
- determine the inverse of a rational function
- use a table of values to draw the graph of a non-linear relation
- interpret a graph to determine unknown information.

The question was attempted by 73 per cent of the candidates, less than 1 per cent of whom earned the maximum available mark. The mean mark was 4.74 out of 15.

Candidates’ performance on this question was generally unsatisfactory.

In Part (a), the majority of candidates were unable to determine the value of x for which a rational function is undefined. Further, although many of the candidates were able to correctly substitute $f(5)$ into $g(x)$, they lacked the algebraic skills to simplify the response. Many candidates, in an attempt to derive the inverse of $f(x)$, reached as far as interchanging $x \; and \; y$ in the original equation. However, they did not demonstrate the algebraic skills to correctly proceed to make y the subject of the formula in their reversed mapping.

In Part (b), the majority of candidates demonstrated a high level of proficiency in using the correct scales and plotting the given points. Nevertheless, a large number of these candidates connected the points with straight lines instead of drawing a parabola as required. Also, a large number of candidates were unable to determine the average speed of the ball during the first two seconds of its motion, and among those who provided an answer, the correct unit of speed was not used. Moreover, the majority of candidates could not determine the speed of the ball at $\mathrm{t} = 3$ since they did not associate this speed with the gradient of the tangent at the greatest height reached by the ball.

Solutions

(a) **(i)** $x = -1$ **(ii)** $gf(5) = \frac{43}{3}$ **(iii)** $f^{-1}(x) = \frac{x-7}{2-x}$

(b) **(ii)** a) $40\ ms^{-1}$ b) $0\ ms^{-1}$

Recommendations

Teachers should strengthen basic algebraic and computational skills of students prior to teaching the functions. Students must be able to differentiate between the composite and the inverse function. The concept of tangent at a point on a curve needs to be reinforced.

<u>Question 10</u>

This question tested candidates' ability to

- use the properties of circles and circle theorems to determine the measures of angles
- use cosine and sine rules to solve problems related to bearings.

The question was attempted by 43 per cent of the candidates, 2.2 per cent of whom earned the maximum available mark. The mean mark was 2.70 out of 15.

Candidates' performance on this question was generally unsatisfactory.

In Part (a), which required the use of the angle properties of triangles and circles, many candidates based their reasoning on the properties of triangles only instead of referring to circle theorems.

In Part (b), candidates lacked proper understanding of bearings. A common response to the calculation of the bearing of P from Q was $180° - (66° + 54°)$ instead of $(66° + 54°)$. In addition, candidates made the assumption that they were working with a right angled triangle and proceeded to employ trigonometric ratios in an attempt to calculate the length of PR and the measure of angle QPR. A simple application of the cosine formula followed by an application of the sine formula was required to complete these calculations.

Solutions

(a) **(i)** **140°** **(ii)** **22°** **(iii)** **110°**

(b) **(i)** $120°$ **(ii)** $PR = 83.64\ km$ **(iii)** $75°$

Recommendations

Teachers should use the appropriate jargon in the teaching of circle theorems. They should assist students in making the determination as to when to use trigonometric ratios and when to use trigonometric formulae.

<u>Question 11</u>

This question tested candidates' ability to

- solve for one unknown in a singular matrix
- use the matrix method to solve an equation with two unknowns
- recognize the position vector of a point in the plane
- write the coordinates of points in the plane as position vectors
- use vector geometry to determine the resultant of two or more vectors
- use the properties of equal vectors to solve problems in geometry.

The question was attempted by 37 per cent of the candidates, less than 1 per cent of whom earned the maximum available mark. The mean mark was 3.90 out of 15.

Candidates performed unsatisfactorily on this question. Most candidates seemed unaware of the condition under which the matrix would not have an inverse, that is, when $|M| = 0$. A few candidates were able to express the simultaneous equations in the required form and some continued to attempt to find a solution although this was not required.

In Part (c), candidates generally wrote the position vectors as required. However, challenges were experienced in determining the coordinates of the point R and identifying the type of quadrilateral.

Solutions

(a) $\boldsymbol{p = \frac{-7}{2}}$

(b) $\begin{pmatrix} \mathbf{4} & \mathbf{-2} \\ \mathbf{2} & \mathbf{3} \end{pmatrix} \begin{pmatrix} \boldsymbol{x} \\ \boldsymbol{y} \end{pmatrix} = \begin{pmatrix} \mathbf{0} \\ \mathbf{4} \end{pmatrix}$

(c) **(i)** $\overrightarrow{\boldsymbol{OP}}$ **is the position vector of the point** $\boldsymbol{P}$

(ii) **a)** $\overrightarrow{\boldsymbol{OP}} = \begin{pmatrix} \mathbf{2} \\ \mathbf{4} \end{pmatrix}$; **b)** $\overrightarrow{\boldsymbol{OQ}} = \begin{pmatrix} \mathbf{8} \\ \mathbf{2} \end{pmatrix}$; **c)** $\overrightarrow{\boldsymbol{PQ}} = \begin{pmatrix} \mathbf{6} \\ \mathbf{-2} \end{pmatrix}$

(iii) $\boldsymbol{R}\ (\mathbf{6}, \mathbf{-2})$

(iv) $\boldsymbol{PQRO}$ **is a parallelogram. It is a quadrilateral with a pair of opposite sides equal and parallel.**

Recommendations

Teachers should encourage students to utilize diagrams to add clarity to their responses when solving problems on Vector Geometry. They should reinforce the concept that a resultant vector can be derived from the sum or difference of two or more position vectors. They should provide students with opportunity to develop the art of identifying the type of quadrilateral formed from a system of vectors in which there are equal or parallel combinations. They should provide students with more practice in writing the matrix equation corresponding to a pair of simultaneous linear equations.

CARIBBEAN EXAMINATIONS COUNCIL

REPORT ON CANDIDATES' WORK IN THE
CARIBBEAN SECONDARY EDUCATION CERTIFICATE® EXAMINATION

JANUARY 2015

MATHEMATICS
GENERAL PROFICIENCY EXAMINATION

GENERAL COMMENTS

The General Proficiency Mathematics examination is offered in January and May/June each year. There was a candidate entry of approximately 9970 in January 2015. Approximately 65 per cent of the candidates earned Grades I–III. The mean score for the examination was 88 out of 180 marks.

DETAILED COMMENTS

Paper 01–Multiple-Choice

Paper 01 consisted of 60 multiple choice items. It was designed to provide adequate coverage of the content with items taken from all sections of the syllabus. This year, no candidate earned the maximum available score of 60. Fifty-nine per cent of the candidates scored 30 marks or more.

Paper 02–Structured Questions

Paper 02 consisted of two sections. Section I comprised eight compulsory questions totalling 90 marks. Section II comprised three optional questions: one each from Algebra, Relation, Functions and Graphs; Measurement, Trigonometry and Geometry; and Vectors and Matrices. Candidates were required to answer any two questions from this section. Each question in this section was worth 15 marks. The mean score for this paper was 54 out of 120 marks. This year, six candidates earned the maximum available mark of 120.

Section I: Compulsory Questions

Question 1

This question tested candidates' ability to

- perform computations on decimals
- solve worded problems involving fractions
- convert from one currency to another given an exchange rate.

The question was attempted by 99 per cent of the candidates, 9.8 per cent of whom earned the maximum available mark. The mean mark was 7.16 out of 11.

Candidates' performance on this question was satisfactory. In Part (a), candidates were generally proficient at computing the square of the decimal and at subtracting from this value their result of the division. However, there were cases where candidates multiplied by 2 instead of squaring, and instances where the division by a decimal was done incorrectly.

In Part (b) (i), most candidates attempted multiplication by $\frac{1}{3}$ even when they failed to recognize that the fraction in the product was the result of $\left(1-\frac{3}{8}\right)$. In Part (b) (ii), while some candidates recognized that to determine the fraction saved required that they subtract from 1, they generally experienced difficulty with determining what to subtract from 1. In Part (c), most candidates multiplied by $1.96 to convert from US$ to BD$, and divided by $700 to determine the exchange rate used.

Solutions

(a) 83.84

(b) (i) $\frac{5}{24}$ (ii) $\frac{5}{12}$

(c) (i) BD$1372 (ii) US$1 = BD$ 1.98

Recommendations

Teachers should provide practice involving computation of fractions in authentic situations. They should continue to provide candidates with opportunities to work with the calculator in performing basic arithmetic operations on rational numbers. Attention should also be given to the difference between the square and the square root, converting from improper fractions and to decimals, estimation and the order of operations.

<u>Question 2</u>

This question tested candidates' ability to

- apply the rules of indices for multiplication of terms
- add two fractional algebraic terms with numerical denominators
- factorize expressions
- solve a linear inequality in one variable
- determine the value of an expression by substituting given values.

The question was attempted by 94 per cent of the candidates, 5.4 per cent of whom earned the maximum available mark. The mean mark was 5.59 out of 12.

The performance of candidates on this question was satisfactory. Most candidates demonstrated an understanding of adding fractions as evidenced by finding a correct common denominator and getting at least one numerator correct. However, a number of candidates simply added the numerator and the denominator with $\frac{4a}{7}$ being the most popular incorrect response. In Part (c) (i), candidates experienced difficulty arriving at the correct factors. They generally did not make the connection that they needed $-4x$ and $-x$ to replace the term $-5x$. Many who chose to use 4 and 1 as the appropriate factors of 4 for this situation proceeded to use incorrect signs in at least one of the two brackets resulting in the factors $(x \pm 4)(x \pm 1)$ with alternating signs in each bracket. In Part (d) (ii), several candidates included 0 in their list of positive integer values for x. Many were unable to correctly solve the inequality and some were unable to correctly list values in Part (d) (ii) based on their answer to Part (d) (i).

Solutions

(a) p^4q^7
(b) $\frac{17}{10}a$
(c) (i) $(x-1)(x-4)$ (ii) $(m-2n)(m+2n)$
(d) (i) $x \leq 5$ (ii) $x = \{1,2,3,4,5\}$
(e) 1.57

Recommendations

Teachers should encourage the use of teaching aids such as algebra tiles to factorize expressions. Manipulation of fractional expressions should be thoroughly reviewed since many candidates indicated incorrect application of the procedure for addition of fractions.

Question 3

This question tested candidates' ability to

- complete a Venn diagram to illustrate the relationship between two sets
- determine the elements in the intersection, union and complement of two sets
- write an expression in x to represent the total number of elements in the Universal set
- formulate and solve a linear equation in one unknown
- use geometric tools to construct a parallelogram.

The question was attempted by 97 per cent of the candidates, 8.0 per cent of whom earned the maximum available mark. The mean mark was 6.74 out of 12.

The performance of candidates on this question was satisfactory. In Part (a), a large percentage of candidates demonstrated knowledge of Venn diagrams accurately completing the diagram from the information provided. However, some candidates experienced difficulty in dealing with the '*x*' to obtain, in terms of *x*, the number in the subsets of C only and D only. The majority of the candidates was able to use 'their' diagram to form both an expression *x* and an equation in *x* for the number of families in the survey. It was however evident that there were those who did not know the difference between an expression and an equation. In addition, many candidates proceeded to solve the equation for *x* even though they were not required to do so.

In Part (b), the majority of candidates was able to draw the lines AB and AD of length 8 cm and 6 cm, respectively. A moderate percentage of candidates constructed the 60° angle but was unable to construct the 120° angle at B. However, many clearly used a protractor and then tried to place the arcs. At least one construction mark was lost as a result. Many candidates simply reproduced the shape through the use of protractors and set squares for which they lost the construction marks. There were a few instances where the completed shape did not resemble a parallelogram even though the shape was provided on the question paper.

Solutions

(a) (ii) $35 - x$ (iii) $35 - x = 30$

Recommendations

Candidates need to be able to distinguish between an expression and an equation.

Attention needs to be given to dealing with ‘both’ common elements of subsets in the set. Students need greater exposure to construction using compasses instead of the protractor to produce angles such as 120°, 150°.

Question 4

This question tested candidates’ ability to

- substitute given values into a linear equation to determine the missing values in a table
- plot coordinates on a Cartesian plane
- draw a straight line graph to show the relationship between sets of ordered pairs
- use a graph to determine given values.

The question was attempted by 96 per cent of the candidates, 23.4 per cent of whom earned the maximum available mark. The mean mark was 8.71 out of 12.

Candidates’ performance on this question was good. They were able to complete the missing values in the table, determine the total charge for a given number of hours worked and the fixed charge. Nevertheless, several candidates interchanged the axes when drawing the graph of total charge against time spent on the job. Furthermore, a large number of candidates disregarded the suggested scale and a few used the values from the table to label the vertical axis and plot these with the corresponding number of hours. Most candidates failed to use the graph to deduce the total charge for 4.5 hours and the time worked to earn \$300.

Solutions

(a) $when\ x = 2, y = 155;\ \ when\ x = 4, y = 235$
(c) (i) \$255 (ii) 5.6 hrs (iii) Fixed charge is \$75

Recommendations

Teachers should ensure that students are familiar with using different scales on graph paper as well as plotting coordinates for various situations and reading coordinates from graphs.

Question 5

This question tested candidates' ability to

- write the coordinates of a point
- draw the image of a triangle after a reflection in the *y*-axis
- describe, using vector notation, the transformation which maps one triangle onto another in the plane
- describe the series of transformations which map one triangle to another in the plane
- explain why two given triangles are congruent.

The question was attempted by 96 per cent of the candidates, 4.3 per cent of whom earned the maximum available mark. The mean mark was 6.48 out of 12.

The performance of candidates on this item was generally satisfactory. They demonstrated high proficiency at writing the coordinates of N and drawing the reflection of triangle LMN in the *y*-axis. However, candidates experienced difficulty providing a description using vector notation of the transformation which mapped $\Delta\ LMN$ onto $\Delta\ PQR$, describing the combination of transformations which mapped $\Delta\ PQR$ onto $\Delta\ GFH$, and stating the properties of congruency.

Solutions

(i) $N(4,\ 5)$ (iii) $\binom{0}{-6}$
(iv) translation parallel to the *y*-axis followed by a reflection in the *y*-axis;
(v) any two cases for congruency of triangles such as same shape, same measure for angles, same length for corresponding sides

Recommendations

Teachers should teach students different ways of describing transformations and combinations of transformations. In addition, they should guide students in deriving the properties of congruency and similarity through discovery methods.

Question 6

This question tested candidates' ability to

- measure and state the length of a line
- determine the scale used on a drawing

- determine the area of a composite figure by taking measurements
- determine the length of the diameter of a semicircle
- calculate the area of a composite figure.

The question was attempted by 87 per cent of the candidates, 2 per cent of whom earned the maximum available mark. The mean mark was 4.07 out of 11.

The performance of candidates on this question was unsatisfactory. In Part (a), most candidates correctly measured and stated the length of PQ. However, they generally recognized that the figure was a compound one comprising of two separate areas and the formulae for finding the area of a rectangle and a triangle were appropriately applied. Nevertheless, they experienced difficulty determining the scale of the drawing and the units were very often ignored, resulting in the popular incorrect answer 1:3. Calculating the actual area of the face of the building required use of the actual length of PQ, but converting PQ to an actual value by multiplying by the scale was often overlooked. In Part (b), candidates knew that the lengths of the straight sides had to be included in the perimeter of the swimming pool. However, some candidates viewed the pool as two separate figures — a circle and a rectangle — so the diameter was included in the final sum.

Solutions

(a) (i) $\mathrm{PQ} = 3.4\ \mathrm{cm}$ (ii) $1:300$ (iii) area of $\mathrm{LMPK} = 256.5\ \mathrm{m}^2$
(b) (i) 3.5 m (ii) 27 m

Recommendations

Teachers should encourage students to pay closer attention to units of measurement. They should provide opportunities for students to solve problems associated with scale diagrams and distinguishing between the area and perimeter.

<u>Question 7</u>

This question tested candidates' ability to

- complete the frequency column in grouped frequency table, given the histogram
- determine the cumulative frequency
- construct a cumulative frequency curve
- determine the median mass, using the graph drawn.

The question was attempted by 94 per cent of the candidates, 11.2 per cent of whom earned the maximum available mark. The mean mark was 5.55 out of 10.

Candidates performed satisfactorily on this question. In Part (a), they demonstrated a high level of proficiency in transferring information from the histogram to the grouped frequency table and in completing the cumulative frequency column. In Part (b), candidates competently used the given scales on the axes provided and to draw the ogive once they had plotted the points correctly on the grid. Nevertheless, a number of candidates used the incorrect coordinates to plot the points for the cumulative frequency curve. Many of them plotted the points by using the upper class limits on the horizontal axis and the class frequency on the vertical axis. Some candidates experienced difficulty reading off the median from their graph. Several candidates arranged the numbers from the frequency column in ascending order and then selected the mid-value from those numbers in an attempt to determine the median.

Solutions

(a) (i), (ii)

Mass (kg)	No. of Parcels	Cumulative Frequency
16–20	15	
21–25		57
26–30	3	

(c) 15 kg

Recommendations

Teachers should ensure that students are aware of the values required to plot and draw a cumulative frequency curve, that is, the upper boundary and cumulative frequency. In addition, they should provide exercises where students need to read values from a graph with different scales. Attention should also be given to the determination of the median from raw data, from grouped data and from the cumulative frequency curve.

<u>Question 8</u>

This question tested candidates' ability to

- draw the fourth figure in a sequence of shapes in which the first three figures in the sequence are given
- complete a table to show the terms in a sequence of numbers
- determine the term of the sequence for which the value of the function is given
- derive an algebraic expression to show the general term of a sequence of numbers.

The question was attempted by 97 per cent of the candidates, 10.3 per cent of whom earned the maximum available mark. The mean mark was 5.58 out of 10.

Candidates' performance on this question was very unsatisfactory. The drawing of the fourth figure in the sequence ranged from excellent for some candidates to poor for many. Many of them produced figures with 14 squares but not reflecting the required shape.

Part (b) was generally well done. Candidates worked out the numerical relation between the number of the figure and the associated number of squares it contained. Some however did not see the relationship between the answer to Part (b) (i) and the number of squares in Part (a). In Part (b) (ii), many candidates gave the answer as ‘23’ which suggests that they multiplied by 2 and added 3 instead of multiplying by 3 and adding 2. In addition, there were those who scored full marks for Parts (b) (i), (ii) and (iii) but could not write the relation in terms of *n*.

Solutions

(b) (i), (ii), (iii) and (iv)

Figure (n)	**No. of Squares**
4	14
10	32
16	50
n	3n+2

Recommendations

Teachers should provide experiences for students to translate verbal statements into algebraic representations of these statements. Moreover, students need to do more work on sequences especially in determining the *n*th term and expressing that term in terms of *n*.

Section II: Optional Questions

<u>Question 9</u>

This question tested candidates’ ability to

- substitute numbers for symbols expressed as functions
- determine the inverse of a linear function
- derive the composite of two linear functions
- express a quadratic function in the form $a(x + h)^2 + k$
- state the minimum value and axis of symmetry of a quadratic function
- sketch the graph of a quadratic function.

The question was attempted by 64 per cent of the candidates, 2.5 per cent of whom earned the maximum available mark. The mean mark was 4.33 out of 15.

The performance of candidates on this question was unsatisfactory. For responses to Parts (a) (i) and (ii), the majority of candidates was able to evaluate f(7). However, in their attempts to determine the inverse function, most of them were unable to move past the interchanging of the variables *x* and *y*. In Part (a) (iii), several candidates expressed fg(*x*) as $\frac{5(x^2-1)-4}{3}$, but were unable to complete simplification of the expression. The most common mistake was in computing –5 – 4. The answer most commonly given was –1.

In Part (b), very few candidates used the factoring method to complete the square. Of those who used this method, few demonstrated competence with the process required to complete the square successfully. Most candidates opted for substitution in the relevant formulae and were, in the majority of cases, able to find *a*, *h*, and *k*. Several, however, did not seem to know that the minimum value required referred to the value of *k* and similarly the connection between the *h* and the equation of the axis of symmetry was not made. Very few candidates wrote the equation as $x = -1$, but rather as $h = -1$.

In (b) Part (iv), most candidates recognized that the curve was a minimum point parabola, and their sketches showed the general shape of such a curve. However, only a minority were able to place the curve in the correct position on the coordinate axes. In several cases the x axis was used as the axis of symmetry. Further, the minimum value and the intercept on the *y*-axis were not used to assist in sketching the graph.

Solutions

(a) (i) $\mathrm{f}(7) = 10\frac{1}{3}$; (ii) $\mathrm{f}^{-1}(x) = \frac{3x+4}{5}$; $\mathrm{fg}(x) = \frac{5x^2-9}{3}$
(b) (i) $\mathrm{f}(x) = 3(x+1)^2 - 5$ (ii) minimum value $= -5$;
(iii) axis of symmetry is $x = -1$

Recommendations

Teachers should assist students in mastering computations involving directed numbers. The algorithm for completing the square must be properly understood by students. Students should be made aware of the information which is immediately available from the quadratic function when represented in the completed square format.

<u>Question 10</u>

This question tested candidates' ability to

- use the sine and cosine rules in the solution of problems involving triangles
- solve problems involving area of a triangle and perpendicular distance from a line
- solve geometric problems using properties of circles and circle theorems.

The question was attempted by 33 per cent of the candidates, 2 per cent of whom earned the maximum available mark. The mean mark was 3.76 out of 15.

Candidates' performance on this question was generally unsatisfactory. In Part (a), the majority of candidates incorrectly identified triangle RSQ as being right-angled. The choice of Heron's (Hero's) formula to compute the area of the triangle was common, but the computational skills to do so correctly were deficient. Often, candidates who were able to make good attempts at calculating the length QS, the measure of the angle T and the area of the triangle, were unable to calculate the perpendicular distance from Q to RS.

In Part (b), it was noted that many candidates did not know the relationship between a radius and a tangent at the point of contact, nor were they able to perceive that angle OJH was a right angle. A common misconception was evident when candidates stated that the angles at K and H are opposite angles of a cyclic quadrilateral.

Solutions

(a) (i) $\text{QS} = 10.8\text{ m}$ (ii) $\text{Q}\widehat{\text{T}}\text{S} = 32.3°$ (iii) area of $\Delta\text{QRS} = 46.8\text{ m}^2$
(iv) perpendicular distance from Q to RS $= 7.8$ m

(b) (i) $\text{O}\hat{\text{J}}\text{H} = 90°$ (ii) $J\hat{O}G = 132°$ (iii) $J\hat{K}G = 66°$ (iv) $\text{J}\hat{\text{L}}\text{G} = 114°$

Recommendations

Teachers of students at this level need to emphasize the strategies needed to handle problems where the triangles are obviously not right-angled. The use of theories involving the properties of circles and lines should be emphasized through greater use of practical work supplemented by the use of protractors. The incorporation of concrete and semi-concrete activities should precede work involving abstract concepts.

Question 11

This question tested candidates' ability to

- write a pair if simultaneous linear equations in the matrix form AX = B
- solve a system of equations using a matrix method
- determine the position vectors given the coordinates of points
- determine a resultant vector given two position vectors
- calculate the modulus of a vector
- use vectors to show that a quadrilateral is a parallelogram.

The question was attempted by 53 per cent of the candidates, 2.5 per cent of whom earned the maximum available mark. The mean mark was 5.27 out of 15.

Candidates performed unsatisfactorily on this question. In Part (a), most candidates were able to write the system of linear equations in matrix form and to calculate the determinant of the 2×2 matrix. However, most candidates were unable to calculate A^{-1} correctly.

Some of the incorrect attempts were $\frac{1}{22}\begin{pmatrix} 4 & -2 \\ -5 & 3 \end{pmatrix}$; $\frac{1}{2}\begin{pmatrix} 4 & -5 \\ -2 & 3 \end{pmatrix}$; $\frac{1}{22}\begin{pmatrix} 4 & -5 \\ -2 & 3 \end{pmatrix}$ and $\begin{pmatrix} 3 & 2 \\ 5 & 4 \end{pmatrix}$.

In Part (b), candidates were able to write down the position vectors, given the coordinates of two points. However, while the formula $\overrightarrow{SR} = \overrightarrow{SO} + \overrightarrow{OR}$ was generally known, several candidates proceeded to use $\overrightarrow{OS}$ instead of $\overrightarrow{SO}$ in their calculations and the result $\overrightarrow{SR} = \begin{pmatrix} -4 \\ 3 \end{pmatrix} + \begin{pmatrix} 6 \\ 2 \end{pmatrix} = \begin{pmatrix} 2 \\ 5 \end{pmatrix}$ was often seen. Further, very few of the candidates knew how to calculate the modulus (length) of a vector when its components are known. The expression $\sqrt{x^2 - y^2}$ was often used and in cases when the formula was known, $\sqrt{-4^2 + 3^2} = \sqrt{-7}$ was commonly seen. In addition, candidates proved that *OSTR* was a parallelogram by simply plotting the point (2, 5) on the grid, without further explanation.

Solutions

(a) (i) $\begin{pmatrix} 3 & 2 \\ 5 & 4 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -1 \\ 6 \end{pmatrix}$ (ii) $x = -8, y = 11.5$

(b) (i) $\overrightarrow{OR} = \begin{pmatrix} 6 \\ 2 \end{pmatrix}$ (ii) $\overrightarrow{OS} = \begin{pmatrix} -4 \\ 3 \end{pmatrix}$ (iii) $\overrightarrow{SR} = \begin{pmatrix} 10 \\ -1 \end{pmatrix}$ (iv) $|\overrightarrow{OS}| = 5$

Recommendations

Teachers need to give students more opportunities to practise calculating determinants and inverses of 2 × 2 matrices and using them to solve simultaneous linear equations.

Students should be allowed to explore the relationship between finding the length of a line segment, the modulus of a vector and the use of Pythagoras' theorem. A graphical approach could be used to reinforce the algebraic approach where students would see the actual sides in the diagrams and apply the formula to calculate the length of the hypotenuse. The techniques required to prove geometrical relationships using vectors should be reinforced.

C A R I B B E A N   E X A M I N A T I O N S   C O U N C I L

REPORT ON CANDIDATES' WORK IN THE CARIBBEAN SECONDARY EDUCATION CERTIFICATE® EXAMINATION

MAY/JUNE 2015

MATHEMATICS
GENERAL PROFICIENCY EXAMINATION

GENERAL COMMENTS

The General Proficiency Mathematics examination is offered in January and May/June each year. There was a candidate entry of approximately 85 000 in May/June 2015 and 57 per cent of the candidates earned Grades I–III. The mean score for the examination was 82 out of 180 marks.

DETAILED COMMENTS

Paper 01 — Multiple Choice

Paper 01 consisted of 60 multiple-choice items. It was designed to provide adequate coverage of the content with items taken from all sections of the syllabus. Approximately 74 per cent of the candidates earned acceptable grades on this paper; the mean score was 35 out of 60 marks. This year, 386 candidates earned the maximum available score of 60.

Paper 02 — Structured Questions

Paper 02 consisted of two sections. Section I comprised eight compulsory questions for a total of 90 marks. Section II comprised three optional questions: one each from Algebra, Relations, Functions and Graphs; Measurement, Trigonometry and Geometry; and Vectors and Matrices. Candidates were required to answer any two of the three questions from this section. Each question in this section was worth 15 marks. The mean score for this paper was 47 out of 120 marks.

Compulsory Section

Question 1

This question tested candidates' ability to

- add and subtract mixed numbers
- add, divide and square rational numbers expressed as decimals
- evaluate the square root of a rational number
- evaluate the tangent of an angle expressed in degrees
- solve problems involving shopping bills and sales tax.

The question was attempted by 99.5 per cent of the candidates, 16.3 per cent of whom earned the maximum available mark. The mean mark was 8.06 out of 12.

The responses of candidates to this question were satisfactory. They demonstrated competence on computations with decimals using a calculator but were less proficient at using the calculator to perform calculations on common fractions and mixed numbers. In many instances, candidates proceeded to round off answers to two or three decimal places even though they were asked to determine the exact values. With reference to the shopping bill, they were able to calculate the unit price of 1 kg of sugar. They also knew that to find the total cost of 4 kg of rice, they should multiply the unit cost by 4. However, they generally experienced difficulty in obtaining the price for a kilogram of rice and produced errors which resulted from not recognizing that 80 cents = \$0.80 and instead wrote \$80. Hence they obtained the solution \$81.60 instead of \$2.40.

Solutions

(a) **(i)** $\frac{137}{30}$ **(ii)** **3.3444** **(iii)** **7.855** **(iv)** **1.7**

(b) **(i)** **X = \$3.60** **(ii)** **Y = \$2.40; Z = \$9.60** **(iii)** **\$25.96**

Recommendations

Teachers should provide students with practice in the proper use of the calculator when performing basic mathematical computations including squares and square roots of numbers and trigonometric operations. They should guide students as to what is expected when *exact* solutions are required.

Question 2

This question tested candidates' ability to

- substitute directed numbers for variables and evaluate the result
- write algebraic expressions for verbal statements
- simplify algebraic fractions
- translate verbal statements into a pair of linear equations which could be solved simultaneously
- factorize algebraic expressions.

The question was attempted by 99 per cent of the candidates, less than 1 per cent of whom earned the maximum available mark. The mean mark was 4.95 out of 12.

Candidates' performance on this item was less than satisfactory. In Part (a) (i), the majority of candidates correctly substituted numbers for the variables, but there were those who could not evaluate $4-2-1$.

In Part (a) (ii), candidates substituted numbers for variables correctly, although a large number of them proceeded to incorrectly evaluate $2(4)^2$ as being equal to 8^2.

In Part (b) (i), several candidates reversed the order of the expression. Instead of writing $500-p$, they wrote $p-500$. The majority of candidates either did not respond to Part (b) (ii) or responded incorrectly. It was common to see $500-q=r$ instead of $500-qr$ as the solution.

In Part (c), candidates recognized that they needed to find the lowest common multiple in order to write the sum as a single fraction. However, once the denominator was found, the numerator proved to be difficult — there were errors in simplification, incorrect use of the distributive law and failure to collect like terms. Hence, $10k + 3(2 - k)$ generated answers such as $9k + 6$ or $7k + 2$.

In Part (d) (i), although candidates were able to write the pair of equations, they could not state what x and y represented. The most popular responses were x represents mangoes and y represents pears.

In Part (d) (ii), a large number of candidates correctly factorized a^3-12a, while only the best prepared were able to correctly factorize $2x^2-5x+3$.

Solutions

(a) **(i) 1** **(ii) 32**

(b) **(i)** $\mathbf{500-p}$ **(ii)** $\mathbf{500-qr}$

(c) $\mathbf{\frac{7k+6}{15}}$

(d) (i) $\mathbf{4x+2y=24,\ 2x+3y=16}$ **(ii)** x **represents the cost of one mango; y represents the cost of one pear**

(e) **(i)** $\mathbf{a(a^2-12)}$; **(ii)** $\mathbf{(2x-3)(x-1)}$

Recommendations

Teachers should ensure that students are proficient at carrying out the four basic rules of arithmetic on directed numbers. They should also be encourage to use brackets to maintain the order of operations. Algebraic expressions should be introduced using actual values, such as 100 ml, instead of *p*, before using the abstract quantity *p*.

Question 3

This question tested candidates' ability to

- determine the number of elements in the complement of two intersecting sets
- list the elements in a defined set
- use information given in a Venn diagram to determine the number of elements in the universal set
- obtain an equation in x for the number of elements in the universal set
- determine the number of elements in a given set
- use a ruler, a pencil and a pair of compasses to construct a right-angled triangle
- use a protractor to determine the measure of an angle
- determine and locate a fourth point in the plane that would extend the right angled triangle to a rectangle.

The question was attempted by 99 per cent of the candidates, 1.7 per cent of whom earned the maximum available mark. The mean mark was 5.04 out of 12.

The performance of candidates on this question was unsatisfactory. In Part (a), a large majority of candidates correctly stated the number of students who played neither the guitar nor the violin. However, an equally large percentage had difficulty identifying the number who played the guitar. Many concluded that $2x$ instead of $2x + x$, played the guitar. Moreover, it was also recognized that some candidates had difficulty substituting a value for an unknown, in that, having arrived at $x = 8$, too many failed to correctly proceed to $2x + x = 2(8) + 8 = 24$. Also, writing an expression followed by an equation in x for the total number of students in the class was problematic for many. Candidates were unclear as to the difference between writing an expression and writing an equation.

In Part (b), some candidates failed to demonstrate competency in using the ruler and compasses to complete the construction of the right-angled triangle. Some drew the line AB as 8 cm rather than 9 cm. A moderate percentage did not construct the right angle using a pair of compasses while there were others who drew the 90 degree angle at the incorrect point.

In Part (b) (ii), some candidates gave the size of angle BAC as a linear while there was evidence that many candidates did not know what a parallelogram looked like and failed to accurately complete the parallelogram.

Solutions

(a) (i) $\mathbf{12}$ **(ii)** $\mathbf{3x + 16}$ **(iii)** $\mathbf{3x + 16 = 40}$ **(iv)** $\mathbf{24}$

(b) (ii) $\mathbf{34°}$

Recommendations

Teachers should engage students in using hands-on resources such as tiles to help distinguish between a whole set, a subset and their relationship to each other.

Students need to learn how to make realistic judgments, for example, the whole is always larger than a part. Teachers should review the basics in algebra as a prerequisite for problem-solving involving Venn diagrams.

Teachers must emphasize the need for a high level of accuracy in measurement and construction. This can be partly achieved through stimulating class discussion of real-life situations, for example, surgery. Also, time must be allowed for students to practise using geometrical instruments in constructing geometrical shapes.

Question 4

This question tested candidates' ability to

- complete the table of values for the function $y = x^2 - 2x - 3$ for a given domain
- plot the graph of a quadratic equation
- use a graph to determine
  (i) the roots of a quadratic function
  (ii) the minimum value of a function
  (iii) the equation of the line of symmetry of a graph.

The question was attempted by 99 per cent of the candidates, 6.2 per cent of whom earned the maximum available mark. The mean mark was 4.40 out of 10.

Candidates performed unsatisfactorily on this item. Many of them incorrectly calculated the values of *y* in the table of values. Some candidates attempted to expand the table of values and separate each term but they were unable to carry out the calculations correctly. Some candidates substituted the values of *x* correctly into the equation but could not perform computations with directed numbers and, hence, were unable to execute the operations correctly. A typical example was: $(–1)^2 –2(–1) –3 = 1 – 2 – 3 = –4$.

Most candidates were able to draw the axes, using the correct scales. However, many of them had difficulty using the scale of 2 cm to represent 1 unit on the *x*-axis but were proficient at plotting the points from the table of values. Attempts at drawing a smooth curve through the points proved difficult for many candidates. Very few candidates used the graph to fill in the information and some misinterpreted the term *values*. Candidates wrote coordinates instead of values.

In Part (d) (i), to determine the values for which $x^2 - 2x - 3 = 0$, many candidates wrote down the two values of *y* which they calculated for the table of values in Part (a). Those candidates who attempted to use the graph had difficulties reading off the values from the *x*-axis correctly, especially if it was not a whole unit.

In Part (d) (ii), come candidates did not appear to know what the minimum value of the graph was and opted not to respond to this part of the question. In Part (d) (iii), candidates could not determine the equation for the axis of symmetry. It was recognizable that candidates, even those who gained 9 out of the 10 marks for the question, either left out this part, wrote back the given quadratic equation or some other equation for the equation of the line of symmetry. Candidates were uncertain how to locate the axis of symmetry and furthermore how to state the line of symmetry.

Solutions

(a)

X	**-1**	**3**
Y	**0**	**0**

(d) **(i)** $\mathbf{-1 \; or \; 3}$ **(ii)** $\mathbf{-4}$ **(iii)** $\mathbf{x = 1}$

Recommendations

When teaching how to graph a quadratic function, emphasis must be placed on students completing a table of values. Students must learn different strategies for completing the tables including substitution into the equation. Students should also be provided with opportunities to use various scales and activities which require reading units correctly from graphs. Further, students should be able to recognize a parabola and the characteristics of a quadratic graph. Identifying examples and non-examples of quadratic graphs could reinforce this.

Teachers need to spend more time developing students' skills in graphical interpretation rather than merely drawing graphs as well as the concept of axis of symmetry and the methods in which it may be derived.

Question 5

This question tested candidates' ability to

- calculate the distance travelled at constant speed for a given interval of time
- calculate time taken in seconds to travel a given distance
- write given scales in the form $1: x$
- use the scale of a map to determine actual distances and areas on the land
- use the scale of a map to determine lengths
- use the scale of a map to determine areas on a map.

The question was attempted by 99 per cent of the candidates, less than 1 per cent of whom earned the maximum available mark. The mean mark was 2.58 out of 12.

The performance of candidates on this question was generally unsatisfactory. In Part (a) (i) most candidates could write the product 54 × 2 ¼, but could not complete the computation to arrive at 121.5. Many of the candidates converted the ¼ of an hour to 15 minutes and then proceeded to multiply 54 × 2.15. Others converted the hours to 135 minutes and then multiplied. For many, the multiplication of 54 by the improper fraction $\frac{9}{4}$ proved a challenge.

Part (a) (ii) proved to be very challenging and few candidates obtained the correct answer or carried out the steps which would lead to the correct answer. A few candidates multiplied 54 by $\frac{5}{18}$ to obtain the speed 15 and then divided 315 by 15 to obtain the time 21 seconds. Few candidates divided 315 by 54 which also would have given the correct solution.

In Part (b) (i) and (ii), determining the ratio in the given format proved difficult. Many candidates could not arrive at a scale of 1:1000 in Part (i) and in Part (ii), candidates left their answer as 2:600 instead of $1: 300$.

In Part (c), the responses showed that candidates were not familiar with this type of question and most candidates omitted this part. For those who attempted it, in Part (i), counting of the squares for QR gave varied answers, for example, 6.1, 7, 5.5 and even 4. Many used the scale 1:2000 and gave their answer as 12 000.

In Part (ii), candidates were not able to give a good estimate of the area and many answers fell outside the range of 17 to 19. Since it was area, some candidates gave answer as 18^2 and likewise for their estimate. Some used the trapezium rule to determine the area: ½(6 + 3) × 4 =18.

In Part (iii), to determine the actual length QR in kilometers, some candidates multiplied the measured length of QR by 2000 giving 12 000 as their answer not realizing that the unit was centimetres. Others tried but could not convert their answer in centimetres to kilometers by dividing by 100 000.

In Part (iv), most candidates did not know that area is calculated as the square of the ratio. Candidates just multiplied 'their' 18 by 2000 rather than 2000^2. Some did use 2000^2 in ingenious ways but did not divide by 100^2 to convert to square metres. Some used the trapezium rule and converted the lengths using the scale 1:2000 and produced solutions as follows:

½(12000 + 6000) × 8000 = 72 000 000 000 cm^2. Having arrived at this answer, candidates should have divided by 10 000 to convert to square metres. However, many did not do the final conversion.

Solutions

(a)	**(i)**	**121.5 km**	**(ii)**	**21 seconds**				
(b)	**(i)**	**1: 1000**	**(ii)**	**1: 300**				
(c)	**(i)**	**6 cm**	**(ii)**	**18 cm^2**	**(iii)**	**0.12 km**	**(iv)**	**7200 m^2**

Recommendations

Teachers should give attention to authentic tasks when teaching scales and scale drawings. Operations with scale factors when determining lengths, areas and volumes must also be reinfroced.

Question 6

This question tested candidates' ability to

- determine the volume of a cylinder given the diameter of its base and its height
- determine the length of the radius of a circle given the measure of its area
- determine the height of a cylinder given its volume and the area of its base
- estimate the gradient of the tangent at a given point on the graph of a quadratic function.

The question was attempted by 99 per cent of the candidates, less than 1 per cent of whom earned the maximum available mark. The mean mark was 2.26 out of 11.

The performance of candidates on this question was unsatisfactory. Most candidates knew the formula for the volume of a cylinder and how to substitute π and r into the formula. Some candidates wrote the units of volume incorrectly. For example, a few candidates wrote $V = 251.2\ cm^3$, others wrote $V = 251.2\ litres$, while a minority of candidates wrote $V = 251.2^3$.

In Part (a) (ii), candidates were required to arrive at $r = \sqrt{\frac{314}{3.14}} = 10m$. Some students, who used $\pi = \frac{22}{7}$ instead of $\pi = 3.14$, got $r = 9.99\ m$. Some candidates started with the answer $r = 10\ m$ and then showed that $3.14 \times 10^2 = 314$. In some cases, candidates correctly stated $r = \sqrt{\frac{314}{3.14}} = \sqrt{100}$ but did not proceed to $r = 10\ m$.

In Part (a) (iii), some candidates did not use the information that $Volume\ of\ tank\ A = 8 \times Volume\ of\ Tank\ B$, and hence could not proceed to determine the value of h.

In Part (b) (i), some candidates confused the concepts of *scale factor* and *scale*, and there were instances when the candidates incorrectly wrote the scale factor as 1: 2.

In Part (b) (ii), most candidates recognized that the transformation was an enlargement. Nevertheless, there were those who incorrectly opined that the transformation was a reflection. In some instances, candidates wrongly argued that rotation accounts for the negative scale factor.

In Part (b) (iii), some candidates incorrectly stated that $P'Q' = \sqrt{13 \times 2} = \sqrt{26}$ instead of $P'Q' = 2\sqrt{13} = \sqrt{4} \times \sqrt{13} = \sqrt{52} = 7.21$ (correct to two decimal places).

In Part (b) (v), some common approaches that candidates used to determine the area of triangle $P'Q'R'$ were as follows:

Area of triangle $P'Q'R' = \frac{1}{2} \times 4 \times 6 = 12$ square units.

Area of triangle $P'Q'R' = (scale\ factor)^2 \times$ Area of triangle $PQR = 4 \times 3 = 12$ square units.

Solutions

(a) (i) $251.2\ m^3$ (ii) 10 m (iii) 6.4 m

(b) (i) 2 (ii) the centre of enlargement lies between object and image

(iii) $2\sqrt{13}$ (iv) 3 square units (v) 12 square units

Recommendations

Candidates are encouraged to examine the properties of each transformation given in the syllabus. They should explore different techniques for finding the area of triangles and other polygons. Changing the subject of a formula is also an important skill that is useful in various mathematical concepts.

<u>Question 7</u>

This question tested candidates' ability to

- complete a table of values with information extracted from a given line graph
- interpret trends shown in a given line graph
- calculate the mean of a set of values
- complete a given line graph to show calculated values.

The question was attempted by 99 per cent of the candidates, 1.3 per cent of whom earned the maximum available mark. The mean mark was 5.28 out of 11.

Candidates performed moderately on this question. Almost all of the candidates scored full marks for Part (a) which required reading information from the graph and completing the related table of values.

In Parts (b) (i) and (ii), many candidates demonstrated a lack of understanding of the term *consecutive months*. Many candidates misunderstood the questions to be asking for the two months between which the highest and lowest increase in sales occurred. In Part (b) (iii), very few candidates were able to identify the feature on the graph that indicated the difference between the greatest and lowest increase in sales, with less than about 10 per cent of candidates identifying the gradient as the feature that indicates the variation in increase in sales.

In Part (c), most candidates demonstrated the correct strategy for calculating the mean sales. A large number of candidates' responses reflected the process of adding the sales for the five months (July to November) then dividing by five to calculate the average sales for that period.

In Part (d) (i), in calculating the December sales, most candidates added the total sales of the months from July to November then subtracted from $130 (the total sales for the six month period). In Part (d) (ii) candidates were generally able to plot their December sales correctly on the graph.

Solutions

(a) August $20 000 October $25 000 November $15 000

(b) (i) August and September (ii) July and August (iii) Gradient of line

(c) $21 800

(d) $(\boldsymbol{i})$ **\$21 000**

Recommendations

Teachers are encouraged to provide students with opportunities to read and interpret information such as titles, keys, scales and footnotes as used on graphs and diagrams. Teachers will find it useful to guide students to develop the practice of interpreting and analysing information given in questions as a first step in responding to questions.

Teachers are encouraged to engage students in activities which allow them to use the concept gradient in situations outside the study of coordinate geometry.

Question 8

This question tested candidates' ability to

- draw the fourth diagram in a given sequence of diagrams
- complete a numerical pattern to determine the total number of unit triangles
- use an arithmetic algorithm to determine the number of unit sides
- derive the general rule representing the patterns in a sequence.

The question was attempted by 99 per cent of the candidates, 15.9 per cent of whom earned the maximum available mark. The mean mark was 5.42 out of 10.

Candidates' responses to this question were satisfactory. However, in Part (a), a large number of candidates could not extend the figure correctly to obtain figure 4 in the sequence. In Parts (b) (i) – (iii), candidates were generally able to determine the correct values for the number of triangles and unit sides, even in instances where they could not get the general rule correct.

Solutions

(b) **(i)** **16 unit triangles; 30 unit sides** **(ii)** **figure 12; 234 unit sides**

$(\boldsymbol{iii})$ **625 unit triangles; 975 unit sides** **(iv)** $\mathbf{n^2}$ **unit triangles;** $\mathbf{\frac{3n(n+1)}{2}}$ **unit sides**

Recommendations

Students are encouraged to use pattern recognition and generalization of patterns in games and puzzles to determine the general rules for patterns and sequences.

Optional Section

Question 9

This question tested candidates' ability to

- draw and interpret graphs of linear functions
- plot a graph given a data set
- interpret data from a given graph
- evaluate the composite of a function
- determine the inverse of a linear function
- apply the distributive law to expand algebraic expressions
- simplify and factorize a quadratic expression.

The question was attempted by 92 per cent of the candidates, 1.7 per cent of whom earned the maximum available mark. The mean mark was 5.22 out of 15.

Candidates' performance on this question was generally unsatisfactory. Many candidates successfully derived the correct percentages for Part (a) (i). Several of them opted to calculate the percentages by first expressing the mark as a fraction of 120. Nevertheless, there were those who misinterpreted the data and instead of plotting a linear relationship of *Percentage vs Mark*, the following were plotted: bar graphs; frequency polygons, line graphs; cumulative frequency curves and in a few instances, two points with no line passing through the points.

In Part (a) (ii), most candidates recognized that the graph was linear and passed through the origin. Many of them successfully derived the correct percentage for Part (a) (iii) either by way of interpolation or by performing the computation $\frac{95}{100} \times 100$. In addition, many candidates successfully derived the correct score for Part (a) (iv) as 102 either by interpolation or by forming an equation. For candidates who produced an answer within the band of 102 ± 2 marks, the full marks were awarded once the lines of interpolation were shown.

Some candidates plotted straight line graphs where the *y* intercept was not 0 even though a correct scale was used and some attempted to draw the straight line freehand instead of using a ruler. Some candidates who correctly plotted the graph, failed to show the *pair of lines of interpolation* to show the answers for Parts (a) (iii) and (a) (iv). Despite this, some candidates correctly derived the answers by either visual inspection of the graph or by forming an equation and solving it.

Candidates who opted to 'break' the *x*-axis by starting off with a number other than 10 did not put the appropriate symbol on the *x*-axis (even though the scale was appropriate). Some candidates did not chose appropriate scales, thereby making it difficult to interpret the graph.

In Part (b) (i), many candidates successfully determined *g*(5). Some, however, appeared not to understand the notation and how to carry out the correct substitution and experienced difficulty in arriving at the correct answer. In Part (b) (ii), some candidates were able to find the inverse. However, a large number of candidates knew they had to interchange the *x* and the *y* but did not know how to transpose. In Part (b) (iii), many candidates experienced difficulty carrying out the correct expansion of $(3x+2)^2$.

They also had difficulty with algebraic simplification: particularly with the expression $\frac{(3x+2)^2+1}{3}$. As a result many of these candidates erroneously produced the result $\frac{9x^2+4}{3}$. Some candidates who correctly produced the expression $3x^2 + 4x + 1$ from $\frac{(3x+2)^2+1}{3}$ were unable to derive the correct quadratic factorization.

Solutions

(a)	**(i)**	$\mathbf{50\%;\ 100\%}$	**(ii)**	**line through $(60, 50)$ and $(120, 100)$**
	(iii)	$\mathbf{79\%}$	**(iv)**	**102 marks**
(b)	**(i)**	**8** **(ii)** $\frac{x-2}{3}$	**(iii)**	$(3x+1)(x+1)$

Recommendations

Teachers need to reinforce the correct reading of scales and maintaining consistency in scales. They should instruct students on the use of the electronic, non-programmable calculator in areas where order of operations is key so that fractions and squares can be solved with relative ease. Students should be taught to assess the validity of an answer.

Further, teachers need to put heavy emphasis on factorization of quadratic expressions where the coefficient of x^2 is not 1 and the expansion of two bracketed expressions, similar to $(3x+2)^2$.

The ideas of a function, inverse of a function and compositions of functions must be introduced to students in the form of arrow diagrams. Authentic representations of mappings and functions should be discussed. Examples are the names and numbers in a telephone directory, the assignment of students to classes in a school and the relationships in families.

Question 10

This question tested candidates' ability to

- use trigonometric ratios to solve problems involving angles of elevation
- use the cosine and sine rules to solve problems related to bearings.

The question was attempted by 65 per cent of the candidates, 2.4 per cent of whom earned the maximum available mark. The mean mark was 3.15 out of 15.

Candidates' performance on this question was generally unsatisfactory.

In Part (a) (i), the line representing a distance of 60 m and a right angle on the diagram were widely known. However, only a minority of candidates was able to label the angles of 35° and 42° to show the angles of elevation. As a consequence, several candidates who calculated BT and BP to be 42 m and 54 m respectively did so from an incorrect diagram and their ratios of tan 35 and tan 42 were incorrect. The angles of 35° and 42° were often inserted on the diagram as angles BTR and BPR respectively. In a few cases, 35 and 42 were placed along the lines TR and PR. In Part (a) (ii), some candidates correctly used the tangent ratio to solve the problem. However, there were those who incorrectly used the sine ratio.

In Parts (b) (i) and (ii), candidates who were unable to calculate angle KLM, did not recognize the relation, alternate angles, between KLM and the bearing. Many indicated on the diagram that the bearing was in fact the angle KLM. In Part (b) (iii), most candidates were able to use the cosine rule correctly and to substitute the correct values of the sides and angle KLM and follow through with their calculations to give a correct answer to the nearest kilometre. There were too many, however, who, on reaching the stage
KM = 20 800 – 19200 cos40, evaluated it to be KM = 1 600 cos40.

In Part (b) (iv), those candidates who used the sine rule were able, generally, to find angle LKM. The weakness here for those who did not, was in making sin K the subject. This also applied to candidates who attempted to use the cosine rule. In Part (b) (v), the majority of candidates was unable to calculate the bearing of M from K. Calculations, such as adding the bearing of 40 degrees to LKM and then subtracting from 180 degrees seemed to indicate that they did not know where the bearing was located.

Solutions

(a) **(ii)** **12 m**

(b) **(ii)** **40°** **(iii)** $\boldsymbol{KM = 78}$ **km** **(iv)** **81°** **(v)** **121°**

Recommendations

Students need to be exposed to a greater variety of problems involving trigonometrical drawings in the areas stated in the syllabus. Some of these drawings can be produced through practical work on the part of the students. In this question, for example, showing the bearing on a diagram or calculating the measure of a bearing should be reinforced. Students also need to be taught how to select the most appropriate ratio to be used in a given situation in order to reduce the number of calculations required.

Question 11

This question tested candidates' ability to

- solve for one unknown in a singular matrix
- use the matrix method to solve an equation with two unknowns
- recognize the position vector of a point in the plane
- write the coordinates of points in the plane as position vectors
- use vector geometry to determine the resultant of two or more vectors
- use the properties of equal vectors to solve problems in geometry.

The question was attempted by 42 per cent of the candidates, 1 per cent of whom earned the maximum available mark. The mean mark was 5.60 out of 15.

Generally, candidates submitted better answers to Part (a) which tested their ability to manipulate 2 × 2 matrices, than Part (b) which tested their working knowledge of vectors.

In Part (a), while many candidates could correctly multiply two 2 × 2 matrices, too many used incorrect strategies, such as multiplying or adding corresponding elements. Many demonstrated that they knew how to compute the adjoint of the matrix M. However, many candidates seemed unfamiliar with the notation $|\overrightarrow{M}|$ for the determinant of the matrix.

In Part (b), the notation $|OR|$ was not familiar to many candidates, and the computation $(-3)^2 = -9$ was also common when the correct method was adopted. Even candidates who could obtain correct solutions to $\overrightarrow{RS}$ and $\overrightarrow{ST}$ were unable to articulate properly that the two were parallel; and they found it even more difficult to conclude that because S was a common point, RST was a straight line. It was good to see that many of the better candidates were able to get the expected result, and for many others this last point was their only failing in the whole question.

Solutions

(a) (i) $\boldsymbol{AB} = \begin{pmatrix} \mathbf{1} & \mathbf{3} \\ \mathbf{2} & \mathbf{7} \end{pmatrix}$ **(ii)** $\boldsymbol{BA} = \begin{pmatrix} \mathbf{5} & \mathbf{7} \\ \mathbf{2} & \mathbf{3} \end{pmatrix}$ **(iii)** $\boldsymbol{A^{-1}} = \begin{pmatrix} \mathbf{3} & \mathbf{-1} \\ \mathbf{-2} & \mathbf{1} \end{pmatrix}$ **(i)** $\boldsymbol{x} = \mathbf{3}$

(b) **(i)** $|\overrightarrow{\boldsymbol{OR}}| = \mathbf{5}$ **(ii)** $\overrightarrow{\boldsymbol{RS}} = \begin{pmatrix} \mathbf{4} \\ \mathbf{-3} \end{pmatrix}$; $\overrightarrow{\boldsymbol{ST}} = \begin{pmatrix} \mathbf{4} \\ \mathbf{-3} \end{pmatrix}$;

(iii) $\boldsymbol{RS}||\boldsymbol{ST}$**; S is a common point; so *R, S* and *T* are collinear**

Recommendations

Teachers should encourage students to utilize diagrams to add clarity to their responses when solving problems on Vector Geometry. They should reinforce the concept that a resultant vector can be derived from the sum or difference of two or more position vectors. They should provide students with the opportunity to develop the art of identifying the type of quadrilateral formed from a system of vectors in which there are equal or parallel combinations. They should also provide students with more practice in writing the matrix equation corresponding to a pair of simultaneous linear equations.

CARIBBEAN EXAMINATIONS COUNCIL

REPORT ON CANDIDATES' WORK IN THE CARIBBEAN SECONDARY EDUCATION CERTIFICATE® EXAMINATION

JANUARY 2017

MATHEMATICS
GENERAL PROFICIENCY EXAMINATION

GENERAL COMMENTS

The General Proficiency Mathematics examination is offered in January and May/June each year. There was a candidate entry of approximately 8926 in January 2017 and 36 per cent of the candidates earned Grades I–III. The mean score for the examination was 67.5 out of 180 marks.

DETAILED COMMENTS

Paper 01 — Multiple Choice

Paper 01 consisted of 60 multiple-choice items. It was designed to provide adequate coverage of the content with items taken from all sections of the syllabus. Approximately 57.5 per cent of the candidates earned Grades I, II and III on this paper; the mean score was 29.87 out of 60 marks. This year, five candidates earned the maximum available score of 60.

Paper 02 — Structured Questions

Paper 02 consisted of two sections. Section I comprised eight compulsory questions for a total of 90 marks. Section II comprised three optional questions: one each from Algebra, Relations, Functions and Graphs; Measurement, Trigonometry and Geometry; and Vectors and Matrices. Candidates were required to answer any two of the three questions from this section. Each question in this section was worth 15 marks. The mean score for this paper was 37.6 out of 120 marks.

Compulsory Section

Question 1

This question tested candidates' ability to

- multiply and divide fractions
- use the calculator to perform a square root calculation
- solve problems involving money and taxes.

The question was attempted by 100 per cent of the candidates, 14.08 per cent of whom earned the maximum available mark. The mean mark was 7.2 out of 11.

In Part (a) (i), most candidates understood the order of operations while others confused addition and multiplication. Many candidates were able to calculate $3\frac{1}{2} \times 1\frac{2}{3}$ correctly. Additionally, some of them flipped the dividend and not the divisor which resulted in inaccuracy. Other candidates only wrote the answer $\frac{25}{18}$, which suggested correct use of the calculator.

For Part (a) (ii), the major area of weakness was $\sqrt{\frac{0.1014}{1.5}}$. Many candidates did not divide and then find the square root of the answer. They opted to calculate $\sqrt{0.1014}$ first and then divided the answer by 1.5

which was inaccurate. Although some candidates used the calculator, they were unable to get the correct solution.

In Parts (b) (i) and (ii), many candidates were able to differentiate when the appropriate operation was required (that is when to multiply or divide). In Part (b) (iii), many candidates multiplied 14 x 2 to get 28. However, if the connection between \$2480.60 and \$44.70 x 2 was not shown, no marks were awarded.

For Part (b) (iv), many candidates calculated 15% of the number of tickets sold and not 15% of the total sum of money collected for the tickets.

Solutions

(a) (i) $\frac{25}{18}$ (ii) 5.21

(b) (i) \$26.10 (ii) \$620.90 (iii) 28 (iv) \$485.25

Recommendations

Teachers should

- encourage the proper use of calculators. (Teachers need to teach students how to use these instruments.)
- emphasize the need to read questions carefully, so that what has been asked is answered completely and correctly.

<u>Question 2</u>

This question tested candidates' ability to

- simplify algebraic fractions
- translate a verbal sentence into an algebraic equation
- factorize a difference of squares
- factorize a quadratic expression of the form $a x^2 + b x + c$
- change the subject of a formula
- expand a completed square and determine coefficients of the resulting quadratic expression.

The question was attempted by 100 per cent of the candidates, three per cent of whom earned the maximum available mark. The mean mark was 3.67 out of 12.

In Part (a), most candidates were able to determine the common denominator and at least one correct term in the numerator. Mistakes were often made when expansion of the numerators was attempted – failure to multiply the second term in expanding was the most common error.

Many candidates who attempted Part (b) were able to get an expression equal to 5 times a variable. Most candidates could not correctly write an expression for the multiplicative inverse.

In Part (c) (i), at least one of the factors was correctly identified. For Part (c) (ii), incorrect factors were used to split the 5x before factorizing by grouping.

In Part (d), most candidates were unable to use inverses to get the correct expression for r. For Part (e), expansion of $(x + 2)^2$ was incorrect in most cases and many candidates could not correctly identify the numbers representing *a* and *b*.

Examples of Common Mistakes

Part (a)

$$\frac{8x+3+3x-4}{12} \qquad \frac{8x+12+3x-4}{12} \qquad \frac{3(x+3)+4(x-4)}{12}$$

$$\frac{6x+9+4x-16}{12}$$

Part (b)

$$5\left(n+\frac{1}{n}\right)$$

Part (c)

(i) $x(x-6)(x+6)$ and $(x-6)+(x+6)$ and even $-36x^2$ were seen.

(ii) $7x^2 - 12$ was frequently seen.

Part (d)

$r = \dfrac{\pi rh}{V}$ and various incorrect versions were seen.

Part (e)

$(x+2)^2 - 3$

$x^2 + 4 = 3$ was frequently seen as the expansion.

Solutions

(a) $\frac{11x}{12}$ (b) $n + \frac{1}{n} = 5n$ (c) (i) $(x - 6)(x + 6)$ (ii) $(2x - 3)(x + 4)$

(d) $r = \sqrt{\frac{V}{\pi h}}$ (e) $a = 4, b = 1$

Recommendations

- Overall, algebraic skills remain weak. Many candidates are unable to expand correctly.
- Few candidates correctly changed the subject of a formula.
- Factorization skills are still unsatisfactory.
- A stronger focus on building the concept of factorization is essential. Candidates also need varied and frequent practice with all types of algebraic expressions.

<u>Question 3</u>

This question tested candidates' ability to

- use information given to complete a Venn diagram
- determine the number of elements in the intersection of two sets
- use a ruler and a pair of compasses to construct a trapezium

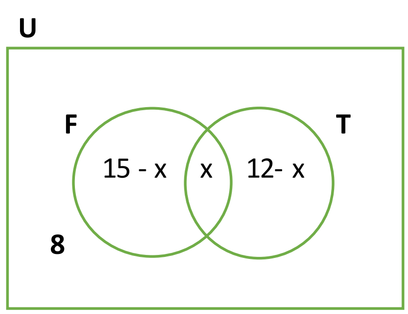
The question was attempted by 100 per cent of the candidates, two per cent of whom earned the maximum two available marks. The mean mark was 5.17 out of 12.

In Parts (a) (i) and (ii), candidates were able to correctly identify where to place the eight students as the number who played neither sport. They were also able to recognize that the value of *x* was seven persons. However, some candidates were not successful in formulating expressions to represent the number of students in each subset (15 – x and 12 - x).

In Part (b), candidates were generally able to construct the required trapezium. However, many of them were unable to construct both 90^0 and 60^0 angles on the diagram. Some candidates also failed to recognize that the line CD (parallel to AB) should also be constructed and not just drawn. The majority of candidates did not attempt to construct the parallel line CD.

Solutions

(a) (i)



(ii) x = 7

Recommendations

Teachers should

- provide more opportunities for students to explore the *only* concept and to write expressions to represent such

- encourage students to draw a rough sketch of diagrams to be constructed before attempting the accurate diagrams
- give additional practice on construction areas/questions which require students to construct parallel lines, perpendicular lines as well as geometrical shapes other than triangles.

Question 4

This question tested candidates' ability to

- evaluate functions and derive the inverse of a given function
- determine the gradient, midpoint and perpendicular bisector of a line segment.

The question was attempted by 100 per cent of the candidates, 3.27 per cent of whom earned the maximum available mark. The mean mark was 3.44 out of 12.

In Part (a) (i), many candidates were able to substitute the values 0 and 5 in g(x) in order to find g(0) and g(5). A few of them, however, had difficulty following through correctly, hence they obtained incorrect final answers. For example, candidates substituted the x value but did not understand that they had to multiply the value by 3, others kept the variable x although they inserted the value for x.

For Part (a) (ii), candidates demonstrated that they understood the order of the composite function fg(5). However, the follow through of substituting and simplifying proved difficult for some. In Part (a) (iii), candidates had difficulty calculating the inverse of a function. Many of them had issues changing the subject of the formula and of those who attempted the question, some did not know how to use the inverse in the question. Candidates had problems substituting and finding the inverse of a function.

Part (b) was poorly answered. Though some candidates knew the formula to use to find the gradient and the midpoint, substitution using the negative numbers posed a problem. Additionally, some candidates had issues identifying which coordinates were x and y.

For Part (b) (ii), in finding the midpoint, some candidates were confused about how to state the coordinates. This led to candidates simply writing two answers for x and y separately; others used the form (y, x).

Many candidates did not attempt Part (b) (iii), and those who did knew the formula to find the equation of a straight line, but were unable to calculate the gradient of the perpendicular bisector. Many candidates resorted to using the same gradient calculated in Part b (i) and the coordinates of one of the given points (as opposed to the midpoint).

Solutions

(a)	(i)	8.5	(ii)	25	(iii)	2
(b)	(i)	-2	(ii)	(4, 3)	(iii)	y = 0.5x + 1

<u>Question 5</u>

This question tested candidates' ability to

- understand the properties and relationship among geometrical objects
- solve geometric problems using properties of similar figures; similar triangles
- determine and represent the location of the image of an object under a transformation
- describe a transformation given an object and its image.

The question was attempted by 100 per cent of the candidates, 0.23 per cent of whom earned the maximum available mark. The mean mark was 2.88 out of 11.

The question was poorly done. The majority of candidates scored five or six out of a total of 11 marks.

Part (a) (i) was poorly done. Some candidates recognized that that the corresponding angles were equal, but very few of them were aware that the corresponding sides had the same ratio/proportion.

Part (a) (ii) was poorly done. Very few candidates were able to determine the length of PQ. Some of them attempted to use trigonometry to do so.

In Part (b), while most candidates were able to draw the reflection of the shape, they came up short in the description of transformation. Though many of them knew a rotation was involved, few of them could state the degree or origin of rotation.

The question was very poorly done. The majority of students scored at most five or six marks. Most candidates were able to identify the coordinates of the point E in Part (b) (i).

Part (b) (ii) was fairly well done. Most candidates recognized that the transformation was a rotation. Some of them recognized the direction and degree of the rotation. However, few candidates recognized that the centre of rotation was the origin.

Part (b) (iii) was well done. Most candidates were able to draw the reflection of the triangle $D''E''F''$ in the x axis. Some of them reflected the object in the x axis instead of the image, while others reflected the diagram in the y axis.

Solutions

(a) (i) equal/congruent ratio/proportion

(iii) 19.2 cm

(b) (i) (4, 2)

(ii) rotation, 90^0 anticlockwise (270^0 clockwise), about origin (0,0).

Recommendations

- Students need to be exposed to the properties of similar triangles; the corresponding angles and the calculation of missing sides using ratios/ proportions.
- Students need to be taught the difference between the object and the image in a transformation. They also need to be taught the location of the x and y axes on the Cartesian plane and be exposed to reflections in each axis.
- More exposure to rotation is needed when doing transformational geometry. Students need more practice in finding the
  - direction of the rotation (clockwise or anticlockwise)
  - degree of rotation
  - origin of the rotation.

<u>Question 6</u>

This question tested candidates' ability to

- use a scale and convert units of length
- determine the length of the hypotenuse of a right-angled triangle
- calculate the area of a circle, square and minor segment of a circle.

The question was attempted by 100 per cent of the candidates, 2.04 per cent of whom earned the maximum available mark. The mean mark was 2.06 out of 11.

In Part (a) (i), most candidates were able to convert the distance on the map to the actual distance, but kept the distance in cm instead of converting to km. They were able to get the 1:25 000 scale correct but were not able to convert from one unit to another. Similarly for Part (a) (ii), candidates knew to divide based on the given scale but could not convert to units on the map.

Diagonal is $11\sqrt{2}$. For those who got it right, the majority used Pythagoras' theorem whilst a few candidates used a trigonometry ratio.

In Part (b) (ii), in order to determine the area of the circle, candidates generally used the correct formula. Many candidates used half of 11 instead of half of $11\sqrt{2}$ as the radius and so ended up missing out on marks even though the correct formula was used.

Part (b) (iii) was generally well done. Most persons who attempted Question 6 got the area of the square correct.

Some candidates ended up with no marks for Part (b) (iv) because they had an incorrect value for the area of the circle, ending up with a negative number or just switching it around to make it positive. Many others left that part blank.

Solutions

(a) (i) 7.95 km (ii) 11 units

(b) (ii) 190.07 cm^2 (iii) 121 cm^2 (iii) 17.27 cm^2

Recommendations

Teachers should

- emphasize the difference between between chord, diameter and radius
- expose students to questions which focus on scales used on maps and converting from one unit to another.

Question 7

This question tested candidates' ability to

- construct a bar chart using data from a table and a given scale
- determine the range of given data
- analyse data from a bar chart to provide answers to questions, giving reasons for answers
- make inference(s) from statistics.

The question was attempted by 100 per cent of the candidates, 0.5 per cent of whom earned the maximum available mark. The mean mark was 4.32 out of 11. Candidates' overall performance on this question was unsatisfactory.

In constructing the bar chart in Part (a), most candidates were able to plot at least four points at the correct vertical heights and were able to obtain the correct scale. However, some candidates depicted the bar chart as a histogram (bars touching).

In Part (b), candidates lacked understanding of the concept of 'range'. For Part (c), most candidates were able to name 2011 as the year with the greatest production of bananas. In Part (d), the majority of candidates were unable to interpret and use the graph to provide answers. Candidates lacked the requisite analytical skills required in this part of the question.

For Part (e), more than 50 per cent of the candidates were able to respond correctly as to why the bar chart is unsuitable for predicting the number of bananas produced in 2016.

Solutions

(b) 235

(c) (i) 2011 (ii) Bar with greatest height

(d) (i) 2011 - 2012 (ii) Consecutive bars with the greatest change in height

(e) Bar charts do not show trends/patterns over time.

Question 8

This question tested candidates' ability to

- draw a fourth diagram in a sequence of three labelled diagrams
- complete a numerical pattern using an arithmetic algorithm to determine either the figure number or the total number of unit squares and the perimeter of the sequence
- derive a general rule as an algebraic expression in terms of (n) to represent the number of unit squares and the perimeter of the figure.

The question was attempted by 100 per cent of the candidates, 3.29 per cent of whom earned the maximum available mark. The mean mark was 4.66 out of 10. Overall, candidates' performance was satisfactory.

For Part (a), many candidates were able to represent the fourth diagram correctly with 13 squares by adding 1 square to each arm. However, the quality of the drawings was for the most part poor, as many candidates had rectangles instead of squares.

Part (b) (i) was well done. Most candidates showed a good grasp of the numerical sequence and the relationship between the perimeter and the number of unit squares in Figure 4. In Part (b) (ii), many candidates were able to obtain the figure number as 12 and the perimeter of 92 either by calculation or by continuing the sequence of drawings until the required answer was found.

In Part (b) (iii), fewer candidates were able to correctly obtain either the number of unit squares as 117 or the perimeter as 236. Many attempted to draw the sequence of diagrams to get an answer.

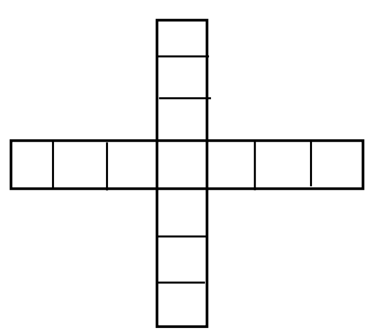
Part (b) (iv) was poorly done. Many candidates just continued a sequence of numbers and were unable to write an algebraic expression in terms of (n) for the number of unit squares and the perimeter. Many also showed 4 and 8 as the multipliers, and wrote 4n as the number of unit squares and 8n as the perimeter. Some candidates gave the correct answer but did not simplify the expressions.

For example, 1 +4n -4 or 4(n-1)+1 for the number of unit squares and 4+8n-8 and 2(4n-3)+2 as the perimeter.

Some wrote an algebraic expression for the relationship between the number of unit squares and the perimeter as 2n+2 which was not asked.

Solutions

(a)



(b)	Figure (n)	No. of unit squares	Perimeter of figure
(i)	4	13	28
(ii)	12	45	92
(iii)	30	117	236
(iv)	n	4n - 3	8n - 4

Recommendations

Teachers should provide students with greater exposure to activities which require them to

- translate verbal statements into algebraic expressions
- determine the n^{th} term for a sequence and to write expressions in terms of a given variable.

Optional Section

<u>Question 9</u>

This question tested candidates' ability to

- represent inverse variation symbolically and to solve problems involving inverse variation
- interpret the graph of a quadratic function to determine the roots and identify the minimum value of the function
- express a quadratic function in the form $a(x + h)^2 + k$
- draw graphs of linear functions
- solve problems involving graphs of linear and non-linear functions.

The question was attempted by 88.4 per cent of the candidates, 1.21 per cent of whom earned the maximum available mark. The mean mark was 1.82 out of 15. This was the most popular of the optional questions.

In Part (a) (i), candidates were able to express *y* in term of *x* and *k*. However there were quite a few cases where candidates used the proportionality symbol ($\propto$) instead of the equal sign (=). For Part (a) (ii), candidates were also able to find a value for the constant *k* using their expression from Part (a) (i), and followed through to find the values of *a* and *b*.

In Part b (i), candidates were able to use the graph to solve the equation; however, too many candidates resorted to solving the equation by factorizing. Others completed a table of values but did not indicate that the required solutions were at the zero values in the table.

In Part (b) (ii), some candidates were able to get the coordinates of the minimum point. In most cases, candidates who earned only one mark got that mark by answering this part of the question correctly.

For Part (b) (iii), some candidates were also able to use the coordinates of the minimum point to obtain the expression in the form $a(x+h)^2 + k$. Instead of simply using the coordinates of the minimum point to write the expression in the form $a(x + h)^2 + k$, quite a number of candidates went about solving the equation the long way and made mistakes.

Part (b) (iv) also saw candidates scoring marks for having a line with a positive gradient, with the line passing through at least one point on the correct line. However, there were too many incorrect answers to this part of the question, as many candidates were not able to demonstrate the simple skill required. Since many of them did not draw the line, they used factorization to solve the equation.

Part (b) (v) was poorly done, as in most cases candidates got marks for correctly factorizing but not for interpreting the intersection of the graphs correctly. Also, even though candidates were able to obtain marks for Part b (i), some candidates were unable to successfully factorize the equation when they chose that option.

Solutions

(a) (i)	$xy = k$	(ii)	6	(iii)	$a = 5, b = 0.3$		
(b) (i)	$x = 2, x = 4$	(ii)	$(3, -1)$	(iii)	$(x - 3)^2 - 1$	(v)	$x = 2, x = 5$

Recommendations

- Students should be taught the difference between the symbols and when to use them. Too many students used the proportionality symbol ($\propto$) in the place of the equal sign (=) in Part (a) (i).
- Teachers should also place emphasis on the need to follow instructions. Too many students used factorization to solve the quadratic equation when the question clearly stated that the graph was to be used.
- All the methods of getting a quadratic function in the form $a(x + h)^2 + k$ should be taught so students are able to choose the *correct* one when the time comes. Too many candidates use the ‘old long way’ when they could have simply used the coordinates of the minimum point to get the answer quite easily.
- There needs to be greater focus on the factorization of quadratic expressions. Getting to $(x - 4)(x - 2) = 0$ or $(x - 5)(x - 2) = 0$ and not knowing how to get the answers is an indication that there is still a skill lacking.
- All candidates at this level should be able to plot the graph of a linear function. Too many drew lines parallel to either the *x* or *y* axes. This indicates that understanding of a positive or negative gradient was lacking. Teachers should teach students to identify this.
- Students’ performance will be greatly improved if more emphasis is placed on teaching them basic skills. Students chose the long way out when answers could have been obtained much more easily, hence giving them more time to do other questions on the exam.

Question 10

This question tested candidates' ability to

- use circle geometry theorems to determine unknown angles giving appropriate reasons
- use the concept of bearings to
  - place a bearing angle on a given diagram
  - place given distances on a given diagram
- calculate the length of an unknown side given two sides and an angle
- use the cosine formula/sine rule to determine an unknown angle when all three sides (and an angle) are known
- determine the bearing of one location from another.

The question was attempted by 76 per cent of the candidates, 0.5 per cent of whom earned the maximum available mark. The mean mark was 1.66 out of 15. The question was poorly done. It was not attempted by many candidates and those who attempted it could not satisfactorily respond to Part (a) and (b).

Far too many candidates obtained a score that was less than 4 out of 15 and very few candidates obtained a score that was greater than 10 out of 15.

Those candidates who attempted Part (a) were able to state the value of H$\widehat{K}$L and calculate the values of J$\hat{O}$K and J$\widehat{\mathrm{H}}$K. Some candidates were unable to satisfactory give reasons for their answers. Alternate angles or corresponding angles were given as reasons for H$\widehat{K}$L being 20^o instead of angles at the circumference in the same segment subtended by the same chord (arc) and angles in a triangle as a reason for the other two angles, even though at least one of these 'angles in a triangle' was incorrect. Most candidates recognized that JOK was an isosceles triangle.

In Part (b) (i), many candidates were able to correctly place the bearing angle and the lengths of the two sides on the diagram and knew that the cosine rule should be used to find the unknown side BC. They were generally able to correctly indicate the bearing 030^0 on the diagram. However, some candidates placed the bearing angle at B$\hat{A}$C. Too often also, 310 km was placed on the line BC rather than on AC.

Even though a majority of the candidates knew that the application of the cosine formula was the appropriate formula to be used to find BC, a number of them subtracted the product $2\times310\times90$ from 310^2+90^2 and then multiplied by cos 60^o. As a result, (104200 – 55800) cos 60^0 = 48400 cos 60 = 24200 was seen regularly.

Candidates had serious problems with Parts (b) (iii) and (iv). These parts were either omitted or the basic definition of a trigonometric function such as sine, cosine or tangent was used to calculate angle ABC. Those who managed to calculate angle ABC could not use this value to determine the required bearing.

Solutions

(a) (i)	20^0	(ii)	80^0	(iii)	40^0
(b) (ii)	276 km	(iii)	104^0	(iv)	106^0

Recommendations

- The rules of circle theorems need to be clearly understood so that they can be applied appropriately. Teachers should ensure that adequate time is spent on the theorems.
- Students should be given sufficient practice in identifying and calculating the angles associated with these theorems, using a variety of diagrams.
- Students should be required to use the correct mathematical language at all times to explain the reasoning for their calculations. Responses such as ‘bow-tie angles’ should not be given as a valid reason why angles subtended at the circumference by the same arc are equal. Vague answers such as “it looks like this”, showing a figure that resembles M, should also be avoided. Neither is opposite angles are equal. These responses were both given for Part (a) (i).
- In addition, teachers need to explain to students the difference between alternate angles and angles in the alternate segment.
- Teachers need to spend more time dealing with questions on bearings and emphasis needs to be placed on the fact that a bearing angle is always measured clockwise with respect to the north line.
- Students should also be given more real world problems dealing with bearings and the calculations involved in solving these problems.
- Trigonometric ratios, the sine and cosine rule are taught at different form levels. It is therefore important that students, in preparing for the CSEC examinations, practise questions which give them the opportunity to decide the appropriate method to be used. In addition, attention must be paid to the correct steps in the calculation after substitution in the cosine rule.

<u>Question 11</u>

This question tested candidates’ ability to

- describe a transformation given an object and its image
- determine and represent the location of
  - the image of an object
  - an object, given the image under a transformation
- describe a transformation, given an object and its image
- associate a position vector OP with a given point P $\begin{pmatrix} a \\ b \end{pmatrix}$ where O is the origin (0, 0)
- determine the magnitude of a vector
- use vectors to represent and solve problems in geometry.

The question was attempted by 36 per cent of the candidates, 0.34 per cent of whom earned the maximum available mark. The mean mark was 3.56 out of 15. The question was poorly done and many candidates did not attempt it. Of those who attempted it, most scored zero, two or three marks. Two candidates scored full marks.

In Part (a) (i), many candidates were able to state correctly the values of c and d in the 2×2 matrix. However, most of these candidates did not use algebra to determine the values of c and d since they were able to recognize that it is a reflection in the x-axis, which simply means a change of sign for the y coordinate.

A number of candidates, however, had 2 and -3 as the values of c and d respectively. Another popular answer was c = 1 and d = 1. It seemed that some candidates did not know how to go about solving for c and d. Candidates gave answers such as $\begin{pmatrix} 2 & 3 \\ 2 & -3 \end{pmatrix}$.

In Part (a) (ii), many candidates were able to correctly write down the image of (-5, 4) under the transformation, T. Nevertheless, a significant number had problems doing so. Many of them omitted this part of the question. Some of them could not multiply a 2×1 vector by a 2×2 matrix and as a result the product was often $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} -5 \\ 4 \end{pmatrix} = \begin{pmatrix} -5 & 0 \\ 0 & -4 \end{pmatrix}$.

Many candidates who attempted Part (a) (iii) had problems recognizing and describing the reflection vector. Many of them confused reflection in the x-axis with reflection in the y-axis. Some candidates knew the reflection matrices but could not identify the one to use in this situation. Many candidates omitted this part of the question.

Candidates had difficulty with Part (a) (iv). They were able to see that it was a reflection in the x-axis and hence would require the use of the matrix for a reflection in the x-axis, as they had previously used. Many candidates omitted this part of the question.

Part (b) (i) was perhaps the best done of the entire question. Candidates were able to correctly write the pair of coordinates in vector form. Only a few of them stated the point as coordinates, which showed their inability to distinguish between vectors and coordinates. A number of candidates were able to find the correct value for vector $\overrightarrow{QR}$. Some candidates presumably, used the graph to get the answer, $\begin{pmatrix} 3 \\ 5 \end{pmatrix}$.

There were candidates who had difficulty finding the value of vector $\overrightarrow{QR}$. They were simply saying $\overrightarrow{QR} = \overrightarrow{OQ} + \overrightarrow{OR}$ = $\begin{pmatrix} 0 \\ 2 \end{pmatrix} + \begin{pmatrix} 3 \\ 7 \end{pmatrix} = \begin{pmatrix} 3 \\ 9 \end{pmatrix}$. Some candidates wrote vector $\overrightarrow{OQ}$ = $\begin{pmatrix} 2 \\ 0 \end{pmatrix}$. Many candidates did not realize that vector $\overrightarrow{QR} = -\overrightarrow{OQ} + \overrightarrow{OR}$ or $\overrightarrow{QR} = \overrightarrow{OR} - \overrightarrow{OQ}$.

In finding the magnitude of vector $\overrightarrow{QR}$ in Part (b) (ii), some candidates understood that they needed to use Pythagoras' theorem but were unable to do so correctly. Some found the difference of the squares rather than the sum of the squares. There were also those who found the sum of the squares and did not apply the square root.

Most candidates who attempted Part (b) (iii) showed an understanding of the magnitude of a vector and were able to calculate it correctly, even when their value of vector QR was wrong. However, some candidates had difficulty with this part of the question. It seemed they did not know how to represent a vector diagram. They simply drew a straight line without using the arrow to show direction. Some of them, in drawing the vector $\overrightarrow{OS}$ = $\binom{7}{4}$, drew a line connecting the points $\binom{7}{0}$ and $\binom{0}{4}$.

Candidates generally did not know how to prove that a shape is a parallelogram. A few of them recognized that once two sides have equal vectors and magnitude, then the shape is a parallelogram At least one candidate used the concept of equal midpoints to show that the shape is a parallelogram. This was the least attempted part of the question.

Solutions

(a) (i) c = 1, d = -1 (ii) (-5, -4) (iii) reflection in x-axis

(iv) $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

(b) (i) $\overrightarrow{OP} = \begin{pmatrix} 4 \\ -1 \end{pmatrix}$, $\overrightarrow{QR} = \begin{pmatrix} 3 \\ 5 \end{pmatrix}$ (ii) 5.83

(iii) $\overrightarrow{QR} = \overrightarrow{PS} = \begin{pmatrix} 3 \\ 5 \end{pmatrix}$ implies $\overrightarrow{QR}$ and $\overrightarrow{PS}$ are parallel (k=1).

Recommendations

- Teachers are encouraged to spend much more time on matrices and vectors, thus equipping students to perform well on such questions.
- Teachers should ensure that students can represent vector diagrams and are encouraged to spend more time teaching vector algebra. Technology may be used to enhance such lessons.
- Teachers are encouraged to spend more time dealing with the transformation matrices for reflections, rotations about the origin (0, 0) and enlargements centre (0, 0). They should use the correct terms when describing these transformations, ensuring that students understand how these transformations affect the object being transformed.
- Teachers should ensure that students know the features of certain shapes, such as trapeziums and parallelograms, and can use vectors to identify them.